Latent Heat Simulation Model of the Generation of Clouds over an Isolated Mountain

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Abstract

A simulation model with the latent heat to observe the formation of clouds by the wind blowing over a mountain is proposed in this study. The flow is assumed to stably stratifying. The incompressible Navier-Stokes equations together with the advection equation of density are chosen for the basic equations. These equations are solved numerically by the standard MAC method. Third order upwind difference is employed for the approximation of the nonlinear terms of the Navier-Stokes equation. Particles of finite number are used to express the clouds. At first, each particle has its own temperature and humidity and is moved according to the velocity of the flow field. The latent heat is integrated into the equation through the position of particle. When the humidity becomes 100%, the particle is assumed to become a small cloud and is displayed. The effect of the latent heat is examined and compared to the result of the simulation without the latent heat [1].

1 Introduction

The atmosphere is greatly affected by the temperature distribution. Even if the atmosphere is stable (i.e. the density decrease with height), we can observe various interesting phenomena occurring in the flow around an obstacle such as a mountain [1]. In this case, vertical motion tends to carry heavier fluid upwards and lighter fluid downwards, and is suppressed. This suppression may take the form of modifying the pattern of the laminar motion or of preventing or modifying its instability. This pattern forms the lee wave downstream of the mountain and complicated flow is formed around the mountain.

The latent heat caused by solidification and evaporation affects sensitively on the formation of clouds around, especially at the top of, a mountain. The crest cloud formed under the critical condition at the top of a mountain is observed as a characteristic cloud.

Although we can find various numerical studies for stable stratified flow around an obstacle [2][3][4], there are few researches concerned with the generation of clouds by such flow. In this study, we compute the stably stratified flow around an isolated mountain under the effect of the latent heat and try to simulate the generation of the typical clouds around a mountain. Comparison to previous simulation of clouds without the effect of the latent heat [5] is carried out in this paper.

2 Numerical Method

2.1 Basic equation

In this study, we take z-axis vertically upwards and suppose that the basic stratification consists of a uniform density gradient $(d\rho_S(z)/dz = -1)$. The pressure $p'(= p - p_S(x))$ and the density $\rho'(= \rho - \rho_S(z))$ represent the modification ones, i.e. the difference from the base pressure $p_S(z)$ and base density $\rho_S(z)$. The basic equations for u, v, w, ρ' and p' in non-dimensional form become as follows [3]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{\partial p'}{\partial x}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{\partial p'}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right) \tag{3}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{\partial p'}{\partial z}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)-\frac{\rho'}{Fr^2} \tag{4}$$

$$\frac{\partial \rho'}{\partial t}+u\frac{\partial \rho'}{\partial x}+v\frac{\partial \rho'}{\partial y}+w\frac{\partial \rho'}{\partial z}=w \tag{5}$$

Equation (1) is the continuity equation, equations (2)-(4) are momentum equations of x, y, and z directions respectively and equation (5) is the advection equation of density which represents the condition of incompressibility.

2.2 Representation of clouds

In order to represent clouds, we consider small virtual particles of the air. Each particle has its own temperature and humidity and is moved by the flow at its position. These particles are put into flow field at various positions on the upstream boundary. If the temperature of the particle becomes smaller than its dew point, the particle is considered to become a cloud and is displayed as a cloud. The dew point of the particle is computed beforehand from its initial temperature and humidity by the empirical relation of Tetens :

$$e = 6.11\times 10^{7.5T/(237.3+T)}\times\frac{h}{100} \tag{6}$$

where e is vapor pressure, T is temperature and h is humidity.

Dew point (T_d) is defined as the temperature at which the humidity becomes 100%. Therefore, (T_d) is obtained by substituting equation (7) into h=100, solving it in terms of T and setting $T = T_d$. Strictly speaking, the equation is obtained for water and it is not accurate for ice. However, we simply assume the water is still liquid under the freezing point. We also assume that the cloud disappears when the temperature exceeds drew point.

In general, the position of the particle does not coincide with the grid points of the calculation. We use the following formula of interpolation to compute the velocity at the position as shown in Fig.1.

$$v' = \frac{\sum_{i=1}^{8} v_i / r_i}{\sum_{i=1}^{8} 1 / r_i} \tag{7}$$

The latent heat is generated at the time of solidification and has the value of

$$2.83 \times 10^6 J \cdot kg^{-1} \tag{8}$$

The mass of vapor involved in unit volume is obtained by saturation vapor curve as follows:

$$4.9784 e^{0.0614 T_d} \quad [g] \tag{9}$$

where T_d is dew point. From equations (8) and (9), the latent heat generated in unit volume is

$$2.83 \times 10^6 (J \cdot kg^{-1}) \times 4.9784 e^{0.0614 T_d} \times 10^{-3} (kg) \quad [J] \tag{10}$$

The energy required to raise the vapor of unit volume to one degree is

$$0.24 (kcal / kg \cdot {}^o c) \times 1.2 (kg / m^3) \times 10^3 / 0.239 \quad [J] \tag{11}$$

Increase in temperature by this energy is obtained from equations (10) and (11):

$$equation(10) \div equation(11)$$
$$= 11.6918 e^{0.0614 T_d} \quad [{}^o C] \tag{12}$$

This increase acts as the buoyancy force to the flow. Numeric constant is multiplied to equation (12) to convert the temperature to buoyancy force and the force is added to the right hand side of equation (4).

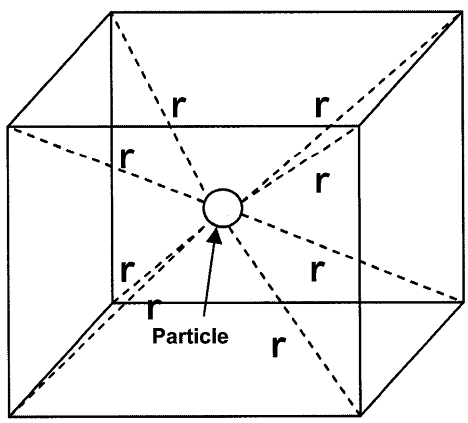


Fig.1 Interpolation

For the realistic simulation, it is desired to use huge number of particles. However, the computation time becomes large at the same time. In order to save the computation time, the function F that represents the cloud is introduced as follows:

$$F(x, y, z) = \sum_{i=1}^{N} \exp\left(-A d_i^2\right) \tag{13}$$

where N is the number of particles, d_i is the distance between the point (x, y, z) and i-th particle and A is numeric constant.

Taking the fact into account that the value of the function become large when the distance from the particle becomes small and where many particles exist its vicinity, this formula is considered to be reasonable. Note that the function F is used only for the visualization of the cloud and does not affect the calculation of the flow field.

2.3 Relation between Froude number and lapse rate

The Froude number (Fr) and the Reynolds number (Re) determine the flow. On the other hand, the initial temperature of the particle is determined by using the lapse rate (Γ) according to their height. In the computation, it is possible to change Fr, Re and Γ independently. However, in the real situation, since both Fr and Γ affect the stability of the atmosphere, there is a relation between them.

The Froude number is defined by

$$Fr = \frac{U}{NH} \tag{14}$$

where U is velocity of uniform wind, H is length scale (height of mountain) and N is the Brunt-Vaisala frequency. On the other hand, as

$$\frac{\partial T}{\partial x} = -\Gamma \quad \text{and} \quad N^2 = \frac{g}{T}\left(\frac{dT}{dx} + \frac{g}{C_F}\right) \tag{15}$$

we can write

$$Fr = \frac{U}{H\sqrt{(g/T)((g/C_F)-\Gamma)}} \tag{16}$$

If the lapse rate is equal to the gradient of adiabatic, the Froude number becomes infinity. Therefore,

$$Fr = B\frac{U}{\sqrt{9.76-\Gamma}} \tag{17}$$

Parameter B depends on the lapse rate and the gradient of adiabatic. In this study, we choose B=0.24 after performing several test calculations.

2.4 Numerical Method

The basic equation is solved by the standard MAC method [7]. Since the Reynolds number is high, the advection terms in the momentum and the density equations are approximated by the third order upwind scheme [6]. The rest of spatial terms are approximated by the second order central difference. Euler explicit method is employed for the time integration.

The overall computational domain is rectangular region of $80km \times 40km \times 12km$ in windward, span-wise and vertical directions respectively as shown in Fig.2.

Fig.2 Computational domain

We assume Mt. Fuji as an isolated mountain; i.e. the shape of the mountain is a cone and its height is 4000m. Since we are interested in investigating the effect of an isolated mountain on the formation of clouds, the mountains surrounding Mt. Fuji are removed. The particles are put on the inflow boundary of x=0km, 10km<y<30km, 0km<z<7km at space intervals of 0.5km. We repeat this procedure in the computations at intervals of unit time.

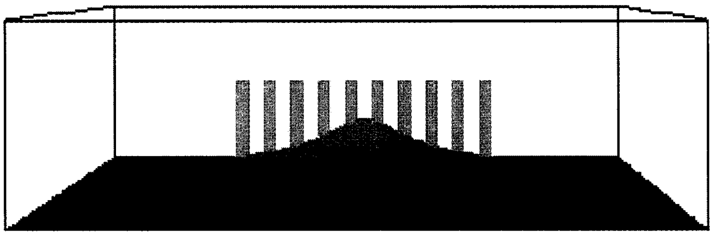


Fig.3 Initial position of particles putting into the domain

Fig.3 is the view from downstream boundary indicating the position of initial particles. Since computation of clouds is time consuming, we cannot use enough grids points, i.e. the total grid points are 61x31x31 in stream-wise, span-wise and vertical direction respectively. In order to perform the computation as accurate as possible, we employ boundary fitted coordinate system. Also the grid is concentrated near the ground and the mountain.

Fig.4 is a grid system in the vertical plane passing the mountain. Initially, the flow is at rest except on the inflow boundary.

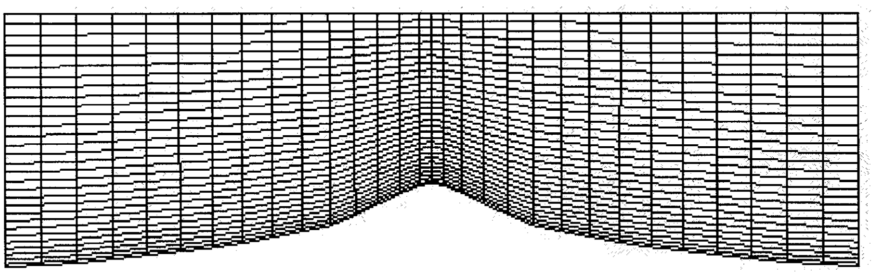


Fig.4 Grid in y-z plane across a mountain

Boundary conditions and the parameters used in this study are summarized in Table 1 and 2.

Table 1 Boundary conditions

	Velocity	Pressure	Density
Upwind	Uniform flow $u = 1, v = w = 0$	Extrapolated $\frac{\partial p'}{\partial x} = 0$	Fixed $\rho' = 0$
Downstream	Extrapolated $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = \frac{\partial w}{\partial z} = 0$	Extrapolated $\frac{\partial p'}{\partial x} = 0$	Extrapolated $\frac{\partial \rho'}{\partial x} = 0$
Top	Slip condition $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = 0, w = 0$	Extrapolated $\frac{\partial p'}{\partial z} = 0$	Extrapolated $\frac{\partial \rho'}{\partial z} = 0$
Bottom	Non-slip condition $u = v = w = 0$	Extrapolated $\frac{\partial p'}{\partial z} = 0$	Extrapolated $\frac{\partial \rho'}{\partial z} = 0$
Lateral	Extrapolated $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = \frac{\partial w}{\partial z} = 0$	Extrapolated $\frac{\partial p'}{\partial x} = 0$	Extrapolated $\frac{\partial \rho'}{\partial x} = 0$

Table 2 Parameters used in this study

Parameter	Value
Grid number	$61(x) \times 31(y) \times 31(z)$
Time step	20000
ΔT	Depends on wind speed (0.015, 0.01, 0.075, 0.005)
Iteration of Poisson equation	20
Error of Poisson equation	0.001
Re	2000
Fr	0.5-2.0
Particle number	$10(x) \times 15(z)$
Humidity	10%-70%
Lapse rate	5℃/km-8℃/km
Temperature on the ground	15℃

3 Results and Discussion

We will show typical results obtained by the method mentioned above. At first, we show the flow around an isolated mountain at $Fr = 1.73$ in order to presents the overview of the results of calculation with the latent heat model.

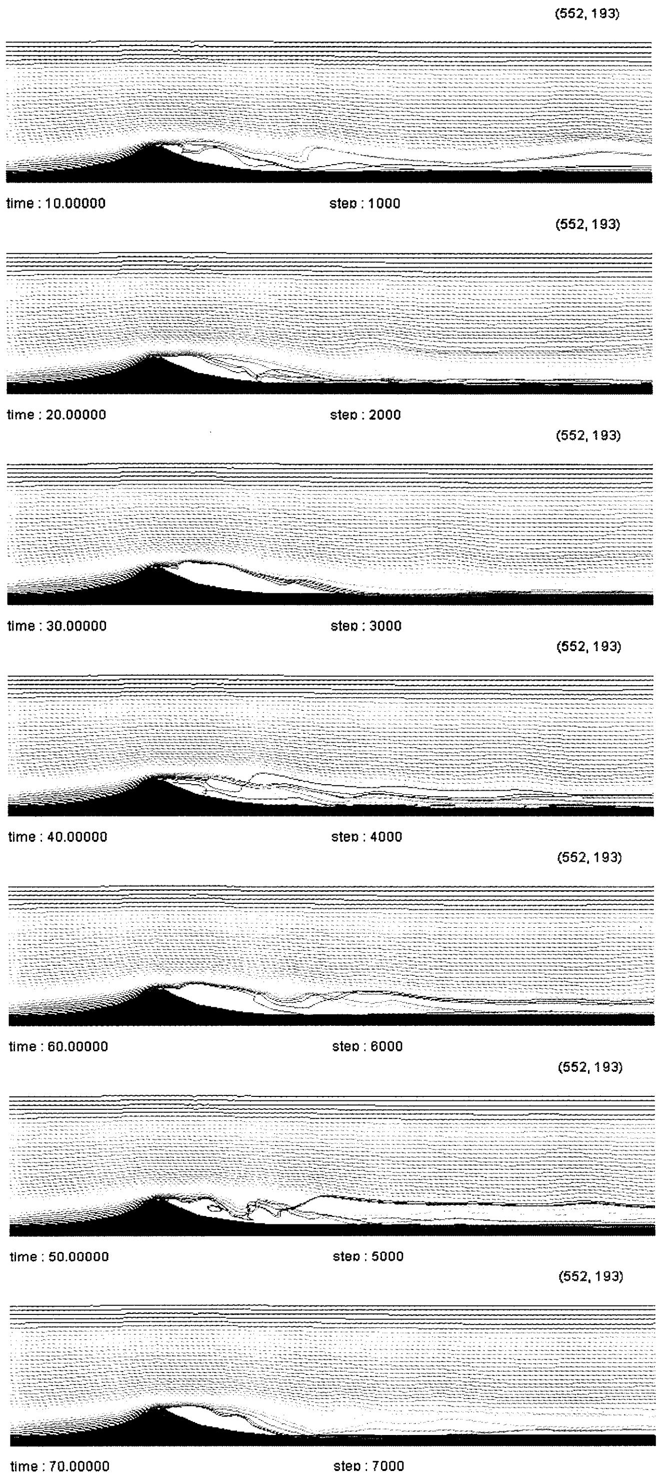
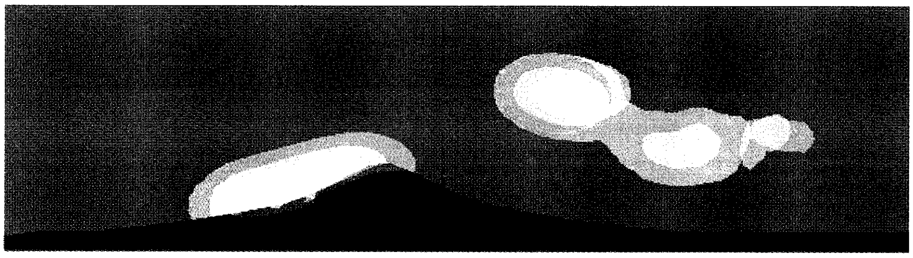
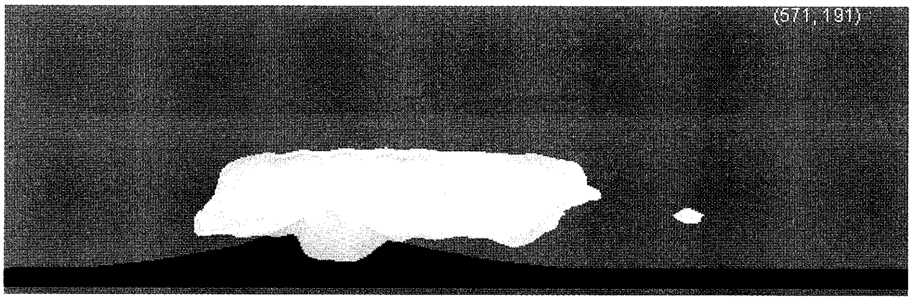


Fig.5 Stream line in x-z plane across a mountain at various times.

Fig.5 shows the flow in the vertical cross section passing the center of the mountain. Various particle paths started at the inflow boundary are shown. Each particle path goes over the mountain and is disturbed strongly showing the flow is not very stable.



(a) Without the latent heat



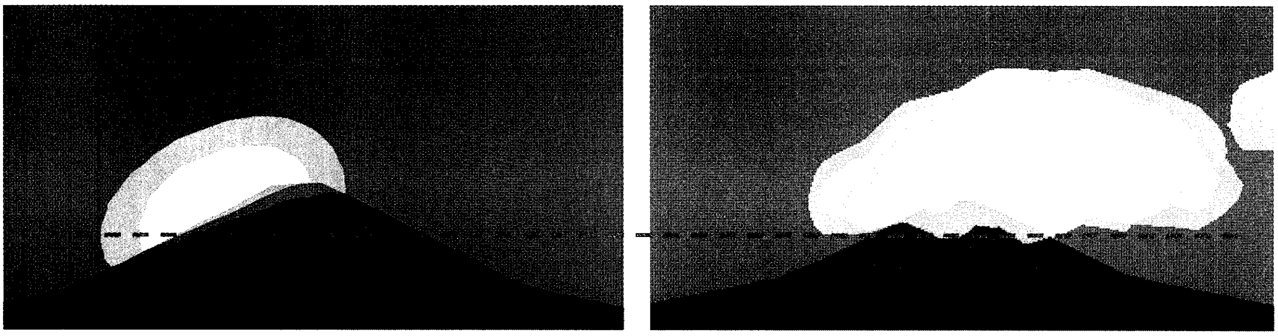
(b) With the latent heat

Fig.6 Formation of clouds

(humidity 50%, lapse rate 8℃/km, wind speed 10m/s, Fr=1.73)

Fig.6 shows the formation of clouds at the humidity is 50%, the lapse rate is 8℃/km, and the wind speed is 10m/s (Fr=1.73) with and without the latent heat. Fig.6 (a), the case without the latent heat [5], shows large cloud that cannot go over the mountain and stays with bump shape at the foot of the mountain. On the other hand, Fig.6 (b), the case with the latent heat, shows the crest cloud formed around the peak of the mountain. This crest cloud that covers uniformly in front of and behind the mountain is formed due to the increase of the temperature caused by solidification of vapor.

Fig.7 shows the clouds generated near the peak of the mountain at the humidity is 30%, the lapse rate is 8℃/km, and the wind speed is 10m/s (Fr=1.20) with and without the latent heat. Fig.7 (a), the case without the latent heat [5], shows the cloud starting to be generated at the foot of the mountain. On the other hand, Fig.7 (b), the case considering the latent heat, shows the cloud being higher than the middle height of the mountain. The bottom of clouds in each case is compared in Fig.7. Red line drawn at the bottom of cloud in Fig.7 (b) clearly shows the differences of height between (a) and (b). This indicates that the cloud is rose up by the latent heat and this critically affects on the formation of the cloud around the mountain.

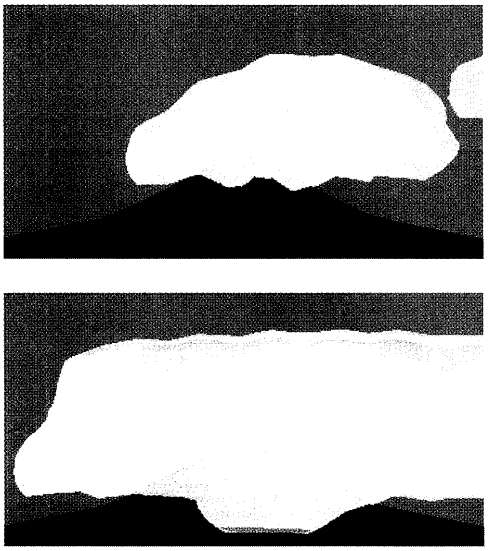


(a) Without the latent heat

(b) With the latent heat

Fig.7 The crest cloud at the peak of the mountain

(humidity 30%, lapse rate 8℃/km, wind speed 10m/s, Fr=1.20)



(a) Humidity is 30%

(b) Humidity is 50%

Fig.8 The effect of humidity on the formation of clouds

Fig.8 shows the effect of humidity on the shape of the clouds. The Froude number and the Reynolds number are fixed (Fr=1.73, Re=2000). Fig.8 (a) and (b) are the results of humidity=30% and 50% respectively. As is expected, the clouds become larger as the humidity increases.

The crest cloud is the cloud lightly covers the top of the mountain as presented in Fig.8 (a). In Fig.8 (b), the case of humidity is 50%, the cloud become large and is different from the crest cloud. As is shown in the results, the suitable condition to form the crest cloud is around 30% of humidity.

4 Summary

In this study, we proposed a simple model with the latent heat to simulate the generation of clouds and formation of the typical clouds above a mountain numerically. The numerical simulation with the latent heat is compared with the result of the same numerical model without the latent heat [5]. Some results show that the latent heat behaves critical role in the simulation. Due to the effect of the latent heat, the crest cloud over a mountain is observed as a result of simulation of the simple numerical model with the latent heat. The effect of humidity is also investigated.

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