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Measurable Norms and Related Conditions in Some Examples

Keiko Harai
Graduate School of Humanities and Sciences, Ochanomizu University,
Tokyo 112-8610, Japan
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Abstract

This short note is the summary of my Master's thesis. We define some norms and cylindrical measures, and investigate the relation between them and seven conditions. The detail will be appear elsewhere.

1 Introduction

The measure theory on infinite-dimensional spaces was established as a certain field of mathematics by Prokhorov, Sazonov and Minlos. In 1962, Gross([5]) introduced the notion of measurable norms. This gives a condition satisfying that the Gauss cylindrical measure extends to a measure. In 1971, Dudley-Feldman-LeCam([3]) defined another notion of measurable norms. If the norm is continuous, then this is a necessary and sufficient condition satisfying that a general cylindrical measure extends to a measure. These two notions are very close, but subtly different. Badrikian-Chevet([1]) presented a Conjecture that the above are equivalent for all cylindrical measures. This was denied by Maeda([9]) in 1984. Also Maeda([10]) showed that these two measurabilities coincide with each other for every generalized rotationally quasi-invariant cylindrical measure. On the other hand there are some conditions around these two notions of measurable norms, and we research into them. In this paper, we study the relation among seven conditions for norms and cylindrical measures made in some examples.

2 Preliminaries

Throughout this paper, let X be a Banach space, X' its topological dual, $(\cdot,\cdot)$ the natural pairing between X' and X, and $\mathscr{B}(X)$ Borel $\sigma-$algebra of X. Let H be a real Hilbert space, $<\cdot,\cdot>$ the inner product on H, $FD(H)$ the family of finite-dimensional subspaces of H, $\mathscr{F}$ the partially ordered set of finite-dimensional orthogonal projections of H and I the identity map.
Let $\{\xi_1,\xi_2,\ldots,\xi_n\}$ be a finite system of elements of X'. Then by Ξ we denote the operator from X into $\mathbb{R}$ mapping x onto the vector $((\xi_1,x),\ldots,(\xi_n,x))$. A set $Z\subset X$ is said to be a *cylindrical set* if there are $\xi_1,\xi_2,\ldots,\xi_n\in X'$ and $B\in\mathscr{B}(\mathbb{R}^n)$ such that $Z=\Xi^{-1}(B)$. Let $\mathscr{R}_{\xi_1,\xi_2,\ldots,\xi_n}$ denote the *σ-algebra* of all cylindrical sets by a fixed $\{\xi_1,\xi_2,\ldots,\xi_n\}$ and $\mathscr{R}$ the *algebra* $\cup\mathscr{R}_{\xi_1,\xi_2,\ldots,\xi_n}$.
We can also denote a cylindrical set on Hilbert space H by $Z=\{x\in H; Px\in F\}\,(P\in\mathscr{F}, F\in\mathscr{B}(PH))$.

Definition 2.1 *A set function μ is called a cylindrical measure if it satisfies the two following conditions:*
(i) $\mu:\mathscr{R}\to[0,1]$
(ii) The restriction to $\mathscr{R}_{\xi_1,\xi_2,\ldots,\xi_n}$ of μ is a probability measure.

Definition 2.2 *The canonical Gauss cylindrical measure on H is the cylindrical measure $\gamma : \mathscr{R} \to [0,1]$ defined as follows :*
If $Z = \{x \in H; Px \in F\}$, then

$$\gamma(Z) = (\frac{1}{\sqrt{2\pi}})^n \int_F e^{-\frac{|x|^2}{2}} dx \ ,$$

where $n = \dim PH$ and dx is the Lebesgue measure on PH.

Remark 1 *If H is an infinite-dimensional space, then γ is finitely additive, but is not $\sigma-$additive. In general, we denote by $\gamma_t(Z) = (\frac{1}{\sqrt{2\pi t}})^n \int_F e^{-\frac{|x|^2}{2t}} dx$ the Gauss cylindrical measure with parameter t. γ_1 is the caonical Gauss cylindrical measure. In this paper, we denote the canonical Gauss cynlindrical measure by γ.*

We define rotationally invariant and rotationally quasi-invariant cylindrical measures.

Definition 2.3 *Let μ be a cylindrical measure on H. Then μ is called a rotationally invariant cylindrical measure if $\mu(C) = \mu(u(C))$ whenever C is a cylindrical set of H and u is a unitary operator of H.*

Definition 2.4 *Let μ be a cylindrical measure on H. Then μ is called a rotationally quasi-invariant cylindrical measure if $\mu \sim_c u(\mu)$ whenever u is a unitary operator of H. "$\sim_c$" means to be cylindrically equivarent and $u(\mu)$ is the image of μ under u.*

Now we present the definitions of measurable norms which are main in this paper. They are introduced by Gross and D.F.L. , i.e. Dudley, Feldman and LeCam.

Definition 2.5 *Let $\|\cdot\|$ be a norm defined on H, then $\|\cdot\|$ is called a μ-measurable($Gross$) norm if for every $\varepsilon > 0$, there exists $P_0 \in \mathscr{F}$ such that $\mu(\{x \in H; \|Px\| > \varepsilon\}) < \varepsilon$ whenever $P \in \mathscr{F}$ and $P \perp P_0$.*

In other words, $\|\cdot\|$ is μ-measurable($Gross$) if for every $\varepsilon > 0$, there exists $G \in FD(H)$ such that $\mu(\{N_\varepsilon \cap F + F^\perp\}) \geq 1 - \varepsilon$ whenever $F \in FD(H)$ and $F \perp G$, where $N_\varepsilon = \{x \in H; \|x\| \leq \varepsilon\}$ and $F^\perp$ is the orthogonal complement of F.

Definition 2.6 *(Abstract Wiener space) Let γ be the canonical Gauss cylindrical measure on H,B be the Banach space which is the completion of H with respect to $\|\cdot\|$ that is γ-measurable($Gross$) and i the inclusion map of H into B. Then the triple (i, H, B) is called an abstract Wiener space.*

After Gross defined a measurable norm, Dudley-Feldman-LeCam defined another measurable norm.

Definition 2.7 *$\|\cdot\|$ is called a μ-measurable(D.F.L.) norm if for every $\varepsilon > 0$, there exists $G \in FD(H)$ such that $\mu(\{x \in H; \|x - F^\perp\| < \varepsilon\}) \geq 1 - \varepsilon$ whenever $F \in FD(H)$ and $F \perp G$.*

In other wards, $\|\cdot\|$ is μ-measurable($D.F.L.$) if for every $\varepsilon > 0$, there exists $G \in FD(H)$ such that $\mu(\{P_F(N_\varepsilon) + F^\perp\}) \geq 1 - \varepsilon$ whenever $F \in FD(H)$ and $F \perp G$, where P_F is the orthogonal projection of H onto F.
Therefore we know that the condition of measurable($Gross$) norms is stronger than that of ($D.F.L.$).

3 Measurable norms and related conditions

In this section, we introduce several conditions approximating to measurable norms.

Theorem 3.1 *Let H be a real Hilbert space, μ be a cylindrical measure on H, $\|\cdot\|$ be a continuous norm defined on H and B be the completion of H with respect to $\|\cdot\|$. Moreover, let Y be the bidual B'' of B with weak*-topology $\sigma(B'', B')$ and j be the inclusion map from H into Y. Then the seven conditions satisfy the following relations:*
$(i) \Rightarrow (ii) \Rightarrow (iii) \Rightarrow (iv)$, $(i) \Rightarrow (ii) \Rightarrow (v) \Leftrightarrow (vi) \Rightarrow (vii)$
If μ is continuous (this means that the charateristic function of μ is continuous on H), then the following conditions satisfy the relations:
$(iii) \Rightarrow (vi)$ *and* $(iv) \Rightarrow (vii)$

(i) *For any $\varepsilon > 0$ there exists $N \in \mathbb{N}$, where $\mathbb{N}$ is the set of all natural numbers, such that $n > m \geq N$ implies*

$$\mu(\{x \in H; \|P_n x - P_m x\| > \varepsilon\}) < \varepsilon$$

for every sequence $\{P_n\} \subset \mathscr{F}$ such that P_n converges strongly to the identity map I, write it $P_n \nearrow I$.
(ii) $\|\cdot\|$ *is a $\mu-$measurable(Gross) norm.*
(iii) *There exists a sequence $\{P_n\} \subset \mathscr{F}$ such that $P_n \nearrow I$, which has the property that for any $\varepsilon > 0$ there exists $N \in \mathbb{N}$ such that $n > m \geq N$ implies*

$$\mu(\{x \in H; \|P_n x - P_m x\| > \varepsilon\}) < \varepsilon.$$

(iv) *There exists a sequence $\{P_n\} \subset \mathscr{F}$ such that $P_n \nearrow I$, which has the property that for any $\varepsilon > 0$ there exist $N_\varepsilon \in \mathbb{N}$ and $n_\varepsilon \in \mathbb{N}$ such that $N \geq N_\varepsilon$ and $n \geq n_\varepsilon$ implies*

$$\mu(\{x \in H; \sup_{1 \leq k \leq n} \|P_k x\| > N\}) < \varepsilon.$$

(v) $\|\cdot\|$ *is a $\mu-$measurable(D.F.L.) norm.*
(vi) $i(\mu)$ *(i.e. $\mu \circ i^{-1}$) is extensible to a measure.*
(vii) $j(\mu)$ *is extensible to a measure.*

Proof. First we proof that (i) implies (ii). Suppose not. Then there exists $\varepsilon_0 > 0$ such that for all $P \in \mathscr{F}$, there exists Q perpendicular to P such that

$$\mu(\{x \in H; \|Qx\| > \varepsilon_0\}) \geq \varepsilon_0 \, , \, Q \in \mathscr{F}.$$

Define a sequence P_n which strongly converges to I inductively. Let $\{e_i\}_{i=1,2,...}$ be an orthonomal basis in H. Let $P_0 = 0$. Suppose we have $P_0, P_1, \ldots, P_{2n}$. Then there exists Q_n perpendicular to P_{2n} such that $\mu(\{x \in H; \|Q_n x\| > \varepsilon_0\}) \geq \varepsilon_0$ and $Q_n \in \mathscr{F}$. Let $P_{2n+1} = P_{2n} + Q_n$ and $P_{2n+2} =$ orthogonal projection onto lin sp $\{P_{2n+1}H, e_{n+1}\}$. Then P_n strongly converges to I, so there exists $N > 0$ such that $n > m \geq N$ implies $\mu(\{x \in H; \|P_n x - P_m x\| > \varepsilon_0\}) < \varepsilon_0$. Choose $n = 2k+1$, $m = 2k$ for some $k > \frac{N}{2}$. Then $P_n - P_m = P_{2k+1} - P_{2k} = Q_k$ and we reach a contradiction.

Secondly we proof that (ii) implies (iii). Define a sequence P_n which strongly converges to I inductively. Let $\{e_i\}_{i=1,2,...}$ be an orthonormal basis in H. Let $P_0 = 0$. Suppose now that $P_0, P_1, \ldots, P_{n-1}$ are defined. There exists $Q_n \in \mathscr{F}$ such that P perpendicular to Q_n implies $\mu(\{x \in H; \|Px\| > \frac{1}{n}\}) < \frac{1}{n}$. Define P_n to be the orthogonal projection onto lin sp $\{Q_n H, P_{n-1}H, e_n\}$. Then P_n strongly converges to I. We show that $\{P_n\}$ satisfies (iii). Given $\varepsilon > 0$, we choose N such that $\frac{1}{N} < \varepsilon$. Then $n > m \geq N$ implies that $P_n - P_m$ is perpendicular to Q_N so that

$$\mu(\{x \in H; \|P_n x - P_m x\| > \varepsilon\}) \leq \mu(\{x \in H; \|P_n x - P_m x\| > \frac{1}{N}\}) < \frac{1}{N} < \varepsilon.$$

Third we proof that (iii) implies (iv). By (iii), there exsists a monotone increasing sequence $\{P_n\}$ which strongly converges to I in $\mathscr{F}$ satisfying that for an arbitrary $\varepsilon > 0$, there exsits n_0 such that $n, m \geq n_0$ implies $\mu(\{\|P_n - P_m\| > \varepsilon\}) < \varepsilon$. (For simplification, we denote $\{x \in H; \|P_n - P_m\| > \varepsilon\}$ by $\{\|P_n - P_m\| > \varepsilon\}$.) By the triangle inequality, we have $\|P_k x\| \leq \|P_{n_0} x\| + \|P_k x - P_{n_0} x\|$. So we have

$$\sup_{n_0 \leq k \leq n} \|P_k x\| \leq \|P_{n_0} x\| + \sup_{n_0 \leq k \leq n} \|P_k x - P_{n_0} x\|.$$

Let N be an arbitrary natural number, then $\sup_{n_0\le k\le n}\|P_kx\|>N$ means $\|P_{n_0}x\|>\frac{N}{2}$ or $\sup_{n_0\le k\le n}\|P_kx-P_{n_0}x\|>\frac{N}{2}$. Therefore it follows that

$$\begin{aligned}\{\sup_{1\le k\le n}\|P_kx\|>N\} &\subset \{\sup_{1\le k\le n_0}\|P_kx\|>N\}\cup\{\sup_{n_0\le k\le n}\|P_kx\|>N\}\\ &\subset \{\sup_{1\le k\le n_0}\|P_kx\|>N\}\cup\{\|P_{n_0}x\|>\frac{N}{2}\}\\ &\cup \{\sup_{n_0\le k\le n}\|P_kx-P_{n_0}x\|>\frac{N}{2}\}.\end{aligned}$$

We have

$$\begin{aligned}\mu(\{\sup_{1\le k\le n}\|P_kx\|>N\}) &\le \mu(\{\sup_{1\le k\le n_0}\|P_kx\|>N\})+\mu(\{\|P_{n_0}x\|>\frac{N}{2}\})\\ &+ \mu(\{\sup_{n_0\le k\le n}\|P_kx-P_{n_0}x\|>\frac{N}{2}\})\\ &\le \mu(\{\sup_{1\le k\le n_0}\|P_kx\|>\frac{N}{2}\})+\mu(\{\|P_{n_0}x\|>\frac{N}{2}\})\\ &+ \mu(\{\sup_{n_0\le k\le n}\|P_kx-P_{n_0}x\|>\frac{N}{2}\}).\end{aligned}$$

For each k , there exists $M_k>0$ such that $(\mu\circ {P_k}^{-1})(|t_k|>M_k)<\frac{\varepsilon}{2^k}$, since $(\mu\circ {P_k}^{-1})$ is a measure on the finite dimensional space. Let $\frac{M}{2}\ge\max\{M_1,M_2,\dots,M_{n_0}\}$ and $N>M$, then $\mu(\{\sup_{1\le k\le n_0}\|P_kx\|>\frac{N}{2}\})<\varepsilon$ and $\mu(\{\|P_{n_0}x\|>\frac{N}{2}\})<\varepsilon$. We can choose N satisfying $\frac{N}{2}>\varepsilon$. By (iii), we have

$$\mu(\{\sup_{n_0\le k\le n}\|P_kx-P_{n_0}x\|>\frac{N}{2}\})<\mu(\{\sup_{n_0\le k\le n}\|P_kx-P_{n_0}x\|>\varepsilon\})<\varepsilon.$$

Therefore for an arbitrary $\varepsilon>0$, there exsist natural number M and n_0 satisfying that $\mu(\{\sup_{1\le k\le n}\|P_kx\| >N\})<3\varepsilon$ for $N>M$ and $n>n_0$.

For $(v)\Leftrightarrow(vi)$, see Dudley-Feldman-LeCam[3]. For $(iv)\Rightarrow(vi)$, see Yan[13].For $(iv)\Rightarrow(vii)$, see Yan[13] and Gong[4]. □

Theorem 3.2 ([12]) *Let μ be a rotationally quasi-invariant cylindrical measure on H. Then the conditions (i), (ii), (iii), (v) and (vi) in Theorem* 3.1 *are equivalent.*

Corollary 1 *Let γ be the canonical Gauss cylindrical measure on H. Then the conditions (i), (ii), (iii), (v) and (vi) in Theorem* 3.1 *are equivalent.*

Corollary 2 *Let μ be a rotationally invariant cylindrical measure on H. Then the conditions (i), (ii), (iii), (v) and (vi) in Theorem* 3.1 *are equivalent.*

4 Some examples on ℓ^2

In this section, suppose $H=\ell^2$. We construct some cylindrical measures and norms and we study the relation to the conditions of Theorem 3.1.

Let $(\ell^2)^*$ be the algebraic dual of ℓ^2, equipped with its weak topology $\sigma((\ell^2)^*, \ell^2)$ and $\mathcal{I}$ be an algebraic basis of ℓ^2 including $\{e_n\}_{n=1,2,\dots}$, where $e_n = (0, 0, \dots, 0, 1, 0, \dots)$, 1 appears in the n-th place. Let $(\cdot, \cdot)$ be a natural pairing between $(\ell^2)^*$ and ℓ^2.

Construction of cylindrical measures

On $(\ell^2)^*$, we take a and b as follows;

$$a \in (\ell^2)^* \text{ s.t } (a, e_n) = 1, \ n = 1, 2, \dots \quad (a, e_\alpha) = 0, \ e_\alpha \in \mathcal{I} \backslash \{e_n\}_{n=1,2,\dots}$$
$$b \in (\ell^2)^* \text{ s.t } (a, e_n) = n, \ n = 1, 2, \dots \quad (a, e_\alpha) = 0, \ e_\alpha \in \mathcal{I} \backslash \{e_n\}_{n=1,2,\dots}$$

Let δ_a, δ_b denote the Dirac measures at the fixed point a, b respectively in $(\ell^2)^*$. Then the induced cylindrical measures μ_a, μ_b on ℓ^2 are follows,

$$\mu_a(\{x \in \ell^2; (< x, \xi_1 >, < x, \xi_2 >, \dots, < x, \xi_m >) \in D\}) = \delta_a(\{x \in (\ell^2)^*; ((x, \xi_1), (x, \xi_2), \dots, (x, \xi_m)) \in D\}),$$

$$\mu_b(\{x \in \ell^2; (< x, \xi_1 >, < x, \xi_2 >, \dots, < x, \xi_m >) \in D\}) = \delta_b(\{x \in (\ell^2)^*; ((x, \xi_1), (x, \xi_2), \dots, (x, \xi_m)) \in D\}),$$

where $\xi_1, \xi_2, \dots, \xi_m \in \ell^2$ and $D \in \mathscr{B}(\mathbb{R}^m)$.

Construcution of norms

We define open, convex, absorbing and circled sets U_1, U_2 and U_3 as follows.
Let $\{\beta_n\}$ be an increasing sequence of non-negative real numbers such that $\beta_n \to \infty$ as $n \to \infty$, and $\{\lambda_n\}$ be the sequence of non-negative real numbers such that $\lambda_{2m} = 0$ for $m = 1, 2, \dots$, $\{\lambda_{2m-1}\}$ be an increasing sequence and $\lambda_{2m-1} \to \infty$ as $m \to \infty$.
We define

$\Gamma_1 =$ the convex hull of the set $\{\pm\beta_n(e_1 + e_2 + \dots + e_n); n = 1, 2, \dots\}$,
$\Gamma_2 =$ the convex hull of the set $\{\pm\lambda_n(e_1 + e_2 + \dots + e_n); n = 1, 2, \dots\}$,
$\Gamma_3 =$ the convex hull of the set $\{\pm\lambda_n(e_1 + 2e_2 + \dots + ne_n); n = 1, 2, \dots\}$.

Denote by B_1 the open unit ball of ℓ^2 and by B_2 the open set $\{x = (x_n) \in \ell^2; \sqrt{\sum_{n=1}^{\infty} (\frac{x_n}{n})^2} < 1\}$.
Let $U_1 = \Gamma_1 + B_1$, $U_2 = \Gamma_2 + B_1$ and $U_3 = \Gamma_3 + B_2$. Then we define $\|\cdot\|_1$, $\|\cdot\|_2$, $\|\cdot\|_3$ and $\|\cdot\|_4$ as a gauge of U_1, U_2, U_3 and B_2, respectively.
In 1984, the couple of μ_a and $\|\cdot\|_2$ was made as an example which is measurable in the sense of D.F.L. but is not measurable in the sense of Gross. In 1999, this was proved that it satisfies the conditions of (iii) and (iv) but dosen't (i) and (ii) in Theorem 3.1. In 1998, the couple of μ_b and $\|\cdot\|_3$ was made as an example which is γ-measurable and μ_b-measurable(*D.F.L.*) but is not μ_b-measurable(*Gross*). We research the relation between an example in 1984 and the Gauss cylindrical measure γ, and between an example in 1998 and the conditions of Theorem 3.1.

Theorem 4.1 ([11]) *$\|\cdot\|_2$ is not γ-measurable.*

Theorem 4.2 *$\|\cdot\|_3$ satisfies the condition (iii) for μ_b.*

Proof. Let P_n be the orthogonal projection from ℓ^2 onto the finite dimensional subspace which is the linear span of $\{e_1, e_2, \dots, e_{2n+1}\}$. Clearly P_n converges to I strongly for $n \to \infty$. For $n > m$, $(P_n - P_m)$ is the orthogonal projection onto the linear span of $\{e_{2m+2}, e_{2m+3}, \dots, e_{2n+1}\}$.
Put $k = \dfrac{1}{\frac{1}{\lambda_{2m+1}} + \frac{1}{\lambda_{2n+1}}}$, then we have

$$k\{(2m + 2)e_{2m+2} + \dots + (2n + 1)e_{2n+1}\}$$
$$= k\{e_1 + 2e_2 + \dots + (2n + .1)e_{2n+1}\} - k\{e_1 + 2e_2 + \dots + (2m + 1)e_{2m+1}\}$$
$$= \tfrac{k}{\lambda_{2n+1}}[\lambda_{2n+1}\{e_1 + 2e_2 + \dots + (2n + 1)e_{2n+1}\}]$$

$$+\tfrac{k}{\lambda_{2m+1}}[-\lambda_{2m+1}\{e_1+2e_2+\ldots+(2m+1)e_{2m+1}\}].$$

By $\frac{k}{\lambda_{2n+1}}+\frac{k}{\lambda_{2m+1}}=1$, we have

$$k\{(2m+2)e_{2m+2}+\ldots+(2n+1)e_{2n+1}\}\in\Gamma_3\subset U_3.$$

Therefore

$$\|(2m+2)e_{2m+2}+\ldots+(2n+1)e_{2n+1}\|_3\le\frac{1}{k}=\frac{1}{\lambda_{2m+1}}+\frac{1}{\lambda_{2n+1}}.$$

Since $\lambda_{2n+1}\to\infty\,(n\to\infty)$, it follows that for an arbitray $\varepsilon>0$ there exists N satisfying

$$\|(2m+2)e_{2m+2}+\ldots+(2n+1)e_{2n+1}\|_3<\varepsilon\ (n>m>N).$$

Since μ_b is the Dirac measure $\delta_{(2m+2)e_{2m+2}+\ldots+(2n+1)e_{2n+1}}$ on the linear span of $\{e_{2m+2},\ldots,e_{2n+1}\}$, we have $\lim_{n,m\to\infty}\mu_b(\{x\in\ell^2;\|P_nx-P_mx\|_3\le\varepsilon\})=1$ for every $\varepsilon>0$. □

Therefore we know that $\|\cdot\|_3$ does not satisfy $(i),(ii)$, but satisfies $(iii),(iv),(v),(vi)$ for μ_b .

Theorem 4.3 ([12]) *$\|\cdot\|_4$ is not μ_b-measurable(D.F.L.).*

Theorem 4.4 ([12]) *$\|\cdot\|_1$ satisfies (iii) for μ_a.*

Theorem 4.5 ([12]) *$\|\cdot\|_3$ and $\|\cdot\|_4$ satisfies (iii) for μ_a.*

5 Remarks

We have the table of a previous chapter. The sign ○ means the norm satisfies the condition with respect to the cylindrical measure, the sign × means it does not satisfy and the blank space means indefiniteness.

norm	cylindrical measure	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)
$\|\cdot\|_1$	γ							
	μ_a			○	○			
$\|\cdot\|_2$	γ	×	×	×		×	×	
	μ_a	×	×	○	○	○	○	○
$\|\cdot\|_3$	γ	○	○	○	○	○	○	○
	μ_a			○	○			
	μ_b	×	×	○	○	○	○	○
$\|\cdot\|_4$	γ	○	○	○	○	○	○	○
	μ_a			○	○			
	μ_b	×	×	×		×	×	

After this it is a subject that we fill up the blank spaces by studying whether $\|\cdot\|_1$ is γ-measurable or not, and $\|\cdot\|_3,\|\cdot\|_4$ are μ_a-measurable in the sense of Gross or not. In this paper we treat only three cylindrical measures, but γ, μ_a and μ_b have entirely different properties. We hope that more universal things are found out among conditions of from (i) to (vii). (For instance, the equivalence of the condtions (i) and (ii), and the equivalence of the conditions from (iii), (iv) and (v).) To the contrary there are a possibility that we make the opposite example.

References

[1] A. Badrikian, and S. Chevet, *Mesures cylindriques, espaces de Wiener et fonctions aléatoires Gaussiennes,* Lecture Notes in Math. 379, Springer-Verlag, Berlin-Heidelberg-New York, 1974.

[2] P. Baxendale, *Gaussian Measures on Function Spaces,* Amer. J. Math. **98**(1976), 891-952.

[3] R.M. Dudley, J. Feldman, and L. LeCam, *On semi-norms and probabilities,and abstract Wiener Spaces,* Ann. of Math. **93**(1971), 390-408.

[4] F. Gong, *A note on generalized Gross and Minlos Theorems,* Dirichet forms and stochastic process (Beijing,1993) 171-173, de Gruyter, Berlin, 1995.

[5] L. Gross, *Measurable functions on Hilbert space,* Trans. Amer. Math. Soc. 105(1962), 372-390.

[6] H.H. Kuo, *Gaussian Measures in Banach Spaces,* Lecture.Notes in Math.463, Springer-Verlag, Berlin-Heidelberg-New York 1975.

[7] M. Maeda, *Measurable norms and rotationally quasi-invariant cylindrical measures,* Hokkaido Math.J.Vol.XII No.1 (1983), 92-104.

[8] M. Maeda, *Some examples of measurable norms,* J. Math. Anal. Appl. 98(1984),158-165.

[9] M. Maeda, *Rotationally invariant cylindrical measures I,* Kodai. Math. J. Vol.6 No.1(1983), 14-25.

[10] M. Maeda, *Generalized rotationally quasi-invariant cylindrical measures,* J. Math. Anal. Appl. 114(1986), 100-110.

[11] M. Maeda, K. Harai and R. Hagihara, *Some examples and connection between cylindrical measures and measurable norms,* preprint.

[12] M. Maeda, K. Harai and M. Shibuya, *Some remarks on six conditions approximating to measurable norms,* preprint.

[13] J.A. Yan, *Generalizations of Gross' and Minlos' Theorems,* Lecture.Notes in Math.1372, Springer-Verlag, Berlin-Heidelberg-New York-London-Paris-Tokyo-Hong Kong, 395-404, 1989.