On the Structure of the Atmospheric Turbulence Near the Ground. III[1)]

Jiro Sakagami (坂上治郎)
Department of Physics, Faculty of Science,
Ochanomizu University, Tokyo

Introduction

We had observed that the atmospheric turbulence is explained by the passage of eddies of Rankine type, and had measured their signs, diameters, vorticities, strengths and lifetimes.(1) However those observations had been confined to two-dimensions, namely we had investigated the horizontal sections of the eddies, so we intended this time to observe their three-dimensional shape. We made some outdoor experiments at the end of May on the beach near Katase, just at the same place and in the same conditions with the former experiments. As we had not any information about adequate schemes for the present observation, we made these experiments as preliminary ones.

Method

We used two kinds of frame-works A and B (Fig. 1) and put puppus wind-vanes on all pins which were put in the frames. In

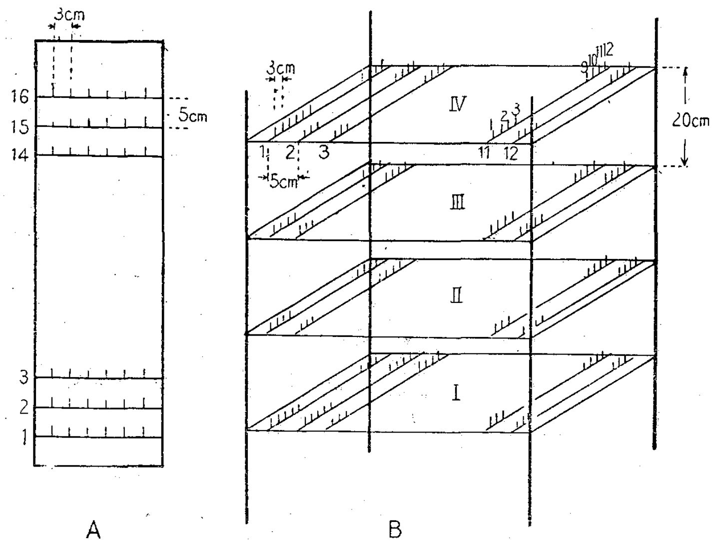

Fig. 1.

three-dimensional case, there were many inconveniences in photographing from upside, as in the former time, so we photographed the

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vanes from a side and measured their deflecting angles by means of photographic surveying. For the investigations of turbulence, as its fluctuation is very rapid, two photographs taken from two separate positions apart at a given distance (base length) must be exactly synchronized, which is a way different from ordinary photographic surveying. So we made a stereo-attachment for a single lens camera.(2) As the lenses used were not for photographic surveying and had some aberrations, the deflecting angles calculated from parallaxes of the photographs had many errors. Therefore we set the photographs in the camera equipped with the stereo-attachment and lighted up each of them from the back, and then both right and left projected rays from a pair of images which correspond to each other were coincided on a movable screen, whose position was thus marked. In such a way we marked the positions of the pins and the puppi, and then we measured the deflecting angle of every vane.

To secure a certain accuracy of the measurements with a camera of small size, we could not use a large and complicated frame, and there has not been any such a camera for large-sized film as is able to photograph cinematographically. So for the experiment I for which the frame A was used, we photographed continuously with a standard cine-camera, and for the experiment II for which the frame B was used, we took snapshots with a 6.5×9 cm. size camera.

Experiment I. It was impossible to place the frame in parallel with the eddies which were invisible and whose inclination was unknown. So in this experiment, it resulted that the frame was placed obliquely to the eddies.

It had been shown(3) that when a Rankine type vortex moved along the main stream, equi-deflection contours had a line of maximum deflection and a line of minimum one, and also there was a plane of no deflecting angle which contained the vortex axis and was perpendicular to the main stream (Fig. 2). These two lines meant the existence of an eddy in the contours, and we represent them with g_1 and g_2 in Fig. 3.[2] So long as a diameter of a vortex does not vary in the direction of its axis, these two lines are in parallel with each other, so that both their elevations are P′L. The angle made between the horizontal projection of g and x-axis in the main stream direction be α, and the angle between g and the horizontal plane be β. The horizontal and vertical traces of the plane of the frame be GH and GJ, and the angle between x-axis and GH be θ, and the angle between g and the horizontal plane be φ. Then g_1 and g_2 intersect with the plane JGH at $S_1(S_1')$ and $S_2(S_2')$. S_1T_1 and S_2T_2 are perpendiculars to the line GH from S_1 and S_2.

[2] This figure is described by the method of the orthogonal projection in descriptive geometry.

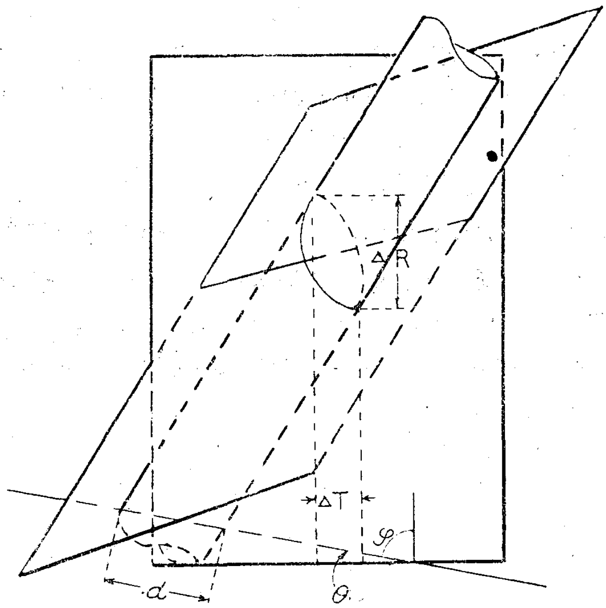

Fig. 2.

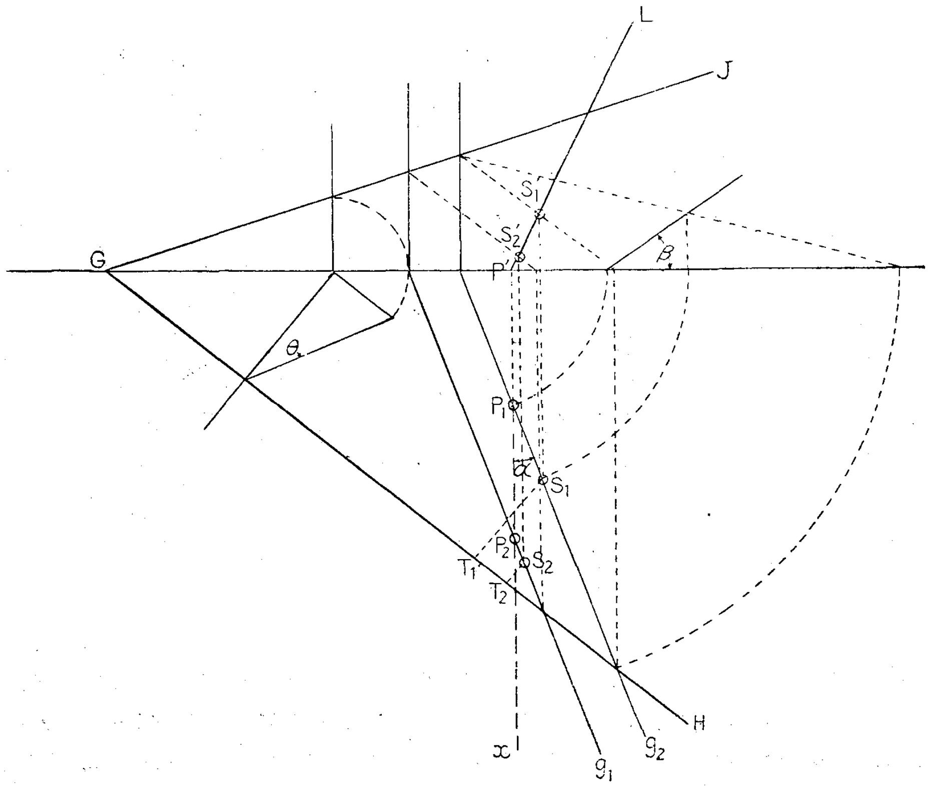

Fig. 3.

By solid geometrical and trigonometrical calculations, we obtain next equations:

$$\overline{T_2T_1}=d\cos\theta\,\{1-\cos\beta(\sin\alpha\tan\theta+\cos\alpha)/A\}, \qquad (1)$$

$$A=\cos\alpha\cos\beta-\sin\alpha\cos\beta\cot\theta+\cot\varphi\sin\beta\,\mathrm{cosec}\,\theta, \qquad (2)$$

where $d=\overline{P_1P_2}$.

The real lengths, on the frame, namely on the plane JGH, of the perpendiculars whose projections on the horizontal plane are S_1T_1 and S_2T_2, be R_1T_1 and R_2T_2, so we obtain

$$\begin{aligned}\overline{R_2T_2}-\overline{R_1T_1}&=(\overline{S_2T_2}-\overline{S_1T_1})\sec\varphi\\&=-d\sin\theta\sec\varphi\{1-\cos\beta(\cos\alpha-\sin\alpha\cot\theta)/\mathrm{A}\}.\end{aligned}\qquad(3)$$

Equation (1) or (3) gives the value of d if the values of θ, φ, α, β and $\overline{T_2T_1}$ or $\overline{R_2T_2}-\overline{R_1T_1}$ are known. Generally in a sea breeze at a place close to a sea-beaten shore, α may be regarded as equal to zero.[3] So we use these equations with $\alpha=0$.

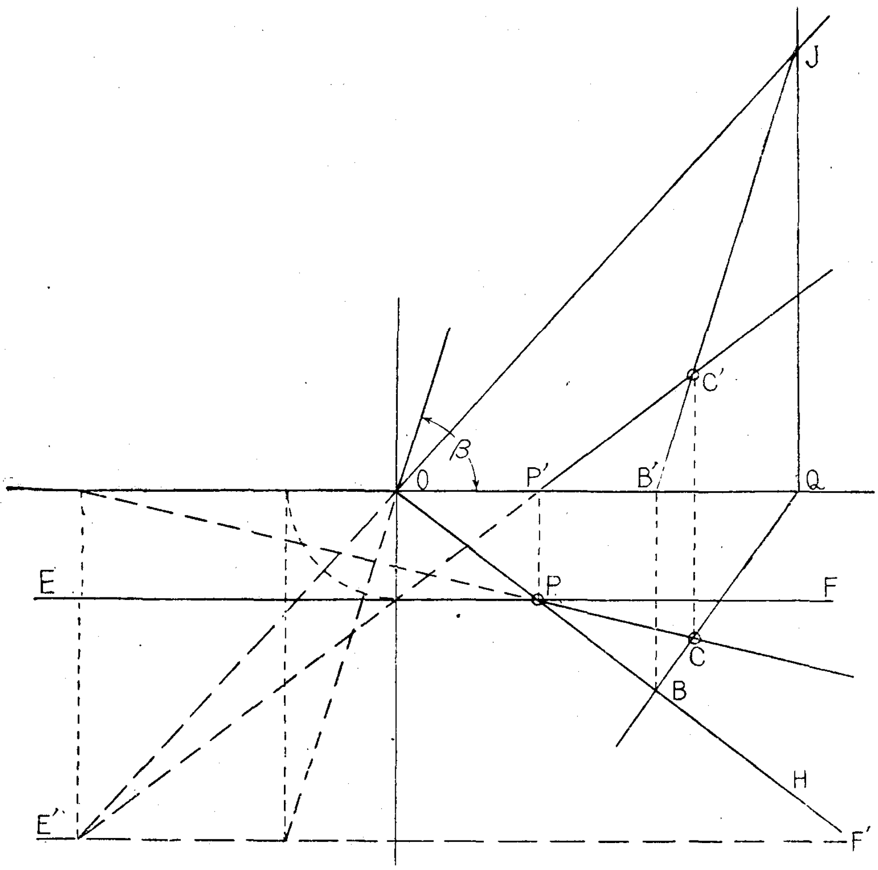

Fig. 4.

In Fig. 4,[4] the plane of the frame be OJH and the plane of no deflecting angle be EF (E′F′) and the angle between this plane and the horizontal plane be β. The line of intersection of OJH and EF (E′F′) be g_3 (g_3'). We want to calculate the inclination ($\tan\psi$) of the line g_3 (g_3') on the plane of OJH, so we make a plane BQJ which is perpendicular to the line OH. The intersecting point of g_3 (g_3') and this plane be C (C′). Real length $\overline{\mathrm{C(C')B}}$ divided by $\overline{\mathrm{PB}}$ is $\tan\psi$, where P is

3) This assumption was justified in Experiment II.

4) In this case, we assumed that $\alpha=0$.

the intersecting point of g_3 and the horizontal plane. So we get

$$\tan\psi=\cos\theta\tan\beta/\sin\varphi(1+\sin\theta\cot\varphi\tan\beta), \qquad (4)$$

or

$$\tan\beta=\sin\varphi\tan\psi/(\cos\theta-\sin\theta\cos\varphi\tan\psi). \qquad (5)$$

So if we measure the angles ψ, θ and φ, we can calculate the value of $\tan\beta$ or the angle β.

By measuring the deflecting angles by projecting method, we obtained equi-deflecting contours of each film, and determined the maximum deflecting points and minimum ones, and consequently, $\overline{T_2T_1}$ and $\overline{R_2T_2}-\overline{R_1T_1}$; and also determined the angle ψ, which was estimated by the most general direction of the equi-deflection contours. The angle θ was determined by the arithmetical mean of all measured angles of the vanes for each film, and φ was measured by the positions of upper and lower rows, which were determined by the projecting method.

We understand from Fig. 5 that so far as the contours represent an eddy, its length must not be shorter than l, and l/d can be calculated by

$$l/d=\sin\varphi/\sin(\beta+\varphi). \qquad (6)$$

If an eddy appears in two pictures, then, by the mean wind speed U and the time interval between these two pictures Δt, we get

$$L\Delta t/U=\sin\varphi/\sin(\bar{\beta}+\varphi), \qquad (7)$$

where $\bar{\beta}$ is the mean value of β for two pictures, and we may conclude that the length of the eddy must be longer than L, but the length L is variable according to experimental conditions.[5] Series of results are shown in Figures 6, 7[6] and Tables 1, 2. We know from Fig. 6 and Table 1 that the values of l/d are 4~6 and those of $\pi/2-\beta(=\beta')$ are about 18°~25° and those of L's are generally 10~20 cm. In the experiment whose results are shown in Fig. 7, the amount of inclination of the frame was too large ($\varphi=74.7°$), so the values of l/d were only about 1.5, and the wind speed was higher than in the other experiment and hardly any eddy appeared in two pictures. Therefore these data were not adequate to investigate the length of the eddies. However, the

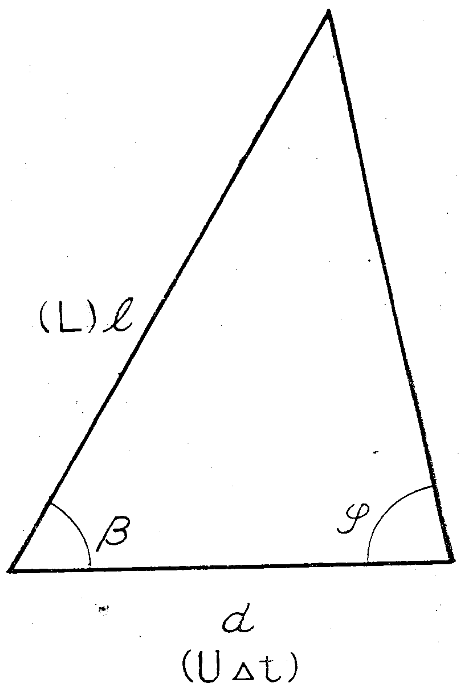

Fig. 5.

[5] In our experimental conditions, eddies appeared in two pictures at most.

[6] In Figures 6, 7, 8 and Tables 1, 2, 3, odd number indicates positive eddy and even number negative one. In Figures 6 and 7, the heights of the lowest ends are 60.2 cm. from the ground.

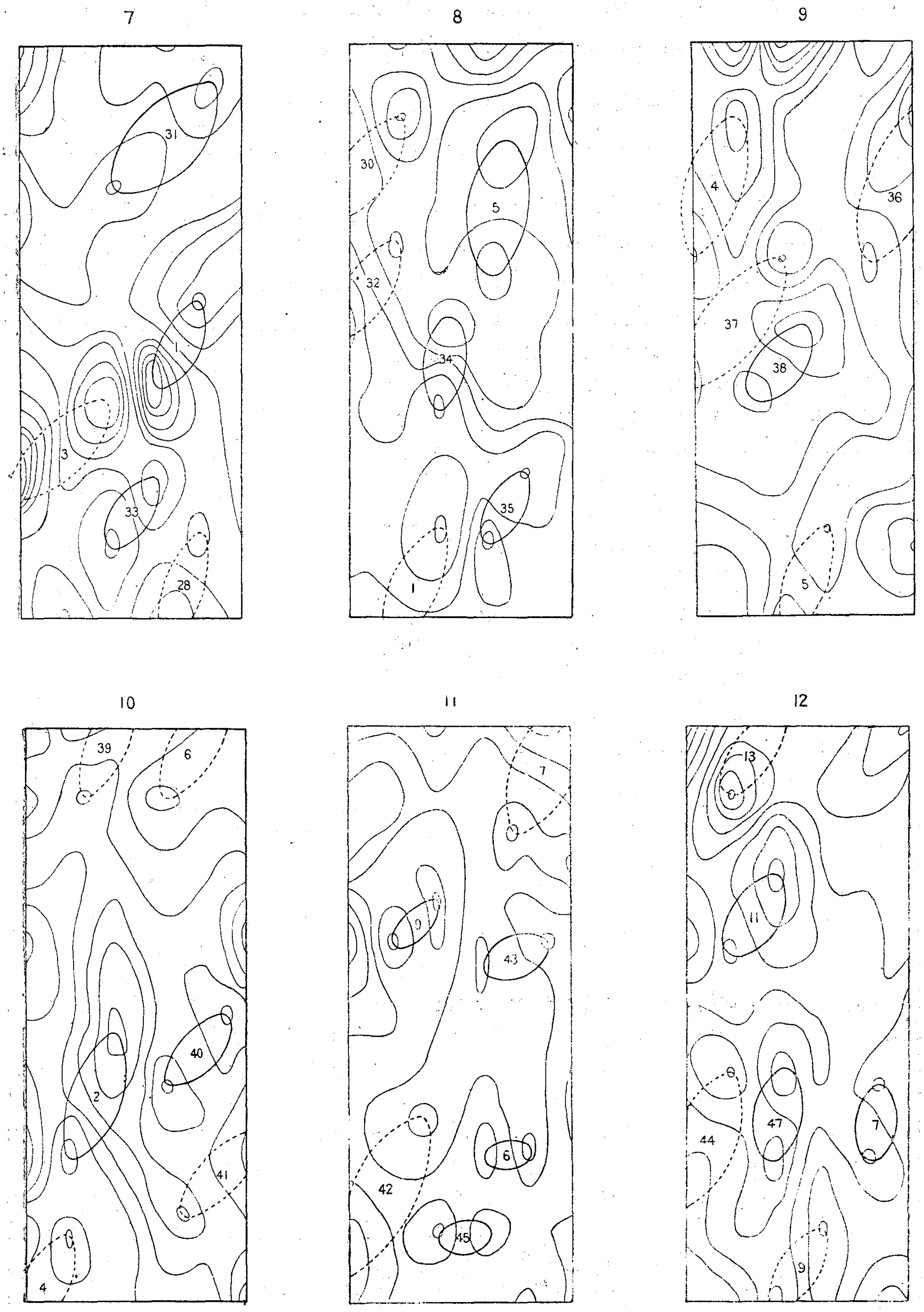
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Fig. 6a (7-12)

Fig. 6 b (13-18)

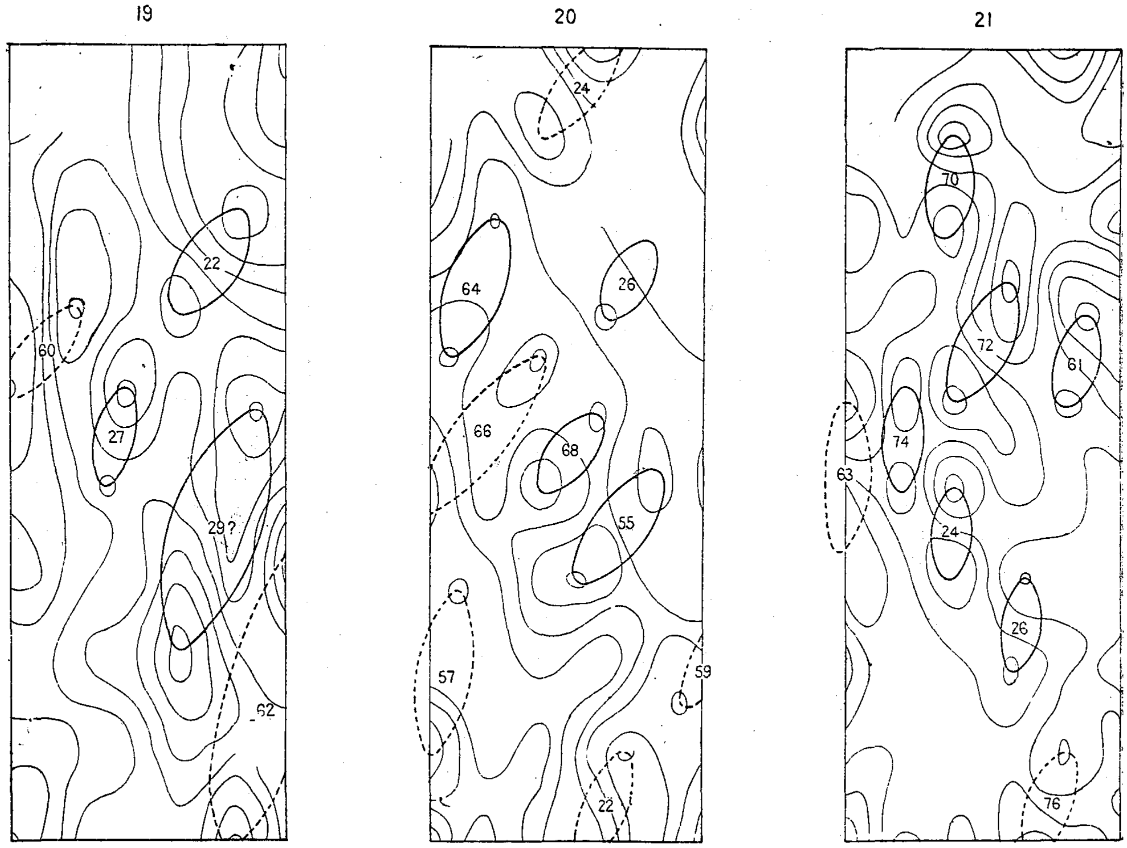

Fig. 6 c (19–21)

Table 1 (a)

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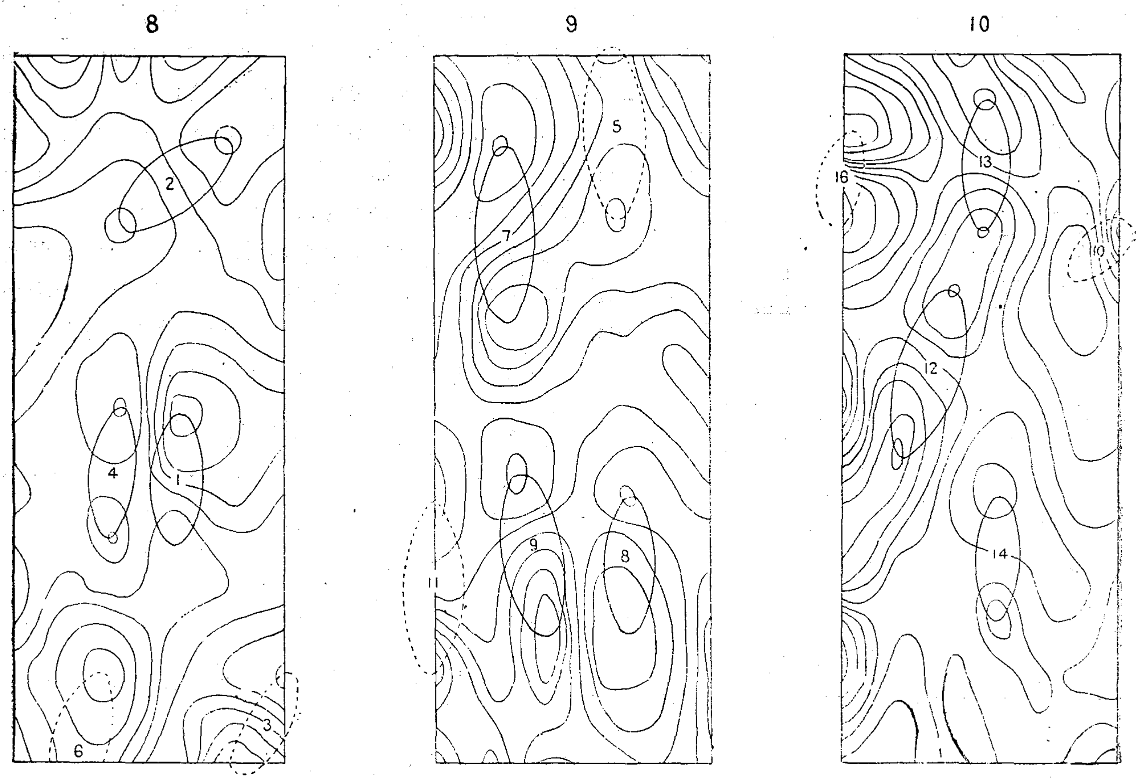
Height (m)	0.5	1.0	1.5
Air Temp. (°C)	24.0	24.0	23.0

No. of Picture	U (m/sec)	t ($\times 10^{-2}$ sec)	l/d	β' (°)
7	1.8	6.0	2.4	29.9
8	1.8	6.0	3.2	22.3
9	1.6	6.5	4.7	16.3
10	1.7	6.5	3.0	23.8
11	1.7	6.0	4·0	18.3
12	1.7	5.5	4.3	17.6
13	1.6	5.5	3.0	23.0
14	1.8	6.5	4.5	16.8
15	1.6	6.0	3.0	23.2
16	1.6	6.0	2.9	23.9
17	1.7	7.0	4.0	19.9
18	1.8	6.5	2.8	24.9
19	1.8	5.5	6.5	12.9
20	1.8	6.0	3.7	19.7
21	1.7		3.9	18.7

Table 1 (b)

No. of Eddy	5	7	8	9	10	12	(14)	15	16	17	(18)
d (cm)	4.2	3.1	6.2	1.8	(3.7)	3.2	3.5	2.7	3.3	3.0	—
L (cm)	12.2	10.5	6.0	10.5	12.2	12.4	4.6	9.2	8.2	16.3	19.5

No. of Eddy	19	22	23	24	25	26	27	(29)
d (cm)	2.7	—	3.0	2.7	3.1	2.0	1.9	5.2
L (cm)	8.2	13.0	13.0	11.8	11.7	8.8	10.0	5.1



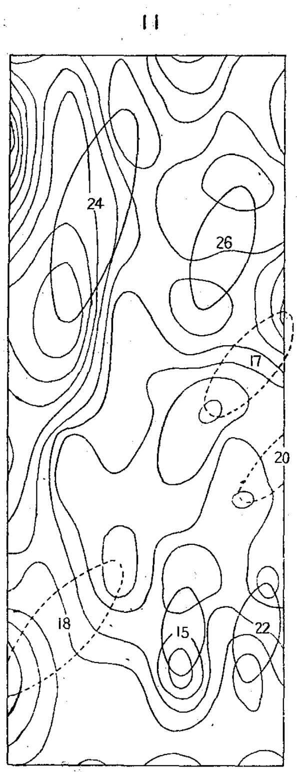


Fig. 7 (8–11)

Table 2

May 25 1255	Height (m)	0.5	1.0	1.5
	Air Temp. (°C)	25.5	25.0	24.7

No. of Picture	U (m/sec)	t ($\times 10^{-2}$sec)	l/d	β' (°)
8	2.2	5.5	1.3	27.0
9	2.2	5.5	1.6	27.4
10	2.2	5.5	1.3	26.7
11	2.2		1.2	22.7

inclination of the eddies (β') was about 20°, just as in the other experiment. In short, the length of an eddy is at least 5 times as long as its diameter and it moves along the main stream inclining forward.

Experiment II. The results are shown in Fig. 8. Eddies which can be observed on two and three stages, namely those whose lengths are surely more than 20 cm. and 40 cm. respectively,[7] are listed in Table 3, and the vorticity (ζ) and strength (S) of each eddy on every stage are also calculated. In that table, the strength

7) As the distance between the stages was 20 cm., we could only ascertain that the length of an eddy which appeared on one stage was not longer than 40 cm. Consequently the distance was too large to investigate the ends of the eddy.

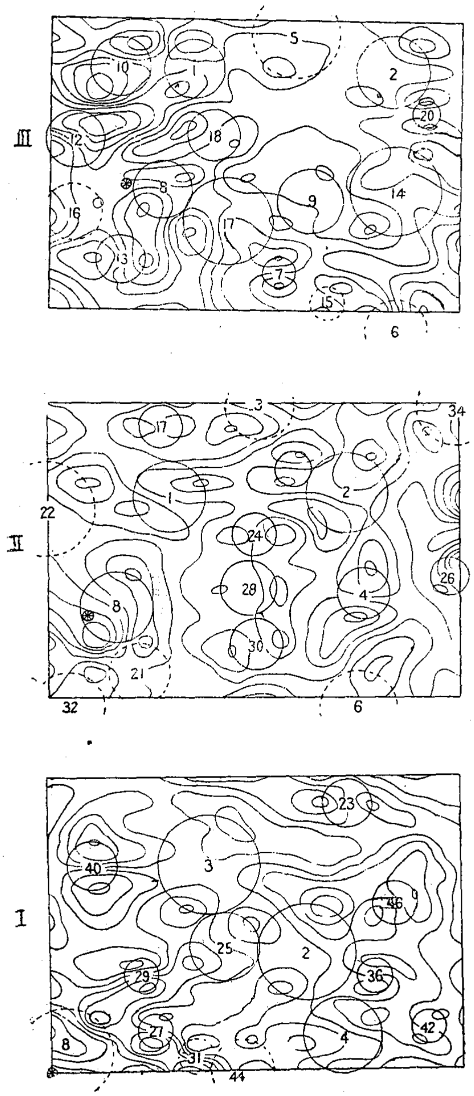


Fig. 8
(Height: III-60, II-40, I-20 cm.)

of an eddy varies with the height of the stage. This can be explained as follows: As our experiments were made in a space very close to the ground, the wind speed varied considerably with height. Nevertheless we measured the wind speed only at one height between the 2nd and the 3rd stages and this value was used for the calculation of ζ and S.[3]

From the theorem of Helmholtz,

$$S'=\zeta\pi d^2/4=k_1 U(z)=\text{constant}. \quad (8)$$

If we adopt approximately the power law of the wind speed,

$$U(z)=U_0(z/z_0)^{\lambda}, \quad (9)$$

so we get

$$S'=k_2 U_0 z^{\lambda}=\text{constant},$$

and

$$U_0=\text{const}\, z^{-\lambda}. \quad (10)$$

As already described, we calculated the strength by a constant value of wind speed U_0, so we get

$$S=k_3 U_0,$$

or

$$S=k_4 z^{-\lambda}. \quad (11)$$

In Fig. 9, $\log S$ and $\log z$ are plotted for several eddies, and we can see that

Table 3

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U=3.1 m/sec

Height (m)	0.5	1.0	1.5
Air Temp. (°C)	24.0	24.0	23.8

No. of Eddy	V_p (m/sec)	d (cm)	ζ ($\times 10^2$ rad/sec)	S ($\times 10^2$ rad·cm²/sec)
III 2	1.116	8.0	0.558	28.1
II 2	1.311	9.5	0.552	39.1
I 2	1.825	10.1	0.723	57.7
II 4	1.277	5.8	0.881	23.3
I 4	1.329	8.2	0.648	34.2
III 1	0.974	6.5	0.599	19.9
II 1	1.038	7.5	0.537	25.7
III 8	1.336	6.5	0.823	26.6
II 8	1.490	7.0	0.852	32.7

the values of λ are all same. In other words, if we consider the vertical change of the wind speed, variation of the strength with height can be corrected, and we can confirm that S=constant.

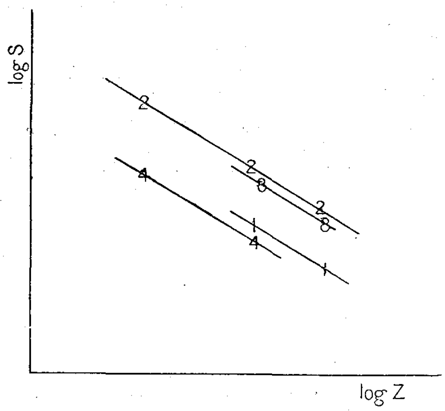

Fig. 9.

If we put the asterisks on the several stages in Fig. 8 one upon another, the cross sections of eddies which appeared on two or three stages fell on the whole one upon another. From this procedure, we concluded that $\beta'=24°$ ca. They were displaced in windward direction to some extent, but they were not displaced in cross-wind direction. This result justified the assumption that $\alpha=0$.

Conclusion

1) Eddies of Rankine type are not short but rather long, and their lengths are several times as long as their diameters.

2) Generally the eddies move along the main stream inclining about 20° forward.

3) Helmholtz's theorem for the strength holds good of each eddy.

4) We could not investigate in detail about the ends of the eddy, but we suppose that at that part the cross section of the eddy will be extremely large and consequently vorticity will be so small that the character of the eddy cannot be recognized any more, and the change of the cross section at the ends will take place in a very short distance.

When we discussed these conclusions with the members of the researching group of atmospheric turbulence, Messrs. S. Nemoto and K. Sôma kindly informed us of their experiences during their observation of fog at Shiozawa, Niigata Prefecture, last winter. They had observed eddies in the atmosphere which were made visible by fog. The observed diameters of the eddies ranged from 3 to 10 cm. and the lengths ranged from 7 to 80 cm. These lengths showed not necessarily the true lengths of the eddies, because they were also determined by the density of fog at the ends of the eddies. Their appearances were cylindrical and the ends were suddenly obscured. These observations may give good supports to our conclusions.

Future planning. As already remarked, these experiments were made as preliminary ones. We want to make further experiments with more appropriate frames and to determine more definite characters of three-dimensional eddies, and especially to make clear the states about the ends of the eddies.

To carry out this research, many assistances were offered by scholars of Ochanomizu University. Miss M. Matsuda exerted herself from the beginning of these experiments and carried out very laborious measurements and analyses of the data. Here the author wants to express his deepest thanks to these persons.

Literature

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