Some Remarks on the Design of a Stereographic Attachment to a Single-lens Camera[1,2]

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Introduction

In the course of the research into the atmospheric turblence, we were obliged to resort to the method of a photographic surveying, but it was impossible to utilize a stereo-camera (especially a stereo-cine-camera), so we intended to make a stereo-attachment to a single-lens camera. Though a stereo-attachment to 'Leica' has been sold and an instruction of making the attachment was reported recently,(1) there has been scarcely any datum for the design of the attachment, so we report in this paper some data for the design.

Fundamental idea

In order to take the stereographs, it was necessary to change the original position of the rays in the object space to another side. For this purpose we used two plane mirrors (Fig. 1). A rectangular prism corresponding to a comparatively large lens is expensive and it has not any particular optical advantage for this apparatus, so we did not adopt it.

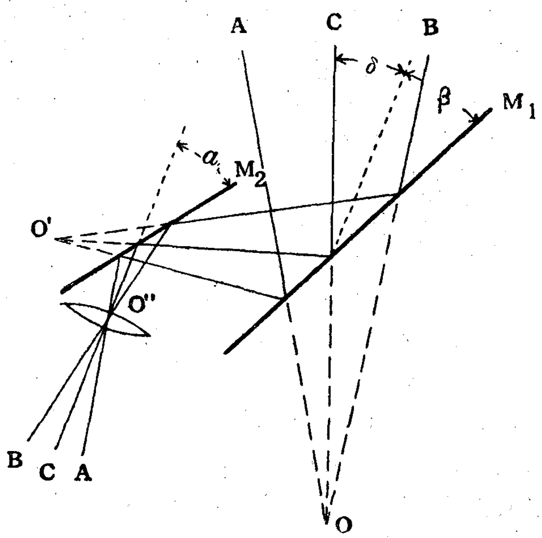

Fig. 1

To obtain two separate stereographic images on the same sensitive film with one exposure, one edge of the beam of light in the image space which corresponded to a suitably restricted beam in the object space was directed to the optical axis of the camera.

We shall consider the design of the righthand setting of the

1) This work was carried on as a part of the research 'On the Structure of the Atmospheric Turbulence Near the Ground', which has been carried on under a Grant in Aid for Fundamental Scientific Research from the Ministry of Education.

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attachment, when there are given the radius r of the camera's lens, angular aparture Φ, base length $2s$ and angle δ, which is made with the ray (c ray in Fig. 1) coming from the middle point of the field, and the optical axis.

Let the angles which the 1st and the 2nd mirrors respectively make with the optical axis be β and α[3] as in Fig. 2. RN_1 is one edge of the beam and N_2S coincides with the optical axis. From the figure, $\gamma=\beta-\varepsilon$ and $\beta+\varepsilon=2\alpha$, so $\gamma=2(\beta-\alpha)$ and $\varepsilon=2\alpha-\beta$. So $\delta=\gamma+\theta/2$ or $\delta=\gamma-\theta/2$ according as N_1R is OA or OB (Fig. 1), where $\theta\equiv\Phi/2$.

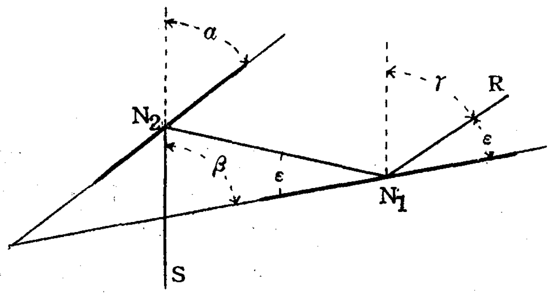


Fig. 2

A) Outward setting

(The case when $O''A'$ coincides with the optical axis. Fig. 3)

Draw the lines OA, OC, OB, O_1A_1, O_1B_1 from the right half of the lens OO_1, where $\angle AOC=\angle COB=\theta/2$, $\angle A_1O_1B_1=\theta$. By setting the 1st mirror M_1M_1', we obtain $O'O_1'$, the image of OO_1, and then by setting the 2nd mirror M_2M_2', we get $O''O_1''$. As the attachment must be symmetrical in respect to the optical axis, the left end of the 2nd mirror must be G. M_3M_3' is the image of M_2M_2' by the 1st mirror. In this case $\delta=2(\beta-\alpha)+\theta/2$, $\angle ADM_1'=2\alpha-\beta$.

In order to prevent an overlapping of the images, a screen is set on the left from O_1A_1, lest any ray which makes a negative angle with $OA(O_1A_1)$ should fall on the lens. The point of intersection K of M_1M_1' and $G'Q$ (Q is the right end of the screen) becomes one end of the 1st mirror.

When the line $O_1''Q$, which is to be defined by the right end of the lens O_1'' and Q, makes an angle smaller than $2\theta(\equiv\Phi)$ with the optical axis, the rays which fall on the lens directly make an image, so a second screen is needed to shut out the rays. A screen KR is set along KG, where R is decided by the intersection of the line KG and $O_2''Q$, (or the line from Q which makes an angle 2θ with the optical axis, if $O_2''Q$ makes an angle larger than 2θ with the optical axis.) The line parallel to OB which passes R', the image of R by the 1st mirror, determines the right ends of the mirrors (N, T). If the 2nd mirror $(M_{21}M_{21}')$ is set nearer the 1st one and the crossing point of $M_{21}M_{21}'$ and F_1O_1' is right from O_1A_1, this arrangement diminishes the quantity of the light, so the right end of the 2nd mirror cannot exceed

3) The sign of the angle is positive when it is measured clockwise from the optical axis.

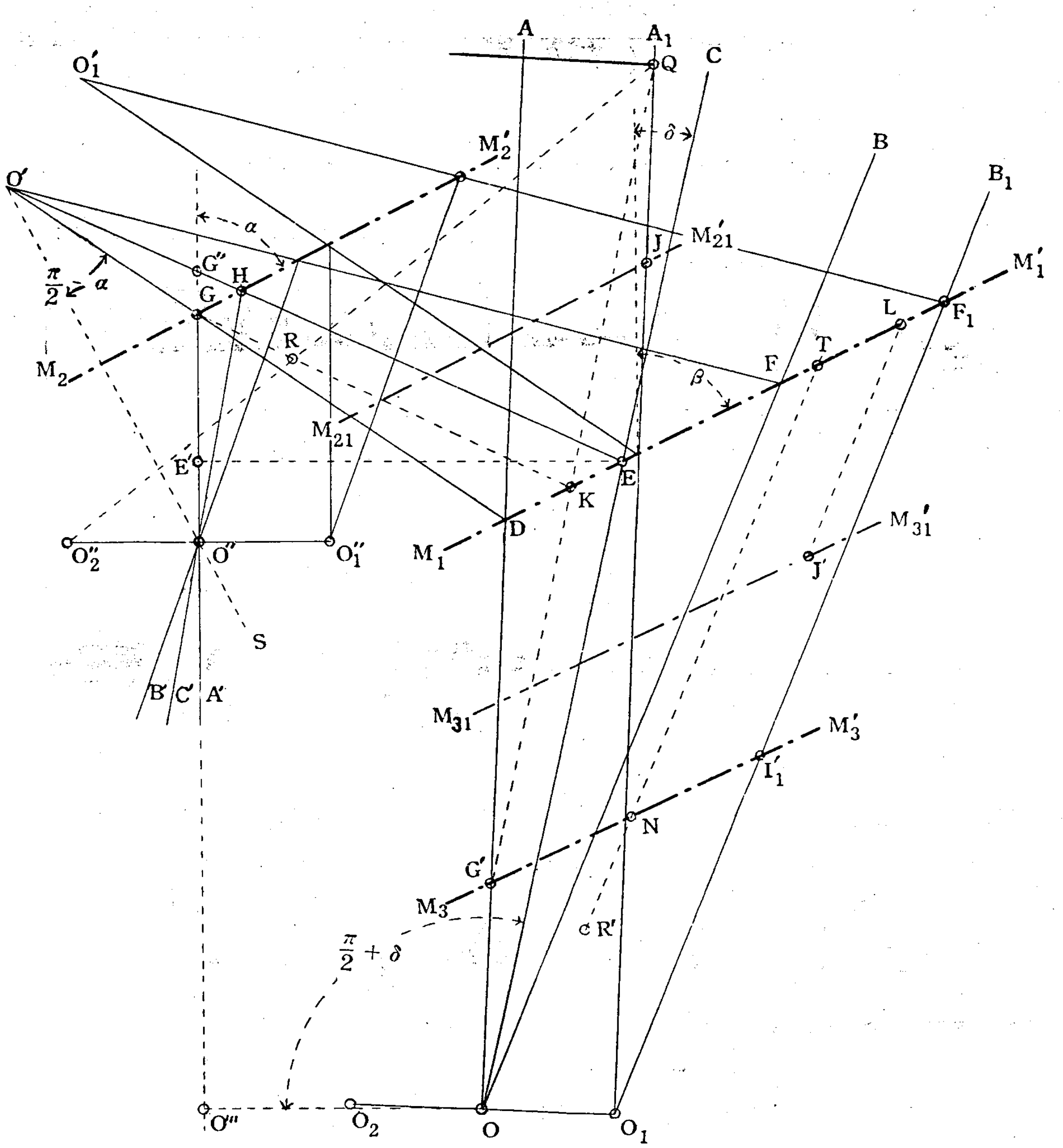


Fig. 3

J. Therefore the right ends of the mirrors are N, T or J', L or I_1', F_1 as the case may be. The window must be set so that all beams of rays which fall on the effective area of the mirror may pass.

Draw OO''' perpendicular to GO'' (or its production) from O, then OO''' is equal to a half length of the base line (s).

$$OO'''=EE'-EO\sin\delta$$

$$EO=OD\frac{\sin(2\alpha-\beta)}{\sin(2\alpha-\beta-\theta/2)}$$

$$EE'=G''E\sin(\angle GG''H)=G''E\sin(2\alpha-\theta/2)$$

$$G''E=O'E-O'G''=OE-O'G''$$

$$O'G''=O'G\frac{\sin 2\alpha}{\sin(2\alpha-\theta/2)}$$

$$G''E = OD\frac{\sin(2\alpha-\beta)}{\sin(2\alpha-\beta-\theta/2)} - OG'\frac{\sin 2\alpha}{\sin(2\alpha-\theta/2)}$$

$$EE' = \left\{OD\frac{\sin(2\alpha-\beta)}{\sin(2\alpha-\beta-\theta/2)} - OG'\frac{\sin 2\alpha}{\sin(2\alpha-\theta/2)}\right\}\sin(2\alpha-\theta/2)$$

$$\therefore \quad OO''' = OD\frac{\sin(2\alpha-\beta)}{\sin(2\alpha-\beta-\theta/2)}\{\sin(2\alpha-\theta/2)-\sin\delta\} - OG'\sin 2\alpha$$

Fig. 4 shows the attachment designed by this arrangement, and Fig. 5 shows the pictures taken by it.

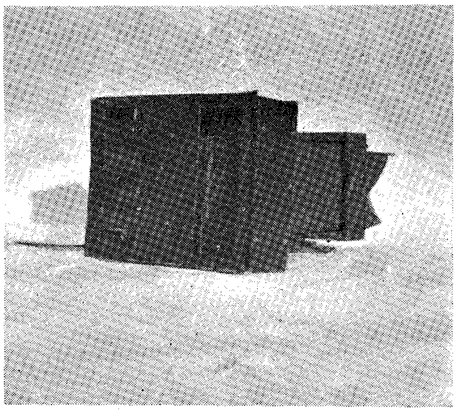
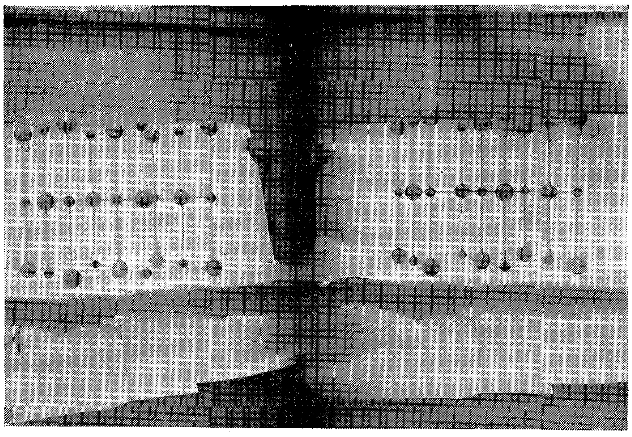
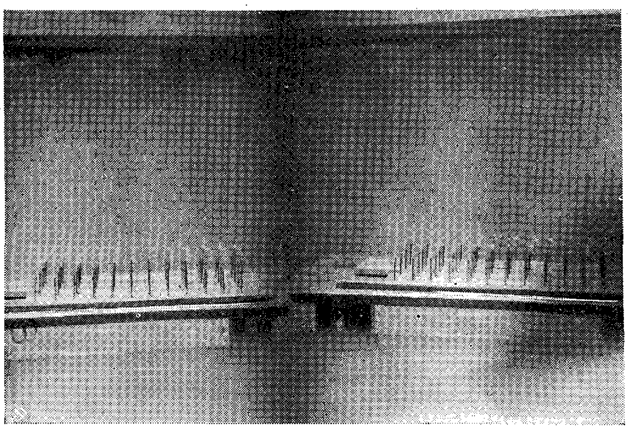


Fig. 4



a



b

Fig. 5

B) Inward setting

(The case when $O''B'$ coincides with the optical axis. Fig. 6)

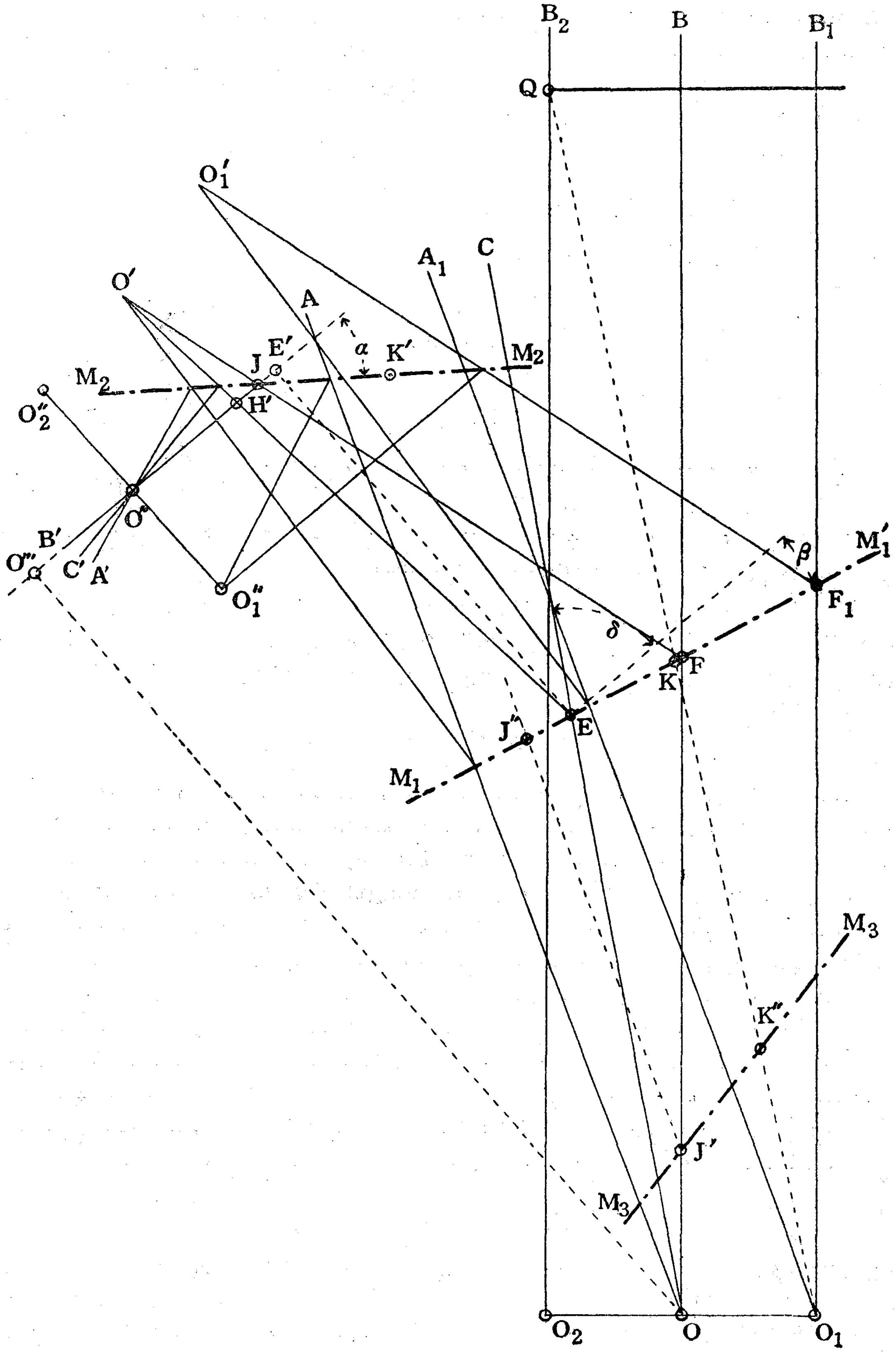


Fig. 6

The drawing can be made just as in the former case. Here $\delta = 2(\beta-\alpha)-\theta/2$, $\angle BFF_1 = 2\alpha-\beta$. A screen is set rightwards from O_2B_2. In some cases a suitable 2nd screen is needed, lest the direct rays from the window should fall on the lens.

Draw OO''' perpendicular to JO'' (or its production) from O. OO''' is equal to a half base length (s).

$$OO''' = EE' - EO \sin\delta$$

$$EE' = H'E \sin(2\alpha+\theta/2)$$

$$H'E = O'E - O'H'$$

$$O'E = OF \frac{\sin(2\alpha-\beta)}{\sin(2\alpha-\beta+\theta/2)}$$

$$O'H' = O'J \frac{\sin 2\alpha}{\sin(2\alpha+\theta/2)}$$

$$\therefore \quad OO''' = OF \frac{\sin(2\alpha-\beta)}{\sin(2\alpha-\beta+\theta/2)} \{\sin(2\alpha+\theta/2) - \sin\delta\} - O'J \sin 2\alpha$$

In this case the quantity of the light is smaller than in the former case, as is clearly seen from the drawing.

For the arrangement of the mirrors in both cases, there are four quantities, namely α, β, OD, $O''G$, $(OF, O''J)$, but when δ and s are given, only one angle and one length must be chosen arbitrarily.

Practical method of the design

When the values of δ and s are given, it is more convenient to design the setting graphically. We consider this procedure in the 1st case, when OD and α are given (cf. Fig. 3). Draw lines OA, OB, OC, O_1A_1, O_1B_1 from OO_1, take the given length OD on OA and draw at D a line M_1M_1' which makes an angle $2\alpha-\beta=\alpha-\delta/2+\theta/4$ with OA. Draw a line OO''' from O which makes an angle $-(\pi/2+\delta)$ with OC and on that line take the length s and, through that point (O'''), draw $O'''O''$ perpendicular to OO'''. Then we make an image (O') of O by M_1M_1', and draw a line $O'S$ which makes an angle $\pi/2-\alpha$ with $O'D$. The point of intersection of $O'S$ and $O'''O''$ is O''. The perpendicular bisector of $O'O''$ makes the position of the 2nd morror. Then draw the paths of light and we can determine the positions of two screens.

Inclination of O″C′ ray

$O''C'$ ray makes an angle $\theta/2$ with the optical axis in the 1st case and $-\theta/2$ in the 2nd. This means that a picture is taken at the end of the base with an angle ω made with the perpendicular to the base, where $\omega = -(\delta-\theta/2)$ in the 1st case and $\omega = -(\delta+\theta/2)$ in the 2nd (Fig. 7).

When ω is negative, the degree of the relief is magnified, and when it is positive, the degree is lessened. When $\omega=0$, then $\delta=\theta/2$ in the 1st case and $\delta=-\theta/2$ in the 2nd, but these settings are unavailable, because the degree of overlapping of the field is not sufficient. Though the attachment is generally used in the state of $\omega\neq 0$, we hardly feel any inconvenience in seeing the pictures by a stereoscope, in so far as $\theta/2$ is less than about 10°.

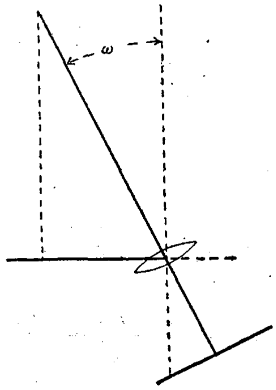


Fig. 7

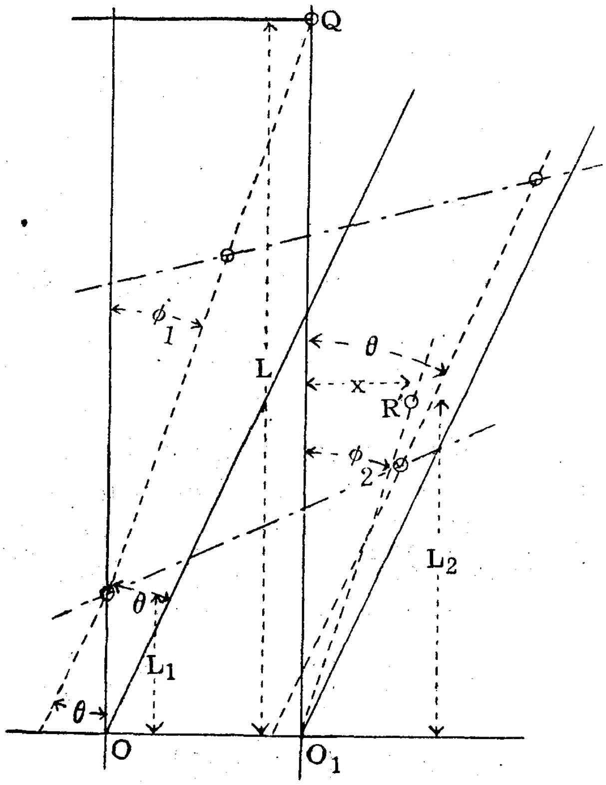


Fig. 8

Quantity of light

We consider the quantity of the light through the attachment of the outward setting. The quantity of the light of the ray which makes an angle φ with the optical axis is proportional to the lens' area, on which the rays fall through the attachment. The notations used below are shown in Fig. 8.

a) $x\geqq 0$

a–1) $\phi_1\leqq\phi_2$

$0\leqq\varphi\leqq\phi_1$

$$H=2\int_{r-L\tan\varphi}^{r}\sqrt{r^2-\xi^2}\,d\xi=r^2[\pi/2-(1-L\tan\varphi/r)\sqrt{1-(1-L\tan\varphi/r)^2}$$
$$-\sin^{-1}(1-L\tan\varphi/r)]$$

$\phi_1\leqq\varphi\leqq\phi_2$

$$H=2\int_{-L_1\tan\varphi\,^{4)}}^{r}\sqrt{r^2-\xi^2}\,d\xi=r^2[\pi/2+\frac{L_1}{r}\tan\varphi\sqrt{1-(L_1\tan\varphi/r)^2}+\sin(L_1\tan\varphi/r)]$$

$\phi_1\leqq\varphi\leqq\theta$

4) If $-L_1\tan\varphi\leqq -r$, the lower limit of the integral should be $-r$.

$$H=2\int_{-L_1\tan\varphi}^{r+x-L_2\tan\varphi}\sqrt{r^2-\xi^2}\,d\xi=r^2[(1+x/r-L_2\tan\varphi/r)\sqrt{1-(1+x/r-L_2\tan\varphi/r)^2}$$
$$+\sin^{-1}(1+x/r-L_2\tan\varphi/r)+\frac{L_1}{r}\tan\varphi\sqrt{1-(L_1\tan\varphi/r)^2}$$
$$+\sin^{-1}(L_1\tan\varphi/r)]$$

a–2) $\phi_1\geqq\phi_2$

$0\leqq\varphi\leqq\phi_2$

$$H=2\int_{r-L\tan\varphi}^{r}\sqrt{r^2-\xi^2}\,d\xi=r^2[\pi/2-(1-L\tan\varphi/r)\sqrt{1-(1-L\tan\varphi/r)^2}$$
$$-\sin^{-1}(1-L\tan\varphi/r)]$$

$\phi_2\leqq\varphi\leqq\varphi_1$

$$H=2\int_{r-L\tan\varphi}^{r+x-L_2\tan\varphi}\sqrt{r^2-\xi^2}\,d\xi=r^2\Big[(1+x/r-L_2\tan\varphi/r)\sqrt{1-(1+x/r-L_2\tan\varphi/r)^2}$$
$$+\sin^{-1}(1+x/r-L_2\tan\varphi/r)-\left(1-\frac{L}{r}\tan\varphi\right)\sqrt{1-\left(1-\frac{L}{r}\tan\varphi\right)^2}$$
$$-\sin^{-1}\left(1-\frac{L}{r}\tan\varphi\right)\Big]$$

$\phi_1\leqq\varphi\leqq\theta$

$$H=2\int_{-L_1\tan\varphi^{4)}}^{r+x-L_2\tan\varphi}\sqrt{r^2-\xi^2}\,d\xi=r^2\Big[\left(1+x/r-\frac{L_2}{r}\tan\varphi\right)\sqrt{1-\left(1+x/r-\frac{L_2}{r}\tan\varphi\right)^2}$$
$$+\sin^{-1}\left(1+x/r-\frac{L_2}{r}\tan\varphi\right)+\frac{L_1}{r}\tan\varphi\sqrt{1-\left(\frac{L_1}{r}\tan\varphi\right)^2}$$
$$+\sin^{-1}\left(\frac{L_1}{r}\tan\varphi\right)\Big]$$

b) $x\leqq 0$ $\qquad\phi_3=\tan^{-1}\dfrac{x}{(L-L_1)}$

$\phi_3\leqq\varphi\leqq\phi_1$

$$H=2\int_{r-L\tan\varphi}^{r+x-L_2\tan\varphi}\sqrt{r^2-\xi^2}\,d\xi=r^2[(1+x/r-L_2\tan\varphi/r)\sqrt{1-(1+x/r-L_2\tan\varphi/r)^2}$$
$$+\sin^{-1}(1+x/r-L_2\tan\varphi/r)-(1-L\tan\varphi/r)\sqrt{1-(1-L\tan\varphi/r)^2}$$
$$-\sin^{-1}(1-L\tan\varphi/r)]$$

$\phi_1\leqq\varphi\leqq\theta$

$$H=2\int_{-L_1\tan\varphi^{4)}}^{r+x-L_2\tan\varphi}\sqrt{r^2-\xi^2}\,d\xi=r^2\Big[(1+x/r-L_2\tan\varphi/r)\sqrt{1-(1+x/r-L_2\tan\varphi/r)^2}$$
$$+\sin^{-1}(1+x/r-L_2\tan\varphi/r)+\frac{L_1}{r}\tan\varphi\sqrt{1-\left(\frac{L_1}{r}\tan\varphi\right)^2}$$
$$+\sin^{-1}(L_1\tan\varphi/r)\Big]$$

As an example we calculated the widths of both mirrors (Fig. 9), and also the distribution of the quantities of the light (H/r^2) against the angle φ of the rays, when α=45, 50, 55, 60, 65, 70 and 75° in the outward setting, where OF=10 cm., s=5 cm. and δ=0° (Fig. 10). We can conclude that there are more advantages when we choose a larger angle for α, because the widths of the mirrors are smaller and the distribution of the quantities of the light on the picture becomes more uniform.

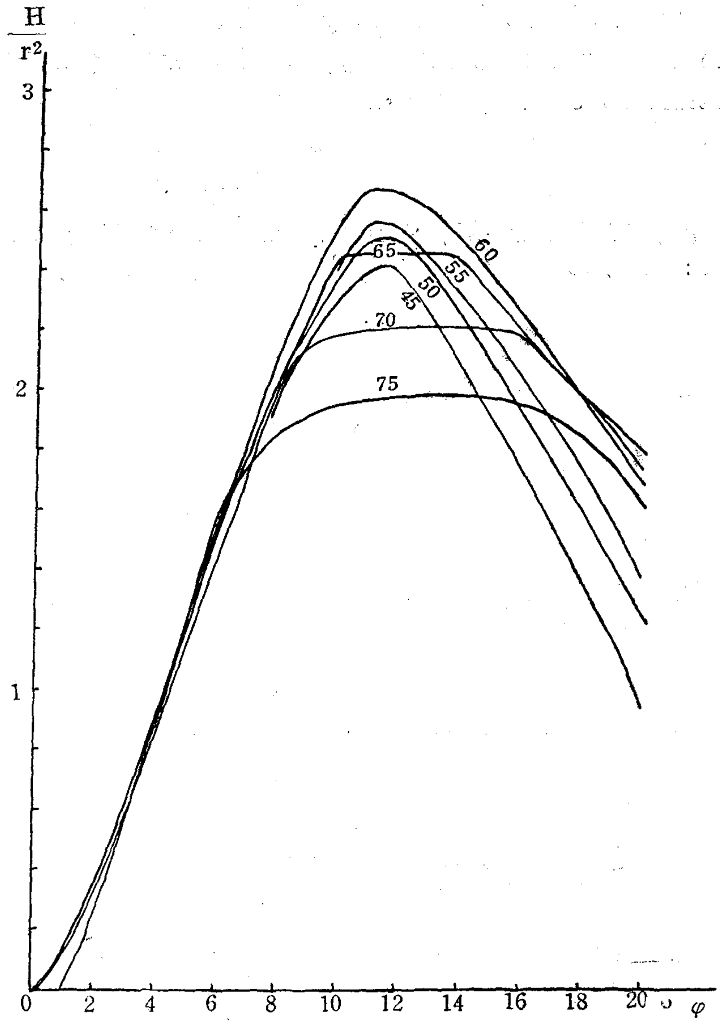


Fig. 9

The effect of the stop upon the picture

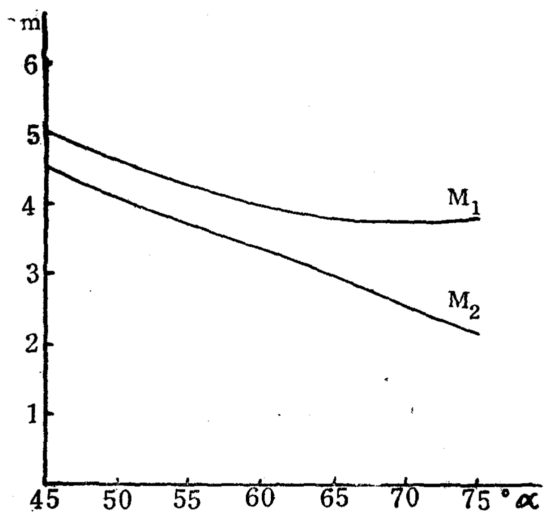


Fig. 10

The 1st screen is set on the left from O_1A_1 in the 1st case, and on the right from O_2B_2 in the 2nd case. When the f-number becomes larger, the effective radius of the lens becomes smaller and O_1 and O_2 come closer to O. Therefore the screen becomes relatively larger and a dark part appears in the middle of the picture. As it is hardly possible to make the screen adjustable, the attachment ought to be designed for the state in which it is used most frequently.

There have been many kind assistances offered by the scholars of the Ochanomizu University, and the writer wants to express his deepest thanks to these persons.

Literature

(1) Namikawa M., "An Apparatus for Taking a Stereographs", Kagaku Asahi 11-9: (1951) 69