CORRECTION TO "ESTIMATES OF THE BESOV NORMS ON FRACTAL BOUNDARY BY VOLUME INTEGRALS"

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Unfortunately, it turned out that the above mentioned paper appeared in Volume 51, Number 1 (2000), pages 1-10 contains a mistake. The proof of Theorem 2 on pages 9-10 should be modified as follows.

PROOF of THEOREM 2. Choose $\eta > 0$ satisfying $(d-\beta)/p + \alpha < \eta < \lambda$ and $\epsilon > 0$. Since f is λ-Hölder continuous on $\overline{D}$, we can find $t_0 > 0$ such that

$$|x-y| < t_0 \text{ implies } \frac{|f(x)-f(y)|}{|x-y|^{\eta}} < \epsilon$$

and $t_0 \leq \operatorname{diam} D$.

Consider a sufficiently small positive real number t satisfying $t < t_0/(80at_1)$, where t_1 is the positive real number defined by (2.4). Put $r = at$. Then $A_t = F_r$. Since $\overline{A_t} \cup \partial D$ is compact, we cover finitely many balls with centers on ∂D and radius $2r$, i.e.

$$\overline{A_t} \cup \partial D \subset \cup_{k=1}^{m} B(w_k, 2r).$$

Using Vitali's covering theorem, we can find a subfamily $\{B(z_j, 2r)\}_j$ of $\{B(z_k, 2r)\}_k$ such that $\{B(z_j, 2r)\}_j$ are mutually disjoint and

$$\overline{A_t} \cup \partial D \subset \cup_j B(z_j, 10r).$$

Using the family, we define functions $\{v_{i,j}\}$ on $\mathbf{R}^d \times \mathbf{R}^d$ as follows. If $B(z_i, 20rt_1) \cap B(z_j, 20rt_1) \neq \emptyset$, then we define $v_{i,j}(x,y) \equiv 0$. If $B(z_i, 20rt_1) \cap B(z_j, 20rt_1) = \emptyset$, then we define

$$v_{i,j}(x,y) = f(z_i)\chi_{B(z_i,10r)}(x) - f(z_j)\chi_{B(z_j,10r)}(y).$$

Let $(x,y) \in \left(B(z_i, 10r) \cap \overline{D}\right) \times \left(B(z_j, 10r) \cap \overline{D}\right)$. If $B(z_i, 20rt_1) \cap B(z_j, 20rt_1) = \emptyset$, we have

$$\begin{aligned}&|v_{i,j}(x,y) - (f(x) - f(y))| \leq |f(z_i) - f(x)| + |f(z_j) - f(y)| \\ &\leq \epsilon|z_i - x|^{\eta} + \epsilon|z_j - y|^{\eta} \leq 2\epsilon(10r)^{\eta} < 2\epsilon|x-y|^{\eta}.\end{aligned}$$

If $B(z_i, 20rt_1) \cap B(z_j, 20rt_1) \neq \emptyset$, then

$$|v_{i,j}(x,y) - (f(x) - f(y))| = |f(x) - f(y)| < \epsilon|x-y|^{\eta}.$$

We also define functions $\{w_{i,j}\}$ on $\mathbf{R}^d \times \mathbf{R}^d$ as follows. If $B(z_i, 2rt_1) \cap B(z_j, 2rt_1) \neq \emptyset$, then we define $w_{i,j}(x,y) \equiv 0$. If $B(z_i, 2rt_1) \cap B(z_j, 2rt_1) = \emptyset$, we define

$$w_{i,j}(x,y) = f(z_i)\chi_{B(z_i,r)}(x) - f(z_j)\chi_{B(z_j,r)}(y).$$

Then we can also estimete

$$|w_{i,j}(x,y) - (f(x) - f(y))| < c_1\epsilon|x-y|^\eta$$

for each pair $(x,y) \in \big(B(z_i,r)\cap\overline{D}\big) \times \big(B(z_j,r)\cap\overline{D}\big)$. Note that each $x \in \overline{D}$ belongs to at most N many members of $\{B(z_i,10r)\}$, where N is a constant depending only on d. Hence, by Lemma 2.1,

$$\begin{aligned}
I_1 &\equiv \iint \frac{|f(x)-f(y)|^p}{|x-y|^{\beta+p\alpha}} d\mu(x)d\mu(y) \\
&\le \sum_{i,j} \int_{B(z_i,10r)\cap\partial D} \int_{B(z_j,10r)\cap\partial D} \frac{|f(x)-f(y)|^p}{|x-y|^{\beta+p\alpha}} d\mu(x)d\mu(y) \\
&\le c_2 \sum_{i,j} \int_{B(z_i,10r)\cap\partial D} \int_{B(z_j,10r)\cap\partial D} \frac{|v_{i,j}(x,y)|^p}{|x-y|^{\beta+p\alpha}} d\mu(x)d\mu(y) \\
&+ c_2\epsilon^p \iint |x-y|^{-\beta-p\alpha+p\eta} d\mu(x)d\mu(y) \\
&\le c_3 \sum_i \int_{B(z_i,10r)\cap\partial D} |f(z_i)|^p d\mu(x) \int_{\{|x-y|\ge 20r\}\cap\partial D} |x-y|^{-\beta-p\alpha} d\mu(y) \\
&+ c_3 \sum_j \int_{B(z_j,10r)\cap\partial D} |f(z_j)|^p d\mu(y) \int_{\{|x-y|\ge 20r\}\cap\partial D} |x-y|^{-\beta-p\alpha} d\mu(x) \\
&+ c_4\epsilon^p.
\end{aligned}$$

Consequently we have

$$I_1 \le c_5 \sum_i |f(z_i)|^p r^{\beta-p\alpha} + c_4\epsilon^p. \tag{3.2}$$

On the other hand, noting that $p\eta - p\alpha - d + \beta > 0$ and $A_t = F_r$, we have

$$\begin{aligned}
I_1' &\equiv \int_{A_t}\int_{A_t} \frac{|f(x)-f(y)|^p}{|x-y|^{d+p\alpha+d-\beta}} dxdy \\
&\ge c_6 \sum_{i,j} \int_{F_r\cap B(z_i,r)} \int_{F_r\cap B(z_j,r)} \frac{|w_{i,j}(x,y)|^p}{|x-y|^{d+p\alpha+d-\beta}} dxdy \\
&- c_7\epsilon^p \int_D\int_D |x-y|^{-d-p\alpha-d+\beta+p\eta} dxdy \\
&\ge c_6 \sum_i |f(z_i)|^p \int_{F_r\cap B(z_i,r)} dx {\sum_j}' \int_{F_r\cap B(z_j,r)} |x-y|^{-d-p\alpha-d+\beta} dy \\
&+ c_6 \sum_j |f(z_j)|^p \int_{F_r\cap B(z_j,r)} dy {\sum_i}'' \int_{F_r\cap B(z_i,r)} |x-y|^{-d-p\alpha-d+\beta} dx - c_8\epsilon^p,
\end{aligned}$$

where $\sum_i'$ (resp. $\sum_j''$) stands for the sum for i (resp. j) satisfying $|z_i - z_j| \ge 4rt_1$.

For each i we denote by z_{j_i} one of points z_j satisfying

$$|z_i - z_{j_i}| = \min\{|z_i - z_j|; |z_j - z_j| \ge 4rt_1\}.$$

Since $\{B(z_j,10r)\}$ is a covering of $\overline{F_r}$, we see that, for $(x,y) \in (B(z_i,r)\cap\overline{F_r}) \times (B(z_{j_i},r)\cap\overline{F_r})$,

$$\begin{aligned}
|x-y| &\le |x-z_i| + |z_i - z_{j_i}| + |z_{j_i} - y| \\
&\le 2r + 4rt_1 + 20r = (22+4t_1)r.
\end{aligned}$$

Consequently we have

$$
\begin{aligned}
I_1' &\geq c_9 r^{-d-p\alpha-d+\beta} \sum_i |f(z_i)|^p \int_{F_r \cap B(z_i,r)} dx \int_{F_r \cap B(z_{j_i},r)} dy \\
&+ c_9 r^{-d-p\alpha-d+\beta} \sum_j |f(z_j)|^p \int_{F_r \cap B(z_j,r)} dy \int_{F_r \cap B(z_{i_j},r)} dx - c_8 \epsilon^p.
\end{aligned}
$$

By Lemma 2.5 we have

$$
I_1' \geq c_{10} r^{\beta - p\alpha} \sum_i |f(z_i)|^p - c_8 \epsilon^p. \tag{3.3}
$$

Combining (3.2) with (3.3), we have the second inequality of (1.4).

We next show the first inequality of (1.4). Since

$$
\begin{aligned}
I_2 &\equiv \int_{F_r} \int_{F_r} \frac{|f(x) - f(y)|^p}{|x-y|^{d+p\alpha+d-\beta}} dxdy \\
&\leq c_{11} \sum_{i,j} \int_{F_r \cap B(z_i,10r)} \int_{F_r \cap B(z_j,10r)} \frac{|v_{i,j}(x,y)|^p}{|x-y|^{d+p\alpha+d-\beta}} dxdy \\
&+ c_{11} \epsilon^p \int_D \int_D |x-y|^{-d-p\alpha+d+\beta+p\eta} dxdy \\
&\leq c_{11} \sum_i |f(z_i)|^p \int_{F_r \cap B(z_i,10r)} dx \int_{F_r \cap \{|x-y| \geq 20r\}} |x-y|^{-d-p\alpha-d+\beta} dy \\
&+ c_{11} \sum_j |f(z_j)|^p \int_{F_r \cap B(z_j,10r)} dy \int_{F_r \cap \{|x-y| \geq 20r\}} |x-y|^{-d-p\alpha-d+\beta} dx \\
&+ c_{12} \epsilon^p,
\end{aligned}
$$

we have, by Lemma 2.5 and Lemma 2.3,

$$
I_2 \leq c_{13} \sum_i |f(z_i)|^p r^{\beta-p\alpha} + c_{12} \epsilon^p.
$$

By the similar methods as in the estimate of I_1' we also have

$$
\begin{aligned}
&\int_{\partial D} \int_{\partial D} \frac{|f(x) - f(y)|^p}{|x-y|^{\beta+p\alpha}} d\mu(x) d\mu(y) \\
&\geq c_{14} \sum_{i,j} \int_{\partial D \cap B(z_i,r)} \int_{\partial D \cap B(z_j,r)} \frac{|w_{i,j}(x,y)|^p}{|x-y|^{\beta+p\alpha}} d\mu(x) d\mu(y) - c_{15} \epsilon^p \\
&\geq c_{16} r^{-\beta-p\alpha} \sum_i |f(z_i)|^p \int_{\partial D \cap B(z_i,r)} dx \int_{\partial D \cap B(z_{j_i},r)} dy \\
&+ c_{16} r^{-\beta-p\alpha} \sum_j |f(z_j)|^p \int_{\partial D \cap B(z_j,r)} dy \int_{\partial D \cap B(z_{i_j},r)} dx - c_{15} \epsilon^p \\
&\geq c_{17} \sum_i |f(z_i)|^p r^{\beta-p\alpha} - c_{15} \epsilon^p.
\end{aligned}
$$

Thus we also have the first inequality of (1.4).

□