

Tables of Topological Characteristics of Multilayered Cyclic Fence Graphs

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Abstract

Matching polynomials of the lower members of five families, $E_{m,n}$, $F_{m,n}$, $G_{m,n}$, $D_{m,n}$, and $X_{m,n}$, of multilayered cyclic fence graphs are given in table format. Isomorphic relations among these graphs are also given.

One of the present authors has defined the series of cyclic fence graphs, E_n , by regular cyclization of the ethylene unit, or path graph, S_2 , so that all the vertices have three neighbors [1]. As an extension of the cyclic fence graph two series, $E_{m,n}$ and $F_{m,n}$, of multilayered cyclic fence graphs (MLCFG) were proposed to be defined as in Fig 1 [2]. Both of them are composed of m $2n$ -membered cycle graphs with periodic bridging. By these definitions we have

$$E_n = E_{1,n} = F_{1,n} \quad (1)$$

It is to be noted here that $E_{m,n}$ and $F_{m,n}$ are all bipartite and cubic. Further, all the $E_{m,n}$ and $F_{m,n}$ graphs can be mapped on the surface of a torus [2].

As the characteristic polynomial of $E_{m,n}$ and $F_{m,n}$ graphs can be factored out into a product of 2×2 determinants, the electronic states of these hypothetical torus networks can easily be calculated. Then it was found that the electronic states of MLCFG's are rapidly converging to that of graphite, or honeycomb lattice of infinitely large size. Actually some families, e.g., $F_{m,3m}$ and $E_{m,m+2}$, etc., have such a peculiar property that all the vertices and edges are equivalent (or transitive) as in the graphite network.

Isomorphism is another characteristics of MLCFG's. For example, besides the general isomorphic relations, such as

$$E_{m,2} = F_{m,2} \quad (2)$$

$$\text{and} \quad E_{m,2m+1} = F_{m,2m+1} \quad (m > 1), \quad (3)$$

a number of sporadic equalities have been found among E and F families, such as

$$E_{2,3} = E_{3,2} = F_{3,2} \neq F_{2,3}. \quad (4)$$

In many cases it is rather difficult to find a general formulas for expressing the general trend of MLCFG's. Further, every MLCFG can be drawn into a number of interesting geometrical shapes or other families of graphs, such as generalized Petersen and generalized Heawood graphs. Chemical importance of several members of MLCFG's has been pointed out not only from graph theoretical [3-5] but also from group theoretical stand points [6]. Thus these new series of mathematically interesting graphs are becoming significant in mathematical chemistry.

Recently one of the present authors defined several new families of MLCFG's, $G_{m,n}, D_{m,n}$ by joining m 2n-membered cyclic graphs in different fashion as shown in Fig. 1 [7]. She has also defined another series, $X_{m,n}$, of MLCFG's as in Fig. 2. All these graphs are cubic, bipartite, and full of hexagons. Among this enlarged set of MLCFG's a number of interesting mathematical properties and relations are being discovered by the analysis of several topological characteristics, especially of matching properties, of these and related graphs. Although systematic analysis is still under way, the matching polynomials [8-11] of the lower members of several families of MLCFG's are tabulated in this paper.

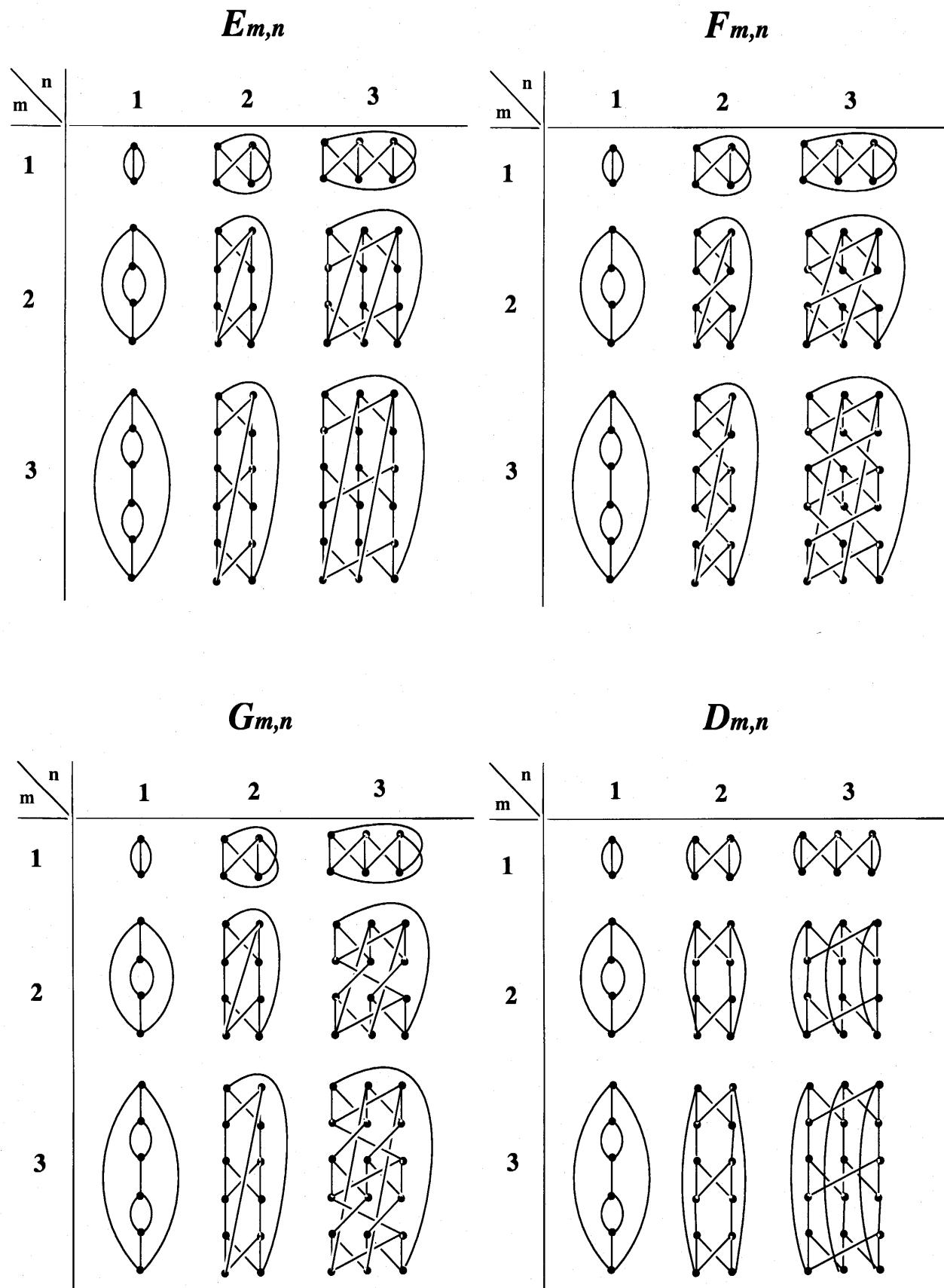
Table 1 lists the set of the non-adjacent numbers, $p(G,k)$'s, of the series of E, F, G, D and X graphs, where $p(G,k)$ is the number of ways for choosing k disjoint edges from given graph G [8]. The perfect matching number $K(G)$ of G is equal to $p(G,m)$ with $m=[N/2]$ as maximum k . By using the set of $p(G,k)$'s for a given graph G the matching polynomial, $M_G(x)$, is expressed as

$$M_G(x) = \sum_{k=0}^{[N/2]} (-1)^k p(G,k) x^{N-2k}. \quad (5)$$

Since there have been found so many entangled isomorphisms in MLCFG's, two tables were prepared in this paper. Table 2 lists those graphs which are isomorphic to $E_{m,n}$, where the table is still incomplete for larger members. Table 3 lists the isomorphic relations among F, G, D and X graphs. Discussion on this problem including the relation with the generalized Heawood graphs will be given elsewhere [12].

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Fig. 1 Four families of MLCFG's, $E_{m,n}$, $F_{m,n}$, $G_{m,n}$ and $D_{m,n}$.

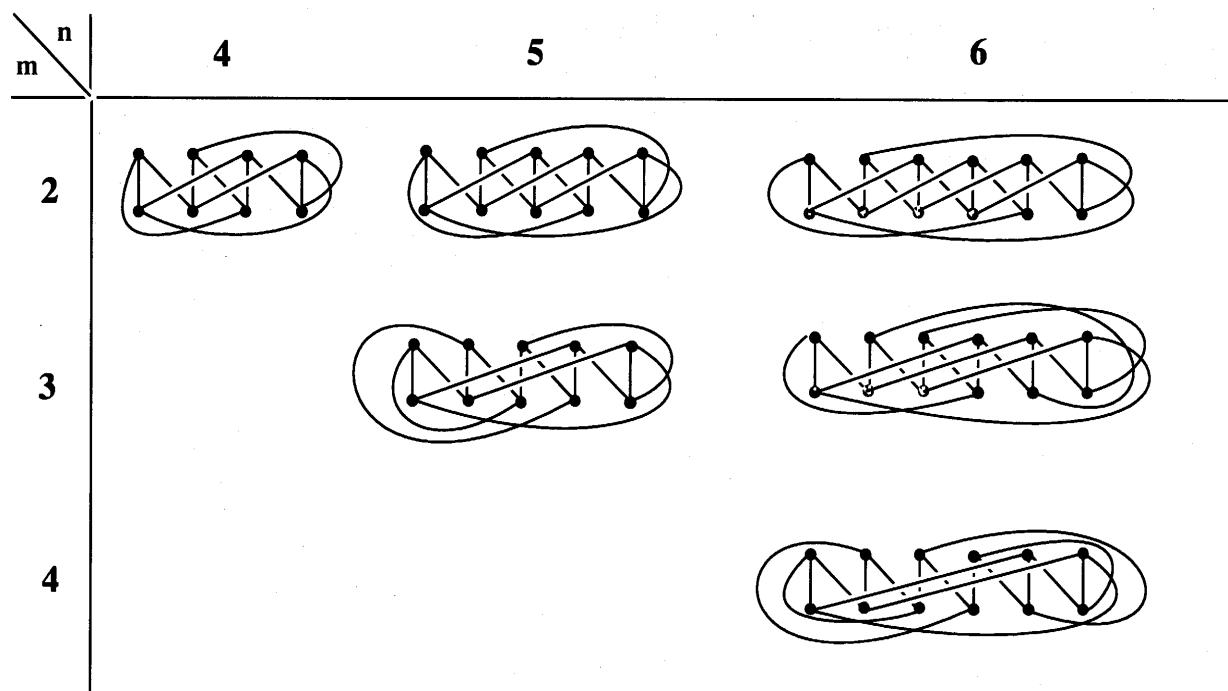
$X_{m,n}$ Fig. 2 $X_{m,n}$ graphs.

Table 1. Non-adjacent numbers, $p(G,k)$'s, of the smaller members of MLCFG's. $\langle E_{1,n} \rangle$

n	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	n
0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
1	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	1
2	18	42	75	117	168	228	297	375	462	553	633	777	900	1032	1173	2
3	6	44	145	336	644	1096	1719	2540	3586	4884	6461	8344	10560	13136	16099	3
4	9	95	420	1225	2834	5652	10165	16940	26625	39949	57722	80835	110260	147050	4	
5	13	192	1085	3880	10656	24626	50380	94128	163943	270004	424839	643568	944146	147050	5	
6	20	371	2588	11097	35645	94457	218124	454220	872648	1571715	2684936	4388567	6	147050	6	
7	31	696	5823	29380	108933	327840	848380	1958004	4130895	8107024	14990889	7	14990889	7	7	
8	49	1278	12535	73238	309828	1049815	3024035	7694295	17750146	37825085	8	37825085	8	8	8	
9	78	2310	26070	173984	831233	3146234	10025995	27984400	70306339	9	27984400	9	9	9	9	
10	125	4125	52752	397410	2124927	8920788	31270632	95307423	10	95307423	10	14990889	10	14990889	10	
11	201	7296	104390	878724	5216160	24132848	92570865	11	92570865	11	14990889	11	14990889	11	11	
12	324	12805	202811	1890665	12370476	62706098	12	62706098	12	14990889	12	14990889	12	14990889	12	
13	523	22330	388005	3974736	28481426	13	28481426	13	28481426	13	14990889	13	14990889	13	13	
14	845	38730	732680	8191433	14	8191433	14	8191433	14	8191433	14	14990889	14	14990889	14	
15	1366	1366	66864	1368143	15	1368143	15	1368143	15	1368143	15	14990889	15	14990889	15	
16	2209	2209	3573	3573	17	3573	17	3573	17	3573	17	14990889	17	14990889	17	
Z	34	108	344	1104	3546	11396	36628	117732	378426	1216380	3909832	12567448	40395794	129844996	417365332	Z

$\langle E_{1,n} \rangle$

n	18	19	20	21	22	23	24	n
0	1	1	1	1	1	1	1	0
1	54	57	60	63	66	69	72	1
2	1323	1482	1650	1827	2013	2208	2412	2
3	19476	23294	27580	32361	37664	43516	49944	3
4	192339	247342	31335	391755	484000	591629	716262	4
5	1347606	1878302	2564152	3436881	4532264	5890369	7555800	5
6	6909867	10530089	15624200	22611330	32025950	44499779	60780420	6
7	26372124	44470165	72305160	113897310	174496828	260846703	381480456	7
8	75489048	142580617	256948925	444681804	742903282	1203230096	1895983449	8
9	162134940	348253185	704514900	1354083500	2489859944	4404724112	7531476256	9
10	259773606	646242801	1489893222	3220804083	6589015818	12850514046	24035924904	10
11	306762048	903866537	2418300660	5968840857	13757761628	29899648422	61745010528	11
12	261954681	940065242	2984254025	8562586942	22570316131	55350165344	127553140316	12
13	157343310	712688138	2759465880	9408265566	28871450674	81084776656	211134619392	13
14	63912861	382048059	1873096780	7797954630	28470085105	93202951359	278275114488	14
15	16593024	140239703	907342584	4774529522	21303470320	83057341135	289370742016	15
16	2530116	33109571	301700575	2099282136	11842995626	56464999908	234474886932	16
17	196992	4639515	65197340	637795200	4755111988	28667288596	145642329408	17
18	5780	336471	8444350	126885521	1327433558	10568110150	67883269808	18
19	9351	573100	15268155	244373976	2724426038	23088797760	19	
20			973686	27443581	466262348	5521693200	20	
21			24478	1650506	49067004	88249760	21	
22				39605	2791993	87308352	22	
23					64081	4713984	23	
24						103684	24	
Z	1341539196	4312135922	13860583628	44552347608	143205490528	460308235562	1479577349604	Z

$\langle E_{1,n} \rangle$

n	25	26	27	28	29	30	n
0	1	1	1	1	1	1	0
1	75	78	81	84	87	90	1
2	2625	2847	3078	3318	3567	3825	2
3	56975	64636	72954	81956	91669	102120	3
4	859600	1023425	1209600	1420069	1656857	1922070	4
5	9577940	12011194	14915232	18355232	22402123	27132828	5
6	81744725	108412889	141963273	183747956	235309016	298395540	6
7	547057075	770735316	1068589557	1460069392	1968505152	2621661540	7
8	2015258800	4384963037	6465934944	9364271434	13340988581	18723153045	8
9	12494647300	20176648570	31801053024	49036141224	74122206147	110026491470	9
10	43329104700	75589028853	128055492837	211287190380	340389790536	536601754497	10
11	121924471100	231415375308	424026554265	752811309992	1299056220552	2184657057060	11
12	278452949850	579642178466	1156865923926	2223839760464	4133312286143	7452434984215	12
13	515002049550	1186610575520	2600311957398	5450356879016	10979313295017	21341722134930	13
14	767871368300	1979110438370	4805588628432	11070898647648	2434050898770	5132456902555	14
15	916553606220	2675221291964	7273760396688	18585595142072	4495333888734	103548389586596	15
16	867402261425	2908795439225	8964657009981	25672397045771	68930737151083	174822698110425	16
17	642587018075	2518663476498	8925381501885	28995619592772	87325960255925	246054242975430	17
18	366531746900	1714290771323	710483400378	26557706874426	90799106269891	287172885977825	18
19	157580451900	902062215600	4462700618214	19519257128868	76830709438873	27602197184310	19
20	49668602770	359251015734	2175313569426	11360190003379	52342741402831	216598035907560	20
21	11064364950	105355153540	805642517970	5148988925656	28330391730517	137271230539180	21
22	1656300900	21942754522	220628628792	1779688366372	11981246520054	69328318288530	22
23	154679100	3088326904	43108443648	456639852984	3877394198786	27444262035840	23
24	7945225	272950405	5722133553	83962982803	934995978879	8340781711465	24
25	167763	13369902	479907585	10540824196	162246168833	1895572480602	25
26	271445	22465026	37695476	840970998	1931420071	311238918285	26
27	28	439206	710649	1469152093	35216389680	27	
28	29	30	1149853	63170903	2559262980	28	
29	30	31	105737280	1860500	29		
30	31	32	1860500	30			
Z	4755836293844	15286778495572	49136533930954	157940723994588	507671987419144	1631820092321064	Z

$\langle E_{2,n} \rangle$

n	2	3	4	5	6	7	8	9	10	11	12	n	
0	1	1	1	1	1	1	1	1	1	1	1	0	
1	12	18	24	30	36	42	48	54	60	66	72	1	
2	42	117	228	375	558	777	1032	1323	1650	2013	2412	2	
3	44	336	1096	2540	4884	8344	13136	19476	27580	37664	49944	3	
4	9	417	2826	10155	26613	57708	110244	192321	313335	483978	716238	4	
5	186	3816	24486	93888	269640	643056	1346922	2563272	4531164	7554456	5		
6	2444	34945	216240	868728	2677912	6898455	15606900	32001046	60745980	6			
7	600	27840	320400	1935240	8052816	26261928	72104280	174158688	380944872	7			
8	33	11045	294045	2946188	17490394	74805381	255414095	739828771	1890330477	8			
9	1750	156116	2985192	27178960	159273174	696376060	2470060142	7488564640	9				
10	42606	1925028	29634536	251509761	1459096222	6495841385	23793689352	10					
11	4788	735392	21972528	290205144	2334064540	13431833388	6070777568	11					
12	129	147784	10563644	239097000	2817288405	21716144384	124146470688	12					
13	12544	3061776	136003104	2521074560	27192919908	202510167456	13						
14	264	475416	50867154	1631365660	26009620164	261446590008	14						
15	31856	11629284	737266624	18642749356	264175878528	15							
16	545	1445967	221522085	9757204688	205788723213	16							
17	79038	41151420	3599497484	121139283624	17								
18	1133	4211530	890931976	52479594020	18								
19	192620	137668212	16147466232	19									
20	2369	11852203	3359530110	20									
21	462770	440054136	21										
22	4977	32437716	22										
23	1098984	10497	23										
24			24										
	Z	108	1092	11068	113232	1160304	11892872	121907900	1249636620	12809643268	131307834428	1345997681224	Z

$$< E_{2,n} >$$

Tables of Multilayered Cyclic Fence Graphs

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n	13	14	15	16	17	18	n
0	1	1	1	1	1	1	0
1	78	84	90	96	102	108	1
2	2847	3318	3825	4368	4947	5562	2
3	64636	81956	102120	125344	151844	181835	3
4	1023399	1420041	1922040	2546568	3312093	4238284	4
5	12009582	18353328	27130608	38982240	54652722	7493721	5
6	108366765	183687784	298318740	467187312	709118467	1047338115	6
7	769926768	1458893296	2620010520	4496783136	7424275224	11850322626	7
8	4375240610	9348421432	18698422350	35383368260	63869365891	110653953903	8
9	20091560684	48878916240	109752159980	230494683360	457414137898	864221800320	9
10	75027760600	210099107876	534263491752	1254300684592	2753224455971	5701805453027	10
11	228563798164	745813441296	5738137917472	2168972950980	32012593805386	11	
12	568333915630	2191232220604	7368317570340	22161799772696	60754155744831	153775736858952	12
13	151343269888	5329033333680	72441635396576	22454910500790	634405585696333	13	
14	1892341602320	10708559636040	50040198571320	200611924040720	709425232684947	2253431153469435	14
15	2506908634568	17715277132784	99848525330076	470501478332192	1917100636553472	6901056400660270	15
16	2652395633901	23992582736024	166115873769075	93303526656474	442920246229236	18227949869022800	16
17	2214149566070	26399294780544	229320932134530	1559978931954976	8735478229916752	41501156943536704	17
18	1435372713229	23363688662608	26098926578015	2189836702106320	14669304991101094	81327807981241888	18
19	707959502016	16419245263008	242836578724860	2566637128264672	20898032811921016	136848184665840848	19
20	258657190857	9014625130588	18277084539422	2494004714573480	25153386227964392	197070190215464224	20
21	67356216390	3786998315200	10982351063440	1991427544255008	25369478305585056	24183327612960768	21
22	12011605775	1185032427600	51812145037080	1292444395675568	21326603594139144	251527340516796832	22
23	1354687412	266587356000	18794549423400	672622646891744	14795411299078024	220271972484078048	23
24	86755487	41071543632	5102966923535	276035627931004	8376544079708747	161122437972886208	24
25	2584998	4040751680	1001047159566	87471013116064	3817615744938162	97491225393425248	25
26	22209	22758704	135209875425	20835985802992	1377175034705261	48227031740871632	26
27	6031872	11733568420	3602030793568	38505391881956	19227486308268432	27	
28	47112	587317275	430840059384	81240456278382	6070429450782233	6070429450782233	28
29	13979850	33294913440	12490138649264	1484663973702927	29		
30	32213600	1494404496	134539428678	273494363235159	30		
31	213313	92603378724	3756212135	36571007947965	31		
32	33	73859934	337466183338	199706245227	32		
33	34	454993	6750502461	105621862	34		
34	35			464527	35		
35	36				36		
Z	Z	13797423654884	141433305286332	1449790944899172	14861377809351036	152339584674031680	Z

$\langle E_{3,n} \rangle$

n	2	3	4	5	6	7	8	9	10	n
0	1	1	1	1	1	1	1	1	1	0
1	18	27	36	45	54	63	72	81	90	1
2	117	297	558	900	1323	1827	2412	3078	3825	2
3	336	1719	4884	10560	19476	32361	49944	72954	102120	3
4	417	5643	26613	80820	192321	391734	716238	1209573	1922040	4
5	186	10557	93888	424404	1346922	3435894	7554456	14913477	27130608	5
6	17	10740	216232	1566375	6898437	22590435	60745956	141910407	298318710	6
7	5328	320304	4094595	26261388	113633613	380943720	1067609403	2620008540	7	
8	1035	293637	7544100	7478415	442468677	1890305421	6453405945	18698361600	8	
9	45	155364	9634000	159222486	1341033407	7488237568	31683827853	109751008820	9	
10	42006	8272926	251280405	3165101646	23790828432	127223585523	534248395782	10		
11	4596	4552050	289534176	5794068462	60690046368	419447173095	2168827456920	11		
12	113	1487960	237817920	815721254	124066177768	1137033331875	7367249246985	12		
13	255240	134437896	8714720742	202240128768	2532155915907	2097105497450	13			
14	18240	49682628	6930690900	260767554768	4618929774213	50012529925695	14			
15	304	11109408	3995011160	262900253024	6866034125373	99748616712516	15			
16	1326861	1608386094	204011808717	8255934355041	16582667678300	16				
17	67230	429141342	11932979784	7950199463001	228649031195820	17				
18	809	70206017	51161517116	6051510288300	25973908271330	18				
19	2137	6216231	15481768248	3580091744040	240979769856360	19				
20	236355	3136535022	1610646422778	180595277841072	20					
21	21	393634520	535558201146	107816071349280	21					
22	22	27020004	126698033709	50386484069820	22					
23	23	804456	20228107833	18027701810160	23					
24	24	5697	2017137582	4798179250880	24					
25	25	110974374	914422845456	914422845456	25					
26	26	2670381	118377261240	435986160	26					
27	27	15273	9637099360	8688000	27					
28	28				28					
29	29				29					
30	30				30					
Z	1092	35392	1158232	37942520	1243998156	40794590352	1337826438480	43873184272216	1438795080306012	Z

$\langle E_{4,n} \rangle$

n	2	3	4	5	6	7	8	11
0	1	1	1	1	1	1	1	0
1	24	36	48	60	72	84	96	1
2	228	558	1032	1650	2412	3318	4368	2
3	1096	4884	13136	27580	49944	81956	125344	3
4	2830	26613	110244	313335	716238	1420041	2546568	4
5	3848	93888	643056	2563272	7554456	18353328	38982240	5
6	2516	216232	2677896	15606880	60745956	183687756	467187280	6
7	632	320304	8052432	72103560	380943720	1458893616	4496780832	7
8	33	293640	17486558	255402680	1890305397	9348374868	35383290660	8
9	155376	27158224	696271000	7488236608	48878124736	230493058144	9	
10	42024	29568536	1458473296	23790811224	210089869444	1254276951824	10	
11	4608	21846384	2331568120	60689863680	745734881024	5737881095584	11	
12	116	10421844	2810386140	124064898808	2190728251656	22159662463592	12	
13	2973264	2507870440	202233907968	5326541542448	7242763693920	13		
14	447704	1614086520	260745977424	106398939869272	200538569324048	14		
15	28208	722189480	262846325984	17686075033088	470190806383520	15		
16	417	213114635	203914808952	23922711419880	931965025597054	16		
17	38347340	1920563532	26267731811664	156970137221024	17			
18	3708990	51050848784	23169772370864	2182929190315184	18			
19	152940	15415411680	16197537720896	253709273559904	19			
20	1537	311114744	8820609382956	2474358731143288	20			
21	387876896	3659412542944	1967338775473056	1967338775473056	21			
22	26339952	1123528483280	1268823447189680	1268823447189680	22			
23	772224	245571556832	654316053315040	23				
24	5508	36213041984	264992052915924	24				
25	3330463584	82386572360864	25					
26	168524160	19094643338032	26					
27	3723776	3173186706272	27					
28	20296	358342274808	28					
29	1000558448	25426255136	29					
30	31	17358176	30					
31		75905	31					
32		75905	32					
Z	11208	1158280	121428984	12742189456	1337313153864	140364589449752	14733021591245816	Z

$\langle E_{5,n} \rangle$

n	2	3	4	5	6
0	1	1	1	1	1
1	30	45	60	75	90
2	375	900	1650	2625	3825
3	2540	10560	27580	56975	102120
4	10160	80820	313335	859575	1922040
5	24556	424404	2563272	9576465	4884
6	35300	1566380	15606880	81704700	27130608
7	28620	4094700	72103560	546394200	192321
8	11715	7545000	255402680	2907760425	716238
9	1910	9638040	69627100	12433208675	1922040
10	65	8283021	1458473312	42051732180	26261568
11	4565805	2331568440	120150135300	2168826688200	301947
12	1497245	2810388780	271986920050	7367240543480	7488236848
13	257685	2507882120	496620791350	20970983048640	109751007080
14	18405	1614116760	727038266823	50012081803650	534248349108
15	309	722236584	845832491855	99746461426516	164468
16	213158635	772491848200	165818578151460	134989092	159239754
17	38371340	544894690100	228625095657300	50080176	23790815544
18	3716110	290542765925	259683329048900	11263716	60689909904
19	153900	113818867875	240877630273740	1354167	2168826688200
20	1569	31522270430	180448994087793	203940582411	7367240543520
21	5849448850	107653930051710	50249431282395	119238535272	2097098305320
22	672159525	389218488	3117291606	260751628584	107654540816700
23	42172675	17941139140800	389218488	50250164915610	228625110674370
24	1145975	4758442711185	26508708	784488	51079770668
25	7633	901658490618	5857	17941782052680	25968338200475
26	115657326855	9282152420	4758845022485	180449370275328	165818580932265
27	27	25	901832681178	389218488	107654540816700
28	411171375	26	115707107955	50250164915610	411982815
29	7984770	27	8013690	784488	9290859220
30	38405	29	38405	17941782052680	38405
Z	115272	37983320	12742357568	4280394878464	1438014826283904

 $\langle E_{6,n} \rangle$

n	2	3	4	5
0	1	1	1	1
1	36	54	72	90
2	558	1323	2412	3825
3	4884	19476	49944	102120
4	26619	192321	716238	1922040
5	94008	1346922	7554456	27130608
6	217188	6898443	60745956	298318710
7	324168	26261568	380943720	2620008540
8	301947	74800755	1890305403	18698361570
9	164468	159239754	7488236848	109751007080
10	46518	251359929	23790815544	534248349108
11	5364	289770732	60689909904	2168826688200
12	129	238272876	124065225860	7367240543520
13	13	134989092	202235518032	2097098305320
14	14	50080176	260751628584	50012081835600
15	15	11263716	262860585424	99746461790856
16	16	1354167	203940582411	165818580932265
17	17	68958	119238535272	228625110674370
18	18	857	51079770668	25968338200475
19	19	15432265656	180449370275328	180449370275328
20	20	3117291606	389218488	389218488
21	21	107654540816700	26508708	26508708
22	22	50250164915610	784488	784488
23	23	17941782052680	5857	5857
24	24	4758845022485	4758845022485	4758845022485
25	25	901832681178	901832681178	901832681178
26	26	115707107955	115707107955	115707107955
27	27	8013690	8013690	8013690
28	28	411982815	411982815	411982815
29	29	9290859220	9290859220	9290859220
30	30	38405	38405	38405
Z	1185888	1245921120	1337447201552	1438018075662624

Tables of Multilayered Cyclic Fence Graphs

 $\langle E_{7,n} \rangle$

n	2	3	4
0	1	1	1
1	42	63	84
2	777	1827	3318
3	8344	32361	81956
4	57715	391734	1420041
5	269822	3435894	18353328
6	870695	22590442	183687756
7	1946720	113633886	1458893616
8	2885479	442473423	9348374875
9	3065286	1341081945	48878125100
10	2019591	3165426117	210089878166
11	795704	5795557971	745735008732
12	165837	8162021945	2190729528967
13	14322	8725638243	5326550782224
14	257	6948059409	10698989792600
15		4013921751	17686280207680
16		1621959087	23923358630535
17		435202005	26269302335164
18		71751421	23172694004194
19		6415143	16201667923420
20		247674	8824982839799
21		2442	3662809808032
22			1125409599132
23			246286072816
24			36390249189
25			3357528132
26			170912098
27			3833956
28			22081
Z	12200592	40869344784	140384697896992

 $\langle E_{8,n} \rangle$

n	2	3	4
0	1	1	1
1	48	72	96
2	1032	2412	4368
3	13136	49944	125344
4	110252	716238	2546568
5	643312	7554456	38982240
6	2681432	60745964	467187280
7	808080	380944104	4496780832
8	17621382	1890313797	35383290668
9	27584016	7488347520	230493058656
10	30444600	23791797936	1254276967120
11	2300048	60696114816	5737881378976
12	11354860	124093960112	22159666611864
13	3403728	202334584800	72427671544928
14	546920	261007268592	200538820952080
15	36848	263353038048	470192235018656
16	513	204641780469	931971456951942
17		119963760456	1556993285225504
18		51610479868	2182996069002160
19		15696979896	2553864446687712
20		3202700886	2474647079216184
21		405754488	1967765433020960
22		28222140	1269321658505584
23		863592	654769471690336
24		7073	265308376745580
25			82552134150432
26			19157886326384
27			3190213812704
28			361435035560
29			25784642848
30			1025006128
31			18194912
32			84481
Z	125522208	1340655987680	14735507208749088

$\langle E_{9,n} \rangle$

k	0	1	2	3	4
0	1	1	1	1	1
1	54	81	108	108	1
2	1323	3078	5562	5562	90
3	19476	72954	181834	181834	3825
4	192330	1209573	4238191	4238191	102120
5	1347264	14913477	74989621	74989621	1922055
6	6904170	141910416	1047224066	1047224066	27131718
7	26317260	1067609916	1848074721	1848074721	298357155
8	75149568	6453419418	110620529632	110620529632	2620836960
9	160714668	31684044186	863832007620	863832007620	18710813985
10	255651804	127225961604	5698145612798	5698145612798	10988972890
11	298375596	419466096072	31984409283508	31984409283508	53543769957
12	250001598	1137146343237	153595268270796	153595268270796	2176855106040
13	145559376	2532672220593	633434886729680	633434886729680	101692058299456
14	56097342	4620754380402	2249010941646044	2249010941646044	7410596225495
15	13273908	6871045048650	6883914911050528	6883914911050528	21160218530550
16	1700487	8266622625666	18171088761786400	18171088761786400	170418184605195
17	92142	7967812389390	41339317328201016	41339317328201016	237484393614750
18	1025	6073708904715	80931844778007680	80931844778007680	50683847961675
19	3601167633999	136014956725057248	21679986737038784	21679986737038784	273527414200235
20	1625403413826	195563436043472832	15783998453313664	15783998453313664	258313971369240
21	542963812734	239497428585562944	117499275	117499275	197986677435597
22	129262837608	248435848395863264	2946969	2946969	10851270484080
23	20811409596	21679986737038784	49058818639967184	49058818639967184	121568897195490
24	2098828863	15783998453313664	46551922336181160	46551922336181160	58819936007505
25			18348316651047172	18348316651047172	18278509414200
26			5703589566652320	5703589566652320	4892744336185
27			1365736155203928	1365736155203928	6147272105235
28			244387064157088	244387064157088	939416869362
29			31389219215216	31389219215216	1243878815406
30			4440179758	4440179758	172573654875
31			2736533262931	2736533262931	10132504560
32			55650305	55650305	15178134920
33			131505	131505	743670165
34			149111781753	149111781753	15728610
35					65537
36					Z

 $\langle E_{10,n}, E_{15,n} \rangle$

Z	E _{10,2}	E _{10,3}	E _{15,2}
0	1	1	1
1	60	90	90
2	1650	3825	3825
3	27580	102120	102120
4	313345	1922040	1922055
5	2563712	27130608	27131718
6	15615560	298318720	298357155
7	72205040	2620009200	2620836960
8	256185710	18698381880	18710813985
9	700473640	109751394240	10988972890
10	1474583652	534253471992	53543769957
11	2376159320	2168876655240	2176855106040
12	2899464590	7367612905670	7410596225495
13	2635004800	20973151573860	21160218530550
14	1741136600	50022093162150	50683847961675
15	808269204	99783417443136	101692058299456
16	250746465	165928125907680	170418184605195
17	48168540	228886055223360	237484393614750
18	5048490	260181695931200	273527414200235
19	225260	241636497159480	258313971369240
20	2049	181362453960660	197986677435597
21		10851270484080	121568897195490
22		50869922172060	58819936007505
23		18278509414200	21953690108640
24		4892744336185	6147272105235
25		939416869362	1243878815406
26		122800802265	172573654875
27		10132504560	15178134920
28		468894540	743670165
29		9894960	15728610
30		61100	65537

$\langle F_{2,n} \rangle$

n	2	3	4	5	6	7	8	9	10	11	12	n
0	1	1	1	1	1	1	1	1	1	1	1	0
1	12	18	24	30	36	42	48	54	60	66	72	1
2	42	117	228	375	558	777	1032	1323	1650	2013	2412	2
3	44	336	1096	2540	4884	8344	13136	19476	27580	37664	49944	3
4	9	420	2830	10155	26613	57708	110244	192321	313335	483978	716238	4
5	192	3848	24486	93888	269640	643056	1346922	2563272	4531164	7554456	5	
6	20	2516	34945	216228	868714	2677896	6898437	1560680	32001024	60745956	6	
7	632	27840	320256	1934988	8052432	26261388	72103560	17415764	380943720	7		
8	11045	293436	2944403	17486566	74798424	255402705	739811424	1890305433	8			
9	33	155024	2978850	27158352	159222684	696271700	2469867708	7488238048	9			
10	65	41832	1913121	29569336	251282196	1458481702	6494436476	2379083700	10			
11	4608	723968	21848880	289542852	2331624900	13424717268	60690136608	11				
12	120	142737	10425988	237842511	2810623965	21690456271	124066802656	12				
13	11690	2976912	134479782	2508512840	27126073986	202243130208	13					
14	229	449400	49725189	1615215500	25884191619	260777837856	14					
15	28592	11134560	723464424	18474603020	262925711072	15						
16	449	1335096	21401195	9599183066	204057468648	16						
17	68544	38719420	3498461472	119388776064	17							
18	884	3792150	848915980	51215626784	18							
19	161260	127052112	15516256512	19								
20	1769	10392393	3151302960	20								
21	22	373802	397669760	21								
22	23	3569	27667488	22								
23		857088	857088	23								
24		7248	7248	24								
Z	108	1104	11208	113232	1157484	11855212	121442320	1244152644	1274689986	130599758840	1338078644232	Z

$\langle F_{2,n} \rangle$

n	13	14	15	16	17	18	n
0	1	1	1	1	1	1	0
1	78	84	90	96	102	108	1
2	2847	3318	3825	4368	4947	5559	2
3	64636	81956	102120	125344	151844	181542	3
4	1023399	1420041	1922040	2546568	3312093	4224686	4
5	12009582	18353328	27130608	38982240	54652722	74594597	5
6	108366739	183687756	298318710	467187280	709118433	1039018679	6
7	769925364	145893616	2620008540	4496780832	7424272572	11719228794	7
8	4375205913	9348374910	18698361615	3538329078	63869268702	109030791688	8
9	20091040242	48878126920	109751009690	230493061216	457411912496	848016335987	9
10	75022462021	210089921468	534248419047	1254277043248	2753188739804	5568956777295	10
11	228525006372	745735633944	2168827837740	5737882776480	14024148830400	31106590276627	11
12	568122812868	219073565556	7367253519375	22159684358152	60750201203743	148586218223036	12
13	1150472668676	5326593956376	20971089776490	72427837103456	224520080198802	609265444282370	13
14	1889592171818	10699215995496	50012744144535	200539997279664	709253485468875	2149916855751326	14
15	2500233592896	17687178752600	99749629945716	470198747612960	1916272006286240	6537582685723282	15
16	2639955998797	23926093938747	165830414887185	931999966780454	4425918204953669	17137397335187508	16
17	2196468265550	26275710808116	228659868886950	1557092867691424	8724735684053878	3870301624454608	17
18	1416422962643	23184237856626	259763850125035	2183274958782640	1464024302144336	75189835799839424	18
19	692904909684	16217563250532	241024354674720	2554491611607392	20832997674731536	125351951216091616	19
20	25000918310	8841535419283	180658243332750	247577845556728	25015212474275172	178727164153909504	20
21	64070854016	3675636775400	107885325653940	1969392933935008	25186776670948532	216982705380417792	21
22	11073445136	1132630205684	50445168021240	1271178144751088	2169324733034872	22366812484174080	22
23	1197001104	249156677000	18065445396180	656430526329568	1455801977937176	192871528394373056	23
24	71917105	37157611687	481623653495	266457865063444	8190005183402800	13901061096111744	24
25	1946958	3486656292	920674388082	83156052602784	3697261004167336	82844093929837472	25
26	14773	183221822	119884243365	19392903651472	1316474115335726	40249742488599704	26
27	4388916	9876274960	3255744624992	361598794839092	15716551562559252	27	
28	30249	458910900	373937580408	74473996547967	4842157620177241	28	
29	29	9830400	27311524512	11081607970358	1150161415467383	29	
30	30	62180	1132774544	1132609984257	204466725388698	30	
31	31	21897696	73967889168	26156742096977	2281154122002	31	
32	32	128129	2761061613	125318206789	32		
33	33	48549246	3822626176	51260700	35		
34	34	264593	3822626176	168384	36		
35	35						
36	36						
Z	13709494587528	14046284469724	1439135001731524	14744893133434896	151071217114812096	1361650209570575616	Z

$\langle F_{3,n} \rangle$

n	2	3	4	5	6	7	8	9	10	n
0	1	1	1	1	1	1	1	1	1	0
1	18	27	36	45	54	63	72	81	90	1
2	117	297	558	900	1323	1827	2412	3078	3825	2
3	336	1719	4884	10560	19476	32361	49944	72954	102120	3
4	417	5643	26613	80820	192321	391734	716238	1209573	1922040	4
5	186	10557	93888	424404	1346922	3435894	7554456	14913477	27130608	5
6	17	10737	216240	1566390	6898443	22590435	60745956	141910407	298318710	6
7	7	5319	320400	4094910	26261568	113633613	380943720	1067609403	2620008540	7
8	8	1026	294045	7546785	74800755	442468677	1890305397	6453405918	18698361570	8
9	9	42	156116	9645985	159239754	1341033407	7488236608	31683826530	109751007080	9
10	10	42606	8303046	251359929	3165101646	23790811224	127223555904	534248349102	10	
11	11	4788	4594710	289770732	5794063462	60689863680	419446771524	2168826687900	11	
12	12	129	1520375	238272879	8157221254	124064898776	1137029647644	7367240536630	12	
13	13	267195	134989110	8714720742	202233907296	2532131707356	20970982953960	13		
14	14	20055	50080221	6930690900	260745971880	4618812165615	50012080918440	14		
15	15	379	11263776	3995011160	262846304512	6865604170191	99746455501856	15		
16	16	1354212	1608386094	203914781589	8254741733127	165818548856505	16			
17	17	68976	429141342	119205719880	7947686799639	228624986813490	17			
18	18	860	70206017	51051262004	6047510505759	259683023083025	18			
19	19	6216231	15416167416	3575336195835	240876980635920	19				
20	20	236355	3111845478	1606501010448	180447963304932	20				
21	21	2137	388258392	532980084456	107652735435760	21				
22	22	26439132	125597876265	50248464621030	22					
23	23	782568	19923687045	17940646863180	23					
24	24	5761	1967009967	4758338490265	24					
25	25	106725843	901698646326	25						
26	26	2535462	115693106865	26						
27	27	15162	9292662540	27						
28	28	412551735	412551735	28						
29	29	8055690	8055690	29						
30	30	39437	39437	30						
Z	Z	1092	35368	1160304	38076560	1245921312	40794590352	1337315574392	43851965148664	1438010024969172

$\langle F_{4,n} \rangle$

k	2	3	4	5
0	1	1	1	1
1	24	36	48	60
2	228	558	1032	1650
3	1096	4884	13136	27580
4	2830	26613	110244	313335
5	3848	93888	643056	2563272
6	2516	216232	2677896	15606880
7	632	320304	8052432	72103560
8	33	293640	17486550	255402695
9	155376	27158096	696271420	1910
10	42024	29567736	1458478346	9638040
11	4608	21843888	2331602300	8283018
12	116	10417700	2810529565	4565790
13	2969616	2508257920	17	1497215
14	446136	1614765000	14	257655
15	27952	722947040	15	18390
16	417	213633125	16	306
17	38550060	3748850	17	213158635
18	155820	1537	18	38371340
19			19	3716110
20			20	153900
			21	113805535505
			22	31513745925
			23	5846012675
			24	671366475
			25	42086325
Z	11208	1158280	121415936	12744960016

 $\langle F_{5,n} \rangle$

n	2	3	4	5
0	1	1	1	1
1	30	45	60	75
2	375	900	1650	2625
3	2540	10560	27580	56975
4	10160	80820	313335	859575
5	24556	424404	2563272	9576465
6	35300	1566380	15606880	81704700
7	28620	4094700	72103560	546394200
8	11715	7545000	255402680	2907760425
9	1910	9638040	696271000	12433208675
10	65	8283018	1458473312	42951732165
11		4565790	2331568440	120150134775
12		1497215	2810388780	271986911900
13		257655	2507882120	496620717300
14		18390	1614116760	727037820225
15		306	722236584	845830322875
16			213158635	772486850025
17			38371340	544884735075
18			3716110	290328877050
19			153900	
20			1569	
21			21	
22			22	
23			23	
24			24	
25			25	
Z	115272	3798324	12742357568	4280337572704

$\langle F_{6,n} \rangle$

n	2	3	4	5
0	1	1	1	1
1	36	54	72	90
2	558	1323	2412	3825
3	4884	19476	49944	102120
4	26619	192321	716238	1922040
5	94008	1346922	7554456	27130608
6	217188	6898443	60745956	298318710
7	324168	26261568	380943720	2620008540
8	301947	74800755	1890305403	18698361570
9	164468	159239754	7488236848	109751007080
10	46518	251359929	23790815544	534248349108
11	5364	289770732	60689909904	2168826688200
12	129	238272879	124065225844	7367240543520
13	134989110	202235517648	20970983050320	1341081945
14	50080221	260751624648	50012081835600	210089878166
15	11263776	262860562832	99746461790856	745735008732
16	1354212	203940502299	165818580932265	165837
17	68976	19238352296	228625110674370	8162021945
18	860	51079497644	25968338820475	2190729528967
19	1543200312	240877802977260	7151421	63
20	3117126630	180449370275328	6415143	4013921751
21	389155256	107654540816700	1621959087	17686280206784
22	26494788	50250164915610	435202005	23923358619559
23	782952	17941782052680	7247674	26269302257660
24	5793	4758845022485	2442	23172693654082
25	901832681178	115707107955	3662806599776	16201666860764
26	9290859220	411982815	1125406391228	8824980621975
27	28	8013690	3830372	246283883888
28	30	38405	22017	36389256645
Z	1185888	1245921312	1337446129440	1438018075662624

 $\langle F_{7,n} \rangle$

n	2	3	4
0	0	1	1
1	1	42	63
2	2	777	1827
3	3	8344	32361
4	4	57715	391734
5	5	269822	3435894
6	6	870695	22590442
7	7	1946720	113633886
8	8	2985479	442473423
9	9	3065286	1341081945
10	10	2019591	3165426117
11	11	795704	579557971
12	12	165837	8162021945
13	13	14322	8725638243
14	14	257	6948059409
15	15	4013921751	10698983792568
16	16	435202005	17686280206784
17	17	7151421	4013921751
18	18	6415143	16201666860764
19	19	247674	8824980621975
20	20	2442	246283883888
21	21	3662806599776	36389256645
22	22	1125406391228	3357243652
23	23	3830372	170865282
24	24	22017	3830372
Z	Z	12200592	40869844784
			14038468424416

$\langle F_{8,n} \rangle$

n	2	3	4
0	1	1	1
1	48	72	96
2	1032	2412	4368
3	13136	49944	125344
4	110252	716238	2546568
5	643312	7554456	38982240
6	2681432	60745964	467187280
7	8080080	380944104	4496780832
8	17621382	1890313797	35383290668
9	27584016	7488347520	230493058656
10	30444600	23791797936	1254276967120
11	2300048	60696114816	5737881378976
12	11354860	124093960112	22159666611864
13	3403728	202334584800	72427671544928
14	546920	261007268592	200538820952080
15	36848	263353038048	470192235018656
16	513	204641780472	931971456951974
17	119963760480	1556993285226528	2182996069016752
18	51610479952	2553864446810080	13273908
19	15696980064	470192235018656	1700487
20	3202701096	2474647079588624	92142
21	405754656	1967765435577376	1025
22	28222224	1269321665415792	6073708904712
23	863616	654769485163616	3601167633972
24	7076	265308395783980	1625403413718
25	82552153598752	542963812482	145751585401490432
26	19157900533072	129262837230	225117609209324384
27	3190221128672	20811409218	232201744986836672
28	361437597480	2098828611	201438472635419072
29	25785221920	117499167	145751585401490432
30	1025083184	2946942	87070436385340160
31	18200032	20706	42417843375381744
32	84609	1236405554563	16598550518781174
Z	125522208	1340655988448	5120165961719438
Z	14735507295752320	3718117213	1215961617990401
Z	34	45286467	215645130108434
Z	35	97401	27424502635264
Z	36	1291399392	43977045561472
Z	37	1415293621827663360	11763609219

 $\langle F_{9,n} \rangle$

n	2	3	4
0	1	1	1
1	54	81	108
2	1323	3078	5560
3	19476	72954	181640
4	192330	1209573	4229249
5	1347264	14913477	74728946
6	6904170	141910416	1041827350
7	26317260	1067609916	11763609219
8	75149568	6453419418	109581706664
9	160714668	31684044186	853529413875
10	255651804	127225961604	5614244070561
11	298375596	419466096072	31415963142250
12	250001598	1137146343237	150360537603792
13	145559376	253267226593	617867219052629
14	56097342	4620754380402	2185336147625512
15	13273908	6871045048650	6661849025210584
16	1700487	8266622625666	17509517544916340
17	92142	7967812389390	3965458289597992
18	1025	6073708904712	77266304081824416
19	3601167633972	129211648432832960	184820215070498560
20	1625403413718	184820215070498560	
21	21	542963812482	
22	22	223201744986836672	
23	23	20811409218	
24	24	2098828611	
25	25	117499167	
26	26	2946942	
27	27	20706	
28	28	1236405554563	
29	29	3718117213	
30	30	45286467	
31	31	97401	
Z	32	1291399392	43977045561472
Z	33	1415293621827663360	11763609219
Z	34	3718117213	
Z	35	45286467	
Z	36	97401	

Tables of Multilayered Cyclic Fence Graphs

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 $\langle F_{10,n}, F_{15,n} \rangle$

	$F_{10,2}$	$F_{10,3}$	$F_{15,2}$
0	1	1	1
1	60	90	90
2	1650	3825	3825
3	27580	102120	102120
4	313345	1922040	1922055
5	2563712	27130608	27131718
6	15615560	298318720	298357155
7	72205040	2620009200	2620836960
8	256185710	18698381880	18710813985
9	700473640	109751394240	109889772890
10	1474583652	534253471992	53543769657
11	2376159320	2168876655240	2176855106040
12	2899464590	7367612905670	7410596225495
13	2635004800	20973151573860	21160218536550
14	1741136600	50022093162150	5068384796675
15	808269264	99783417443136	101692058299456
16	250746465	165928125907680	170418184605195
17	48168540	22886055223360	237484393614750
18	5048490	260181695931200	273527414200235
19	225260	241636497159480	258313971369240
20	2049	181362453960660	197986677435597
21		108512704840080	121568897195490
22		50869922172060	58819936007505
23		18278509414200	21953690108640
24		4892744336185	6147272105235
25		939416869362	1243878815406
26		122800802265	172573654875
27		10132504560	15178134920
28		468894540	743670165
29		9894960	15728610
30		61100	65537
Z	13286195328	1442602340446464	1531443389475072

$\langle G_{2,n} \rangle$

n	2	3	4	5	6	7	8	9	10	11	12	n		
0	1	1	1	1	1	1	1	1	1	1	1	0		
1	12	18	24	30	36	42	48	54	60	66	72	1		
2	42	117	228	375	558	777	1032	1323	1650	2013	2412	2		
3	44	336	1096	2540	4884	8344	13136	19476	27580	37664	49944	3		
4	9	417	2834	10160	26613	57708	110244	192321	313335	483978	716238	4		
5	186	3880	24556	93888	269640	643056	1346922	2563272	4531164	7554456	7554456	5		
6	2588	35300	216240	868714	2677896	6898437	15606880	32001024	60745956	60745956	60745956	6		
7	696	28620	320400	1934988	8052432	26261388	72103560	174157764	380943720	380943720	380943720	7		
8	49	1175	294045	2944403	17486562	74798415	255402680	739811391	1890305397	1890305397	1890305397	8		
9	1910	156116	2978850	27158288	159222486	696271000	2469866586	7488236608	9	9	9	9		
10	65	42606	1913121	29568936	251280405	1458473312	6494419701	23790811224	10	10	10	10		
11	4788	723968	21847664	289534176	2331568440	13424571936	60689863680	11	11	11	11	11		
12	129	142737	10424188	237817920	2810388780	21689646946	124064898776	12	12	12	12	12		
13	11690	2975824	134437896	2507882120	27123036292	202233907296	13	13	13	13	13	13		
14	229	449288	49682628	1614116760	25876353580	260745971880	14	14	14	14	14	14		
15	28656	11109408	722236584	18460678148	262846304512	15	15	15	15	15	15	15		
16	449	1326861	213158635	9582343804	203914781589	16	16	16	16	16	16	16		
17	67230	38371340	3484980356	119205719880	17	17	17	17	17	17	17	17		
18	809	3716110	842070834	51051262004	18	18	18	18	18	18	18	18		
19	153900	124986224	15416167416	19	19	19	19	19	19	19	19	19		
20	1569	1005881	3111845478	20	20	20	20	20	20	20	20	20		
21	21	348546	388258392	21	21	21	21	21	21	21	21	21		
22	22	2953	26439132	22	22	22	22	22	22	22	22	22		
23	23	782568	5761	23	23	23	23	23	23	23	23	23		
24	24	Z	108	1092	11396	115272	1160304	11855212	121437700	1243998156	12742357568	130534386852	1337315574392	Z

$$< G_{2,n} >$$

n	13	14	15
0			
1	78	1	1
2	2847	3318	90
3	64636	81956	3825
4	1023399	1420041	102120
5	12009582	18353328	1922040
6	108366739	183687756	27130608
7	769925364	1458893616	298318710
8	4375205874	9348374868	262008540
9	20091038448	48878124736	18698361570
10	75022424620	210089869472	10975107080
11	228524538320	745734882256	534248349162
12	568118887258	2190728276170	2168826690900
13	1150449336588	5326541834600	7367240605310
14	1889490837806	10698942197636	20970983908680
15	2499906715280	17686088154840	50012089929300
16	2639167693383	23922765345955	99746516669776
17	2195048462698	26267896108692	228626166554220
18	1414528182951	23170146037542	259686473955200
19	691058927832	16198171828468	240884731231740
20	248717661283	8821406775657	165818857487010
21	63454898754	3660145844448	180461300489337
22	10878570521	1124011843080	107670190599390
23	1158973140	245793493904	50265619392945
24	67770339	36281200333	17953070506620
25	1732198	3343634308	4764795147615
26	11029	169970710	90401993466
27		3798116	116241663345
28		21345	9371815520
29			418779285
30			8273250
	Z	13700953260968	140368150057236
			14380925568937952
			41297

$\langle G_{3,n} \rangle$

n	2	3	4	5	6	7	8	9	10	n
0	1	1	1	1	1	1	1	1	1	0
1	18	27	36	45	54	63	72	81	90	1
2	117	297	558	900	1323	1827	2412	3078	3825	2
3	336	1719	4884	10560	19476	32361	49944	72854	102120	3
4	417	5643	26613	80820	192321	391734	716238	1209573	1922040	4
5	186	10557	93888	424404	1346922	3435894	7554456	14913477	27130608	5
6	17	10740	216232	1566380	6898455	22590456	60745964	141910407	298318710	6
7	5328	320304	4094700	26261928	113634432	380944104	1067609403	2620008540	7	
8	1035	293637	7545000	74805381	442482852	1890313797	6453405945	18698361570	8	
9	45	155364	9638040	159273174	1341177152	7488347520	31683827853	109751007080	9	
10	42006	8283021	251509761	3166051371	23791797936	127223585523	534248349162	10		
11	4565805	290205144	5798369325	60696114816	419447173095	2168826690900	11			
12	113	1497245	239097000	8170897140	124093960112	113703331875	7367240605310	12		
13	257685	136003104	8745496620	202334584800	2532155915907	20970983908680	13			
14	18405	309	50867154	6979486899	261007268592	4618929774213	500112089929300	14		
15	11629284	4048672257	263353038048	6866034125373	99746516669776	15				
16	1445967	1648192644	204641780469	8255934355041	165818857487010	16				
17	79038	448221480	119963760456	7950199463001	228626166554220	17				
18	1133	75741918	51610479868	6051510288300	259686473955200	18				
19	7092330	15696979896	3580091744040	240884731231740	19					
20	299040	3202700886	1610646422778	180461300489337	20					
21	3432	405754488	535558201146	107670190599390	21					
22	2822140	126698035709	50265619392945	17953070506620	22					
23	863592	20228107833	2017137582	4764795147615	24					
24	7073	110974374	90401993466	116241663345	25					
25	2670381	15273	9371815520	418779285	28					
26	8273250	41297	8273250	41297	30					
Z	1092	35392	1158232	37983320	1249636620	41012271228	1340655987680	4387318427216	1438092568937952	Z

$< G_{4,n} >$

n	2	3	4	5
0	1	1	1	1
1	24	36	48	60
2	228	558	1032	1650
3	1096	4884	13136	27580
4	2830	26613	110244	313335
5	3848	93888	643056	2563272
6	2516	216232	2677896	15606880
7	632	320304	8052432	72103560
8	33	293640	17486558	255402695
9	155376	27158224	696271420	9
10	42024	29568536	1458478346	10
11	4608	21846384	2331602300	11
12	116	10421844	2810529565	12
13	2973264	2508257920	447704	13
14	1614765000	1614765000	14	14
15	28208	722947040	417	15
16	417	213633125	18	16
17	38550060	3748850	19	17
18	155820	155820	20	18
19	1537	1537	21	19
20			22	20
Z	11208	1158280	121428984	23
			12744960016	24
				25

 $< G_{5,n} >$

n	2	3	4	5
0	1	1	1	1
1	30	45	60	75
2	375	900	1650	2625
3	2540	10560	27580	56975
4	10160	80820	313335	859575
5	24556	424404	2563272	9576465
6	35300	1566380	15606880	81704700
7	28620	4094700	72103560	546394200
8	11715	7545000	255402680	2907760425
9	1910	9638040	696271000	12433208675
10	65	8283021	1458473312	42951732180
11	4565805	2331568440	120150135300	
12	1497245	2810388780	271986920050	
13	257685	2507882120	496620791350	
14	18405	1614116760	727038266825	
15	309	722236584	845832091855	
16	213158635	772491848200		
17	38371340	38371340	544894690100	
18	3716110	3716110	290542765925	
19	153900	153900	113818867875	
20	1569	1569	31522270430	
21	5849448850	5849448850	672159525	
22	42172675	42172675	1145975	
23	7633	7633		
Z	115272	37983320	12742307568	4280394878464

$\langle G_{6,n} \rangle$

n	2	3	4	5
0	1	1	1	1
1	36	54	72	90
2	558	1323	2412	3825
3	4884	19476	49944	102120
4	26619	192321	716238	1922040
5	94008	1346922	7554456	27130608
6	217188	6898443	60745956	298318710
7	324168	26261568	380943720	2620008540
8	301947	7480755	1890305403	18698361570
9	164468	159239754	7488236848	109751007080
10	46518	251359929	23790815544	534248349108
11	5364	289770732	60689909904	2168826688200
12	129	238272876	124065225828	7367240543520
13	134989092	202235517264	2097098305320	3065286
14	50080176	260751620712	50012081835600	2019591
15	11263716	262860540240	99746461790856	11
16	1354167	263940422187	165818580932265	795704
17	68958	119238169320	228625110674370	12
18	857	51079224620	259683388200475	165337
19	15431734968	240877802977260	165818580932265	13
20	3116961654	180449370275328	165818580932265	14
21	389092536	107654540816700	165818580932265	14322
22	26481636	50250164915610	165818580932265	15
23	781800	17941782052680	165818580932265	257
24	5793	4758845022485	165818580932265	4013921751
25	901832681178	9290859220	165818580932265	1621959087
26	115707107955	411982815	165818580932265	435202005
27	28	8013690	38405	71751421
28	30	12200592	40869844784	6415143
Z	11858888	1245921120	1337445059056	1438018075662624

 $\langle G_{7,n} \rangle$

n	2	3	4
0	1	1	1
1	42	63	84
2	777	1827	3318
3	8344	32361	81956
4	57715	391734	1420041
5	269822	3435894	18353328
6	870695	22590442	183687756
7	1946720	113633886	1458893616
8	2985479	442473423	9348374875
9	3065286	1341081945	48878125100
10	2019591	3163426117	210089878166
11	795704	579557971	745735008732
12	165337	8162021945	2190729528967
13	14322	8725638243	5326550782224
14	257	6948059409	10698989792600
15	4013921751	17686280207680	17686280207680
16	1621959087	23923358630535	23923358630535
17	435202005	26269302335164	26269302335164
18	71751421	23172694004194	23172694004194
19	6415143	16201667923420	16201667923420
20	247674	8824982839799	8824982839799
21	2442	3662809808032	3662809808032
22	22	1125409599132	1125409599132
23	23	246286072816	246286072816
24	24	36390249189	36390249189
25	25	3357528132	3357528132
26	26	170912098	170912098
27	27	3833956	3833956
28	28	22081	22081
Z	12200592	40869844784	140384697896992

< $G_{8,n}$ >

n	2	3
0	1	1
1	48	72
2	1032	2412
3	13136	48944
4	110252	716238
5	643312	7554456
6	2681432	60745964
7	8080080	380944104
8	17621382	1890313797
9	27584016	7488347520
10	30444600	23791797936
11	23000048	60696114816
12	11354860	124093960112
13	3403728	202334584800
14	546920	26107268592
15	36848	263353038048
16	513	204641780469
17		119963760456
18		51610479868
19		15696979896
20		3202700886
21		405754488
22		28222140
23		863592
24		7073
Z	125522208	1340655987680

< $G_{9,n}$ >

n	3	4
0	1	1
1	54	81
2	1323	3078
3	19476	72954
4	192330	1209573
5	1347264	14913477
6	6904170	141910416
7	26317260	1067609916
8	75149568	6453419418
9	160714668	31684044186
10	255651804	127225961604
11	298375596	419466096072
12	250001598	1137146343237
13	145559376	2532672226593
14	56097342	4620754380402
15	13273908	6871045048650
16	1700487	8266622625666
17	92142	7967812389390
18		1025
19		
20		
21		
22		
23		
24		
Z	1291399392	43977645563008

< $G_{10,n}$ >

n	2	3
0	1	1
1	60	90
2	1650	3825
3	27580	102120
4	313345	1922040
5	2563712	27130608
6	15615560	288318720
7	72205040	2620009200
8	256185710	18698381880
9	70473640	109751394240
10	1474583652	53453471992
11	2376159320	2168376655240
12	2899464590	7367612905670
13	2635004800	20973151573860
14	1741136600	50022093162150
15	808269264	99783417443136
16	250746465	165928125907680
17	48168540	22886055223360
18	5048490	260181695931200
19	225260	241636497159480
20	2049	181362453960660
21	10851270484080	
22	50869922172060	
23	18278509414200	
24	4892744336185	
25	939416869362	
26	122800802265	
27	10132504560	
28	468894540	
29	9894960	
30	61100	
Z	13286195328	1442602340446464

$\langle D_{2,n} \rangle$

n	2	3	4	5	6	7	8	9	10	11	12	n
0	1	1	1	1	1	1	1	1	1	1	1	0
1	12	18	24	30	36	42	48	54	60	66	72	1
2	42	117	228	375	558	777	1032	1323	1650	2013	2412	2
3	44	336	1096	2540	4884	8344	13136	19476	27580	37664	49944	3
4	9	417	2830	10160	26619	57715	110252	192330	313345	483989	716250	4
5	186	3848	24556	94008	269822	643312	1347264	2563712	4531714	7555128	5	
6	2516	35300	217188	870695	2681432	6904170	15615560	32013509	60763212	6		
7	632	28620	324168	1946720	8080080	26317260	72205040	174328176	381213192	7		
8	33	11715	301947	2985479	17621382	75149568	256185710	741372786	1893167103	8		
9	1910	164468	3065286	27584016	160714668	700473640	2480019652	7510130896	9			
10	46518	2019591	30444600	255651804	1474583652	6542724474	23915539416	10				
11	5364	795704	2300048	298375596	2376159320	13595438720	61229199600	11				
12	129	165837	11354860	250001598	2899464590	22141847002	125853276268	12				
13	14322	3403728	145559376	2635004800	28017226292	20679793968	13					
14	257	546920	56097342	1741136600	27186751482	269695069560	14					
15	36848	13273908	808269264	19862312624	276236810768	15						
16	513	1700487	250746465	10651140973	219023353023	16						
17	92142	48168540	4046192282	131832433896	17							
18	1025	5048490	1034840521	58670965628	18							
19	225260	225260	164994368	18618773496	19							
20	2049	14440657	4001581818	540650	539065304	21						
21	21	4097	40059132	4077928	8193	23						
22	22											
23	23											
24	24											
Z	108	1092	11208	115272	1185888	12200592	125522208	1291399392	13286195328	136691243712	1406309006208	Z

$$< D_{2,n} >$$

n	13	14	15
0	1	1	1
1	78	84	90
2	2847	3318	3825
3	64636	81956	102120
4	1023412	1420055	1922055
5	12010388	18354280	27131718
6	108389840	183717884	298357155
7	770331692	1459483128	2620836960
8	4380116273	9356366229	18710813985
9	20134291658	48957825308	109889772890
10	75309934635	210696056466	53543769657
11	229997698424	749331587228	2176855106040
12	574012044736	2207618499667	7410596225495
13	1168979639288	5339873950832	21160218534550
14	1935385157992	10889424243880	50683847961675
15	2589266354168	18146139782288	101692058299456
16	2775171810723	24813266322579	170418184605195
17	2355304795714	27642631056012	237484393614750
18	1558685397685	24849438864922	273527414200235
19	788146742604	17803105834332	258313971369240
20	296385250252	10003251473621	197986677435597
21	79897249588	4317126971432	121568897195490
22	14657785480	1392251021532	58819936007505
23	1693071900	323269828728	21953690108640
24	108306731	51289732375	6147272105235
25	2981862	5142607876	1243878815406
26	286490974	172573654875	
27	6881252	15178134920	
28	32769	743670165	
29		15728610	
30		65537	
Z	14468410468992	148854128491008	1531443389475072

$\langle D_{3,n} \rangle$

n	2	3	4	5	6	7	8	9	10	n
0	1	1	1	1	1	1	1	1	1	0
1	18	27	36	45	54	63	72	81	90	1
2	117	297	558	900	1323	1827	2412	3078	3825	2
3	336	1719	4884	10560	19476	32361	49944	72954	102120	3
4	417	5643	26613	80820	192321	391734	716238	1209573	1922040	4
5	186	1057	93888	424404	1346922	3435894	7554456	14913477	27130608	5
6	17	10737	216232	1566380	6898443	22590442	60745964	141910416	298318720	6
7	5319	320304	4094700	26261568	113633886	38094104	1067609916	2620009200	7	
8	1026	293637	7545000	74800755	442473423	1890313797	6453419418	18698381880	8	
9	42	155364	9638040	159239754	1341081945	7488347520	31684044186	109751394240	9	
10	42006	8283021	25135929	3165426117	2379179736	127225961604	534253471992	10		
11	4596	4565805	289770732	5795557971	60696114816	419466096072	2168876655240	11		
12	113	1497245	238272879	8162021945	124093960112	1137146343237	7367612905670	12		
13	257685	134989110	8725638243	202334584800	2532672226593	20973151573860	13			
14	18405	50080221	6948059412	261007268592	4620754380402	50022093162150	14			
15	309	11263776	4013921772	263353038048	6871045048650	99783417443136	15			
16	1354212	1621959150	204641780469	8266622625666	165928125907680	16				
17	68976	435202110	119963760456	7967812389390	22888605523360	17				
18	860	71751526	51610479868	6073708904712	260181695931200	18				
19	6415206	247695	15696979896	3601167633972	241636497159480	19				
20	2445	3202700886	1625403413718	181362453960657	108512704840050	20				
21	405754488	542963812482	129262837230	50869922171925	21					
22	28222140	863592	20811409218	18278509413840	22					
23	7073	2098828611	4892744335555	23						
24	117499167	939416868606	939416868606	24						
25	2946942	2946942	122800801635	25						
26	27	20706	10132504200	26						
27	28	468894405	468894405	27						
28	29	9894930	9894930	28						
29	30	61097	61097	29						
30				30						

$$< D_{4,n} >$$

n	2	3	4	5	6	7
0	1	1	1	1	1	1
1	24	36	48	60	72	84
2	228	558	1032	1650	2412	3318
3	1096	4884	13136	27580	49944	81956
4	2830	26613	110244	313335	716238	142041
5	3848	93888	643056	2563272	7554456	18353328
6	2516	216232	2677896	15606880	60745956	183687756
7	632	320304	8052432	72103560	380943720	1458893616
8	33	293637	17486550	255402680	1890305403	9348374875
9	155364	27158096	696271000	7488236848	48878125100	
10	42006	29567736	1458473296	23790815544	210089878166	
11	4596	21843888	2331568120	60689909904	745735008732	
12	113	10417700	2810386140	12405225844	2190729528967	
13		2969616	2507870440	202235517648	5326550782224	
14		446136	1614086520	260751624648	10698989792600	
15		27952	722189480	262860562832	17686280207680	
16		417	213114635	203940502299	23923358630535	
17		38347340	119238352296	26269302335164		
18		3708990	51079497644	23172694004194		
19		152940	15432000312	16201667923420		
20		1537	3117126630	8824982839799		
21		389155256	3662809808032			
22		26494788	1125409599132			
23		782952	246286072816			
24		5793	36390249189			
25			3357528132			
26			170912098			
27			3833956			
28			22081			
Z	11208	1158232	121415936	12742189456	1337446129440	140384697896992

$\langle D_{5,n} \rangle$

n	2	3	4	5	6
0	1	1	1	1	1
1	30	45	60	75	90
2	375	900	1650	2625	3825
3	2540	10560	27580	56975	102120
4	10160	80820	313335	859575	1922040
5	24556	424404	2563272	9576465	27130608
6	35300	1566380	15606880	81704700	298318710
7	28620	4094700	72103560	546394200	2620008540
8	11715	7545000	255402680	2907760425	18698361570
9	1910	9638040	69627100	12433208675	109751007080
10	65	8283021	1458473296	42951732165	534248349108
11	4665805	2331568120	120150134775	2168826688200	
12	1497245	2810386140	271986911900	7367240543480	
13	257685	2507870440	496620717300	20970983048640	
14	18405	1614086520	727037829225	50012081803650	
15	309	722189480	845830323875	99746461426516	
16	213114635	772486850025	165818578151460		
17	38347340	544884735075	228625095657300		
18	3708990	290528877050	259683329048900		
19	152940	113805535050	240877630273740		
20	1537	31513745925	180448994087793		
21	671366475	5846012675	107653930051710		
22	42086325	671366475	50249431282395		
23	1143525	17941139140800			
24	7623	4758442711185			
25		901658490618			
26		115657326855			
27		9282152420			
28		411171375			
29		7984770			
30		38405			
Z	115272	37983320	12742189456	4280337572704	1438014826283904

$\langle X_{2,n} \rangle$

n	4	5	6	7	8	9	10	11	12	13	14	15	16	17	n
0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
1	12	15	18	21	24	27	30	33	36	39	42	45	48	51	1
2	42	75	117	168	228	297	375	462	558	663	777	900	1032	1173	2
3	44	145	336	644	1096	1719	2540	3586	4884	6461	8344	10560	13136	16099	3
4	9	95	417	1218	2826	5643	10155	16929	26613	39936	57708	80820	110244	147033	4
5	13	186	1050	3816	10557	24486	50193	93888	163644	269640	424404	643056	943551	15	
6	17	336	2444	10740	34945	93258	216240	451438	868728	1566390	2677912	4379523	6		
7	7	24	600	5328	27840	105270	320400	834834	1935240	4094910	8052816	14912349	7		
8	8	33	1035	11045	67672	294045	1012453	2946188	7546785	17490394	37393574	8			
9	9	45	1750	22044	156116	772031	2985192	9645985	27178960	68736338	9				
10	10	65	2915	42606	344981	1925028	8303046	29634536	91455835	10					
11	11	91	4788	80223	735392	4594710	21972528	86198357	11						
12	12	129	7787	147784	1520375	10563644	55699803	12							
13	13	185	12544	267195	3061776	23511629	13								
14	14	264	2055	475416	6026942	14									
15	15	379	31856	834190	15										
16	16	545	50303	16											
17	17	785	785	785	785	785	785	785	785	785	785	785	785	785	17

 $\langle X_{3,n} \rangle$

Z	108	344	1092	3462	11068	35392	113232	362454	1160304	3714676	11892872	38076560	121907900	390308036	Z	
0	5	6	7	8	9	10	11	12	13	14	15	16	17	n		
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	
2	15	18	21	24	27	30	33	36	39	42	45	48	51	1		
3	75	117	168	228	297	375	462	558	663	777	900	1032	1173	2		
4	145	336	644	1096	1719	2540	3586	4884	6461	8344	10560	13136	16099	3		
5	95	417	1225	2830	5643	10155	16929	26613	39936	57708	80820	110244	147033	4		
6	13	186	1085	3848	10557	24486	50193	93888	163644	269640	424404	643056	943551	5		
7	17	371	2516	10740	34945	93258	216232	451425	868714	1566375	2677896	4379506	6			
8	31	632	5328	27840	105270	320304	834639	1934988	4094595	8052432	14911890	7				
9	33	1035	11045	67672	293637	1011348	2944403	7544100	17486558	37388321	8					
10	45	1750	22044	153364	769080	2978850	9634000	27158224	68703035	9						
11	91	4596	78117	723968	4552050	191321	8272926	29568536	91327570	10						
12	113	7345	142737	1487960	10421844	55232796	11690	255240	2973264	23085456	13					
13	159	229	304	18240	447704	5805976	417	775608	775608	15						
14	229	304	304	28208	447704	5805976	417	43707	43707	16						
15	304	304	304	28208	447704	5805976	417	581	581	17						
16	304	304	304	28208	447704	5805976	417	581	581	17						
17	344	1092	3546	11208	35392	113232	362454	1158232	3704068	11855212	37942520	121428984	388651556	Z		

$\langle X_{4,n} \rangle$

n	6	7	8	9	10	11	12	13	14	15	16	17	n
0	1	1	1	1	1	1	1	1	1	1	1	1	0
1	18	21	24	27	30	33	36	39	42	45	48	51	1
2	117	168	228	297	375	462	558	663	777	900	1032	1173	2
3	336	644	1096	1719	2540	3586	4884	6461	8344	10560	13136	16099	3
4	420	1218	2830	5652	10160	16929	26613	39936	57708	80820	110244	147033	4
5	192	1050	3848	10656	24556	50193	93888	163644	269640	424404	643056	943551	5
6	20	336	2516	11097	35300	93258	216232	451438	868728	1566380	2677896	4379506	6
7	24	632	5823	28620	105270	320304	834834	1935240	4094700	8052432	14911890	7	
8	33	1278	11715	67672	293640	1012453	2946188	7545000	17486558	37388321	8		
9	78	1910	22044	155376	772031	2985192	9638040	27158224	68703035	9			
10	65	2915	42024	344981	1925028	8283018	29568536	91327570	10				
11	91	4608	80223	735392	4565790	21846384	85889202	11					
12	116	7787	147784	1497215	10421844	55232796	12						
13	13												
14	14												
15	15												
16	16												
17	17												
Z	1104	3462	11208	36628	115272	362454	1158280	3714676	11892872	37983224	121428984	388651556	Z

 $\langle X_{5,n} \rangle$

n	7	8	9	10	11	12	13	14	15	16	17	n
0	1	1	1	1	1	1	1	1	1	1	1	0
1	21	24	27	30	33	36	39	42	45	48	51	1
2	168	228	297	375	462	558	663	777	900	1032	1173	2
3	644	1096	1719	2540	3586	4884	6461	8344	10560	13136	16099	3
4	1225	2826	5643	10160	16940	26619	39936	57708	80820	110244	147033	4
5	1085	3816	10557	24556	50380	94008	163644	269640	424404	643056	943551	5
6	371	2444	10740	35300	94457	217188	451438	868714	1566380	2677912	4379523	6
7	600	5328	28620	108933	324168	834834	1934988	4094700	8052816	14912349	7	
8	33	1035	11715	73238	301947	1012453	294403	754500	17490394	37393574	8	
9	45	1910	26070	164468	772031	2978850	9638040	27178960	68736338	9		
10	65	4125	46518	344981	1913121	8283021	29634536	91455835	10			
11	201	5364	80223	723968	4565805	21972528	86198857	11				
12	129	7787	142737	1497245	1053644	5569803	12					
13	185	11690	257685	3061776	23511629	13						
14	229	18405	475416	6026942	14							
15	309	31856	834190	15								
16	545	50303	16									
17	785	17										
Z	3546	11068	36392	115272	378426	1185888	3714676	11855212	37983320	121907900	390308036	Z

Table 2. List of graphs isomorphic to $E_{m,n}$.

$m \setminus n$	2	3	4	5
1	$F_{1,2}, G_{1,2}$	$F_{1,3}, G_{1,3}$	See $E_{2,2}$	$F_{1,5}, G_{1,5}$ $X_{2,5}, X_{3,5}$
2	$E_{1,4}, F_{1,4}, F_{2,2}, G_{1,4},$ $G_{2,2}, D_{2,2}, X_{2,2}$	See $E_{3,2}$	$X_{2,8}, X_{5,8}$	$F_{2,5}, X_{2,10},$ $X_{3,10}$
3	$E_{2,3}, F_{3,2}, G_{2,3}, G_{3,2},$ $D_{2,3}, D_{3,2}, X_{2,6}, X_{3,6}$	$G_{3,3}, X_{2,9}, X_{3,9},$ $X_{5,9}$	$G_{3,4}, D_{3,4}$ $D_{4,3}, X_{3,12}$	$X_{3,15}$
4	$F_{2,4}, F_{4,2}, G_{4,2}, D_{2,4},$ $D_{4,2}, X_{3,8}, X_{4,8}$	$F_{4,3}, G_{4,3}, X_{4,12}$	$G_{4,4}, X_{3,16},$ $X_{4,16}$	$D_{4,5}, D_{5,4}$
5	$F_{5,2}, G_{2,5}, G_{5,2}, D_{2,5},$ $D_{5,2}, X_{4,10}, X_{5,10}$	$G_{3,5}, G_{5,3}, D_{3,5},$ $D_{5,3}, X_{5,15}$	$F_{5,4}, G_{2,10},$ $G_{5,4}$	$G_{5,5}$
6	$F_{6,2}, G_{6,2}, D_{2,6}, X_{5,12}$	$G_{6,3}$	*	$F_{6,5}, G_{6,5}$ *
7	$F_{7,2}, G_{7,2}, D_{2,7}$	$F_{7,3}, G_{7,3}$	$G_{7,4}, D_{4,7}$ *	*
8	$F_{8,2}, G_{8,2}, D_{2,8}$	$G_{3,8}, G_{8,3}, D_{3,8}$ *	*	*
9	$F_{9,2}, G_{9,2}, D_{2,9}$	$G_{9,3}$	*	*
10	$F_{10,2}, G_{10,2}, D_{2,10}$	$F_{10,3}, G_{10,3}$ *	*	*

$m \setminus n$	6	7	8	9
1	$F_{1,6}, F_{2,3}, X_{4,6}$	$F_{1,7}, X_{3,7}, X_{5,7}$	$F_{1,8}, G_{2,4}$	$F_{1,9}, X_{4,9}$
2	$F_{3,4}, G_{2,6}, X_{2,12}$	$X_{2,14}, X_{4,14}$	$X_{2,16}, X_{5,16}$	$G_{3,6}$
3	$G_{2,9}$	$F_{3,7}$	*	$G_{3,9}$

* Not yet completely surveyed.

Table 3. List of isomorphic MLCFG's without $E_{m,n}$.

N	Isomorphic graphs	N	Isomorphic graphs
12	$F_{2,3}, X_{4,6}$	30	$F_{3,5}, X_{2,15}$ $F_{5,3}, X_{4,15}$
14	$X_{2,7}, X_{4,7}$	32	$F_{4,4}, D_{4,4}$
18	$F_{3,3}, D_{3,3}$	36	$F_{3,6}, F_{6,3}, D_{3,6}$
22	$X_{2,11}, X_{3,11}, X_{4,11}$	40	$F_{4,5}, G_{4,5}$
26	$X_{2,13}, X_{4,13}, X_{5,13}$	48	$F_{6,4}, D_{4,6}$
28	$F_{2,7}, G_{2,7}, X_{3,14}, X_{5,14}$	50	$F_{5,5}, D_{5,5}$