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Practical Method for Estimation of Ground Level Concentration of Matter Emitted from an Elevated Areal Source

Jiro Sakagami

Department of Physics, Faculty of Science,
Ochanomizu University, Tokyo
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Introduction. In order to discuss the air pollution problems in urban areas, it is necessary to calculate the concentration of matter emitted from elevated areal sources. So we intended to have a practical formula for such sources.

Formula. We used the author's diffusion formula[1], and the areal source is at the height of h and extends from $-L_1$ to L_2 in wind direction (x-direction) and from $-Y_1$ to Y_2 in cross wind direction (y-direction), and z-direction is vertically upward. (Fig. 1)

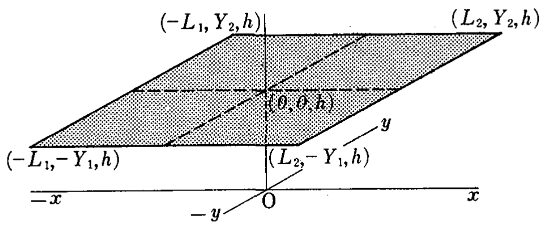

Fig. 1. Areal source and coordinate system.

The concentration is given by the next equation:

$$C=\frac{q}{u}\int_{-L_1}^{L_2}d\xi\int_{-Y_1}^{Y_2}\frac{e^{-\frac{(y-\eta)^2}{A}}}{\sqrt{A\pi}}\cdot\frac{e^{-\frac{h+z}{B}}}{B}J_0\left(i\frac{2\sqrt{hz}}{B}\right)d\eta \qquad (1)$$

$$\left.\begin{aligned} A(x)&=q_A(\varphi_A x+e^{-\varphi_A x}-1)\\ B(x)&=q_B(\varphi_B x+e^{-\varphi_B x}-1)\end{aligned}\right\} \qquad (2)$$

where q is the source intensity, u is the wind speed at the height of the source and $J_0(i\xi)$ is the first kind Bessel function of the order zero with imaginary argument, and q_A, q_B, φ_A, φ_B are the diffusion parameters[2].

For the ground level concentration, we have

$$C=\frac{q}{u}\int_{-L_1}^{L_2} d\xi \int_{-Y_1}^{Y_2} \frac{e^{-\frac{(y-\eta)^2}{A(x-\xi)}}}{\sqrt{A(x-\xi)\pi}} \frac{e^{-\frac{h}{B(x-\xi)}}}{B(x-\xi)} d\eta$$

$$=\frac{q}{u}\int_{x-L_2}^{x+L_1} d\zeta \int_{-Y_1}^{Y_2} \frac{e^{-\frac{(y-\eta)^2}{A(\zeta)}}}{\sqrt{A(\zeta)\pi}} \frac{e^{-\frac{h}{B(\zeta)}}}{B(\zeta)} d\eta \qquad (3),$$

where $x-\xi=\zeta$. In order to obtain a practical formula, we consider an approximation. The function $\exp(-h/B)/B$ has its maximum value when $h=B(\zeta_0)$, namely $\zeta_0=B^{-1}(h)$, where B^{-1} is the inverse function of B, we have

$$C \doteqdot \frac{q}{u\sqrt{\pi}}\int_{x-L_2}^{x+L_1} \frac{e^{-\frac{h}{B(\zeta)}}}{B(\zeta)} d\zeta \int_{-Y_1}^{Y_2} \frac{e^{-\frac{(y-\eta)^2}{A(\zeta_0)}}}{\sqrt{A(\zeta_0)}} d\eta$$

$$=\frac{q}{2u}\left[\Phi\left(\frac{y+Y_2}{\sqrt{A(\zeta_0)}}\right)-\Phi\left(\frac{y-Y_1}{\sqrt{A(\zeta_0)}}\right)\right]\int_{x-L_2}^{x+L_1} \frac{e^{-\frac{h}{B(\zeta)}}}{B(\zeta)} d\zeta \qquad (4),$$

where

$$\Phi(\zeta)=\frac{2}{\sqrt{\pi}}\int_0^{\zeta} e^{-t^2}\, dt \qquad (5).$$

Generally, the air pollution problems in urban areas are considered in large scale, we usually assume as

$$\zeta \gg 1, \quad \text{and} \quad Y_1,\ Y_2 \gg 1 .$$

$$\left.\begin{aligned} A(\zeta)&=q_A(\varphi_A\zeta+e^{-\varphi_A\zeta}-1)\doteqdot q_A\varphi_A\zeta \\ B(\zeta)&=q_B(\varphi_A\zeta+e^{-\varphi_A\zeta}-1)\doteqdot q_B\varphi_B\zeta \end{aligned}\right\} \qquad (6),$$

$$\Phi\left(\frac{y+Y_2}{\sqrt{A_0}}\right)-\Phi\left(\frac{y-Y_1}{\sqrt{A_0}}\right)\doteqdot 2 \qquad (7),$$

so, eq. (4) becomes

$$C=\frac{q}{u}\int_{x-L_2}^{x+L_1} \frac{e^{-\frac{h}{B(\zeta)}}}{B(\zeta)} d\zeta \qquad (8).$$

If we put $\dfrac{h}{q_B\varphi_B\zeta}=\lambda$, we have

$$\int_{x-L_2}^{x+L_1} \frac{e^{-\frac{h}{q_B\varphi_B\zeta}}}{q_B\varphi_B\zeta} d\zeta=\frac{1}{h}\int_{\frac{h}{q_B\varphi_B(x+L_2)}}^{\frac{h}{q_B\varphi_B(x-L_1)}} \lambda e^{-\lambda}\frac{h}{q_B\varphi_B}\frac{d\lambda}{\lambda^2}$$

$$=\frac{1}{q_B\varphi_B}\int_{\frac{h}{q_B\varphi_B(x+L_2)}}^{\frac{h}{q_B\varphi_B(x-L_1)}} \frac{e^{-\lambda}}{\lambda} d\lambda \qquad (9).$$

So we obtain

$$C=\frac{q}{u}\frac{1}{q_B\varphi_B}\int_{\frac{h}{q_B\varphi_B(x+L_2)}}^{\frac{h}{q_B\varphi_B(x-L_1)}}\frac{e^{-\lambda}}{\lambda}d\lambda$$

$$=\frac{q}{u}\left[-E_i\left(\frac{-h}{q_B\varphi_B(x+L_2)}\right)+E_i\left(\frac{-h}{q_B\varphi_B(x-L_1)}\right)\right] \qquad (10),$$

where

$$-E_i(-\mu)=\int_\mu^\infty \frac{e^{-t}}{t}dt \qquad (11).$$

This formula should be used as $x \geqq L_1$.

Example. We calculated the ground level concentrations at $y=0$ and $x=1\sim80$ km in two conditions of stability, namely $\zeta=0.4$ (stable) and $\zeta=-0.2$ (unstable). The results are shown in Fig. 2. The concentration in remote leeward region decrease as x^{-1}.

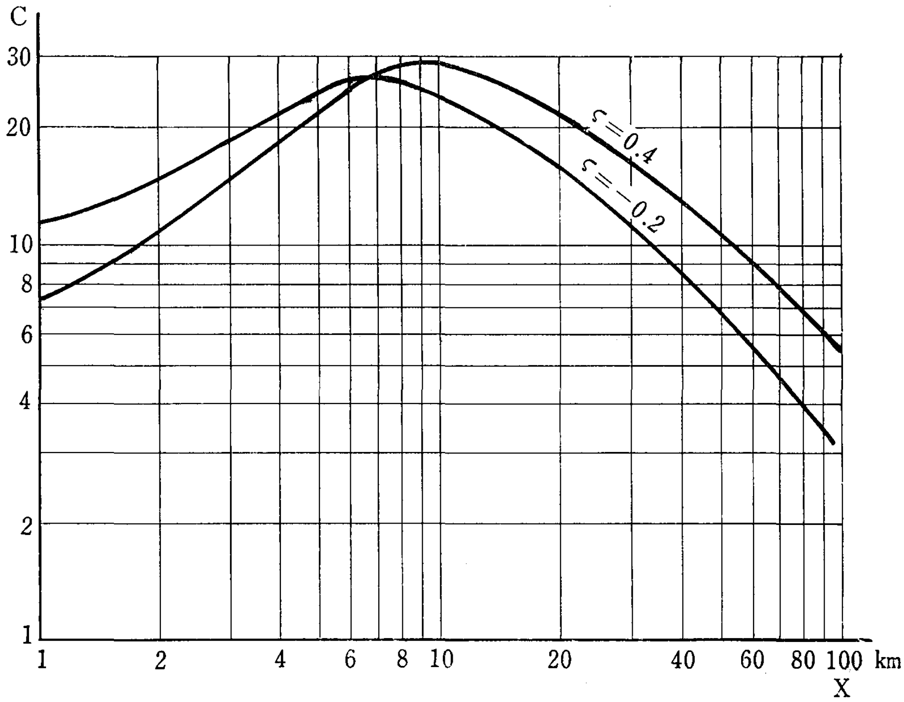

Fig. 2. Concentration distribution along x ($q/u=1$).

Literatures

1) Sakagami, J.: On the Turbulent Diffusion in the Atmosphere Near the Ground, 1954, Natural Science Report, Ochanomizu Univ., 5 (1), pp. 79–91.
2) Sakagami, J.: On the Relations between the Diffusion Parameters and Meteorological Conditions, 1960, ditto, 10 (1), pp. 19–30.