Numerical study of the performance of straight-wing vertical-axis-wind-turbine with two and three blades

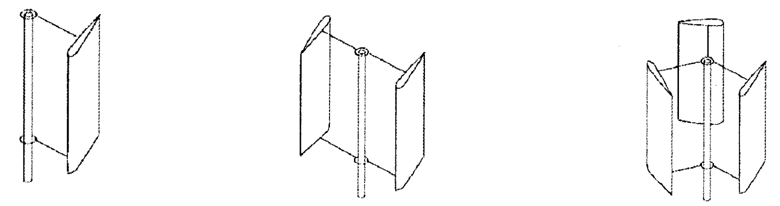
Yoko Mizukami and Tetuya Kawamura

Department of information science, Ochanomizu University
(Received October 8, 2008)
(Revised November 19,2008)

Abstract
Two-dimensional flows around the straight wing vertical axis wind turbine (SW-VAWT) are investigated by means of numerical simulations. The flows around the turbine with two or three blades as well as one blade are analyzed. Incompressible Navier-Stokes equations are solved by the fractional step method. The rotational coordinate system combined with the boundary fitted coordinate system is employed. The third order upwind difference is chosen to approximate the non-linear terms. The torque and power coefficients are computed for various tip speed ratios. Moreover, the effect of the number of blades on the flow field and force acting on the wind turbine is investigated. The agreement with experimental data is satisfactory showing effectiveness of the numerical method.

1. Introduction

Wind energy is one of the most promising renewable energy and is usually obtained by wind turbines. We can classify wind turbines into two types, that is horizontal axis type and vertical axis one. The typical example of the former is the propeller turbine. Disadvantage of this type is that the performance of the turbine is greatly affected by the direction of the wind. On the other hand, the vertical axis type is free from this disadvantage, i.e. it can be rotated by the wind of any direction. We are now seeking potentiality of the vertical axis wind turbine and try to investigate the performance of those kinds of turbine by numerical simulations. In this study, we focus on the straight wing vertical axis wind turbine (SW-VAWT) since this turbine rotates quickly and is suitable for generating electricity like popular propeller turbines. Although we can find several numerical and experimental studies about this turbine [1][2][3][4][5], we are trying to obtain more reliable results by numerical simulations based on the finite difference method. The objective of this study is to simulate the flow fields numerically around SW-VAWT to show the effectiveness of the numerical method and to obtain fundamental data for its design. The number of the blade of SW-VAWT is usually two or more. Therefore, we simulate the flow field around the SW-VAWT that has two or three blades as well as one blade. Figure 1 shows schematic figures of these turbines.



a) N=1 b) N=2 c) N=3

Fig. 1

2. Numerical method

The flow around the wind turbine is well determined by the incompressible Navier-Stokes equation, since the rotation speed is not high compared with the sound speed. We used rotational coordinate system that rotates at the same speed of the blade, so that the flow field can be computed in the fixed grid system. In the coordinate system that rotates constantly with the same angular velocity of the rotor, the basic equations become as follows:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}-\omega^2 X+2\omega V=\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)-\frac{\partial P}{\partial X}$$

$$\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}-\omega^2 Y-2\omega U=\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)-\frac{\partial P}{\partial Y}$$

where X,Y and U,V are the position and velocity components in rotational coordinate system. There are following relations between the stationary coordinate and the rotational one:

$$x = X\cos\theta + Y\sin\theta \qquad X = x\cos\theta - y\sin\theta$$

$$y = -X\sin\theta + Y\cos\theta \qquad Y = x\sin\theta + y\cos\theta$$

$$u = U\cos\theta + V\sin\theta + \omega y \qquad U = u\cos\theta - v\sin\theta - \omega Y$$

$$v = -U\sin\theta + V\cos\theta - \omega x \qquad V = u\sin\theta + v\cos\theta + \omega X$$

where x,y and u,v are the position and velocity components in the stationary coordinate system and θ is rotation angle. These equations are solved by the fractional step method. This method consists of following three steps:

$$\frac{\mathbf{v}^* - \mathbf{v}^n}{\delta t} = -(\mathbf{v}^n \cdot \nabla)\mathbf{v}^n + \frac{1}{\mathrm{Re}}\nabla^2\mathbf{v}^n - \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}) - 2\boldsymbol{\omega}\times\mathbf{v}^n$$

$$\nabla^2 P^{n+1} = \frac{1}{\delta t}(\nabla\cdot\mathbf{v}^*)$$

$$\mathbf{v}^{n+1} = \mathbf{v}^* - \delta t \nabla P^{n+1}$$

where v* is temporal velocity and δ t is time increment. In order to impose boundary conditions precisely on the curved blade (no-slip on the blade in a rotating coordinate system), the boundary-fitted coordinate system is employed. The transformed equations are solved by the finite difference method. All spatial derivatives except nonlinear terms are approximated by the central differences. Time integration is performed by the Euler explicit method. Nonlinear terms are approximated by the third order upwind finite difference since it provides a stable solution without any turbulence models even at considerably high Reynolds number. The third order upwind finite difference is shown as follows:

$$\left(f\frac{du}{dx}\right)_i \sim f_i \frac{-u_{i+2} + 8u_{i+1} - 8u_{i-1} + u_{i-2}}{12\Delta x} + |f_i| \frac{u_{i+2} - 4u_{i+1} + 6u_i - 4u_{i-1} + u_{i-2}}{4\Delta x}$$

3. Grid system

According to our experience, using suitable grids is the key point for reliable computations. Therefore we pay special attention on the grid generation around the blade of the turbine. After some preliminary computations, we determine to use O-type grid of 192×96 in circumferential and radial direction respectively. The shape of the blade is NACA0012. The distance between the blade and the nearest grid points is 2.5× 10 c, where c is aerofoil chord length. Trailing edge is rounded as is shown in the Fig.2. Figure 3 shows the computational region near the whole blade for the simulation of the turbine with one blade. The outer boundary is a circle of radius 10c where c is the cord length of the blade.

In order to simulate the flow around a turbine with two blades, we divide whole circular region into two half-circle regions. Assuming the edge of each region as outer boundary, we generate the grid system by the similar manner to one blade case. Straight outer boundary of each region is connected smoothly, so that the whole grid system is also smooth. The number of grid is 384×96. The boundary values of velocity and pressure on the straight outer boundary is simply set to the average values of the nearest two grid points on the circumferential grid line. For the grid system of three blades, the above-mentioned method can be adopted except for the shape of the outer region. In this case, the region is the fan-shaped region. The total number of grid points is 576×96. Figure 4 and 5 are the grid systems of two blades and three blades wind turbine.

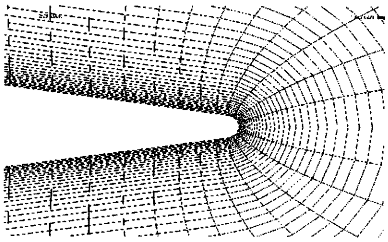


Fig. 2

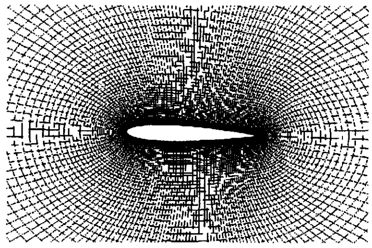


Fig. 3

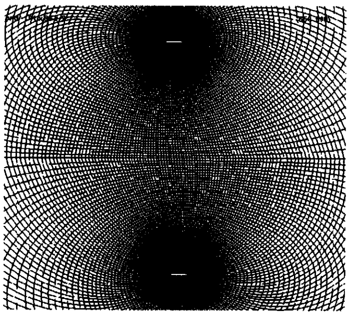


Fig. 4

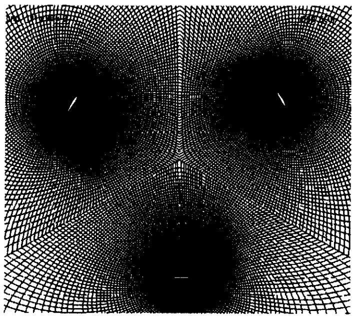


Fig. 5

4. Results

The performance of a wind turbine is determined by tip speed ratio λ, the Reynolds number Re and solidity σ =Nc/R where N is number of the blade and R: radius of the turbine, ν : kinetic viscosity of the air and u_∞: velocity of uniform flow. A solidity σ is set to 0.2. We choose Re= 250000. This corresponds to the situation that the diameter of turbine is 1 meter and the speed of wind is 5 meter per second. Torque coefficients Ct and power coefficients Cp are computed by the torque T which is determined by the pressure fields. Coefficients Ct and Cp are defined as:

$$Ct = \frac{T}{0.5\rho A U_\infty^2} \qquad Cp = \lambda Ct$$

where ρ is the density of the air.

4.1 Torque and Power coefficient for single turbine

At first, we calculate the flow around SW-VAWT with single blade in order to verify the computation in this study. Figure 6 shows the relation between rotation angle from rest and torque coefficient. Since the geometry of the region becomes the same after the rotation of 360 degree, the torque curve has the same periodicity. When the angle measured from y-axis (see fig.8) becomes 90+360n (n: rotation number), the torque reaches its maximum.

Figures 7 shows the effect of tip speed ratio on Cp. The power coefficient Cp have peak values around tip speed ratio λ =8. Although these results are obtained by two-dimensional calculations, qualitative agreement with experiments is reasonable [6].

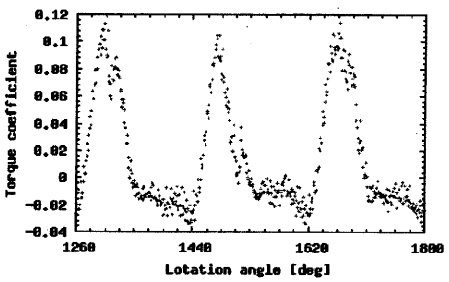


Fig. 6

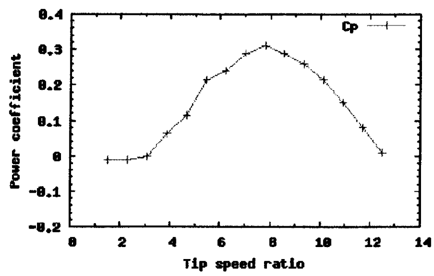


Fig. 7

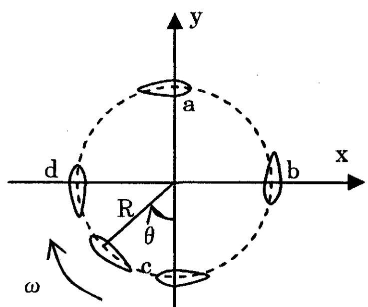


Fig. 8

4.2 Pressure distributions for single blade turbine

A schematic diagram of the rotational motion of the SW-VAWT is shown in Fig. 8. An attack angle changes periodically with rotation. The amplitude of attack angle becomes smaller when the tip speed ratio increases as shown in Fig.9. Figure 10 shows an example of the flow fields (pressure and velocity distribution) around the blade at typical four positions a,b,c,d as shown in Fig.9. Here the torque reaches maximum value at the position d. The series of left hand side figures are the results for λ =4.7 and the right one for 4.0. Various sizes of vortices are generated and shedding. This shows occurrence of large separation that depends on the flow direction.

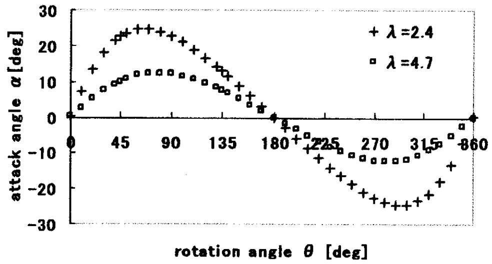
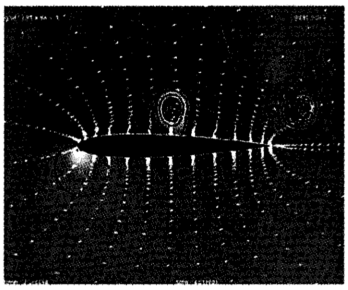
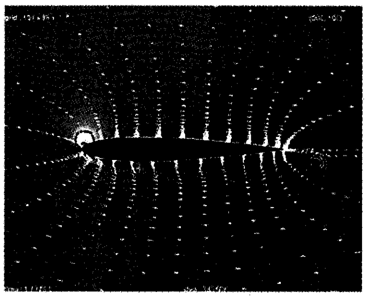


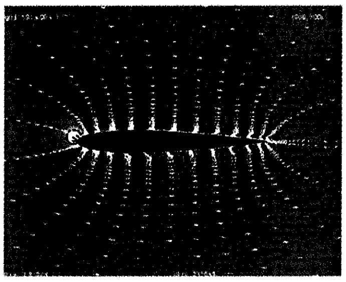
Fig. 9



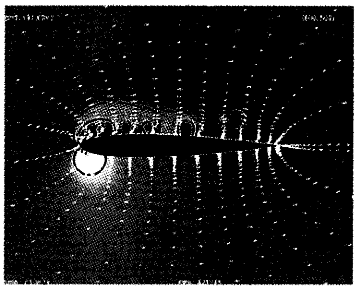
a)at position a.



b)at position b.



c)at position c.



d)at position d.

Fig. 9 single blade (λ =4.7)

4.3 Torque and Power coefficient for two or three blades turbine

Figure 11 shows the relation between rotation angle from rest and torque coefficient for two blades turbine. Since the geometry has the period of 180 degree, the torque coefficient has the same period. The torque reaches its maximum value at 180n+a. Figure 12 shows results of three blades turbine. In this case, the period becomes 120 degree as is expected. The torque becomes its maximum at the degree of 120n+b.

Figure 13 indicates the effect of the tip-speed ratio on the power coefficient for two and three blades. The result of single blade is also plotted in the same figure for comparison. The peak value is almost the same for every case. On the other hand, the optimum tip-speed ratio decreases from 8 to 6 as the number increases from 1 to 3. This tendency is also verified in experiments. Moreover, the range of tip-speed ratio suitable for the turbine becomes narrow with increase of the number of the blade.

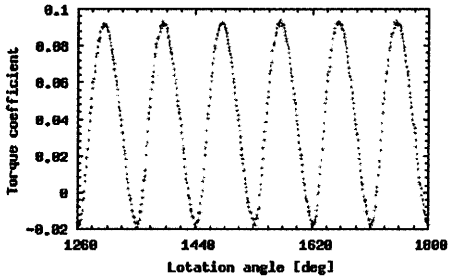


Fig. 11

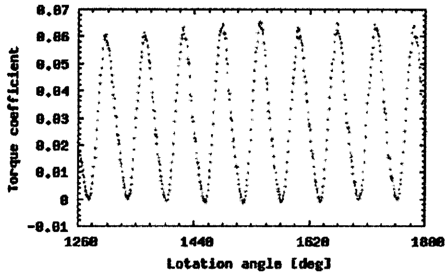


Fig. 12

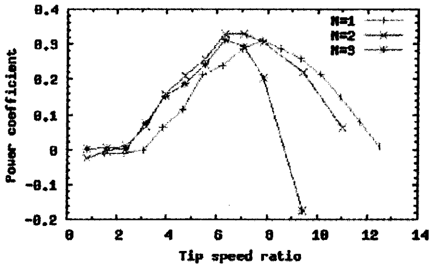


Fig. 13

4.4 Pressure distributions for two or three blades turbine

Figure 14 and 15 show an example of the flow fields (pressure and velocity distribution) of the turbine with two and three blades respectively. Various size of vortices are generated and shedding. This shows that large separation depends on the flow direction. These vortices flow out downstream direction and some times collapse the blade making the flow field complex.

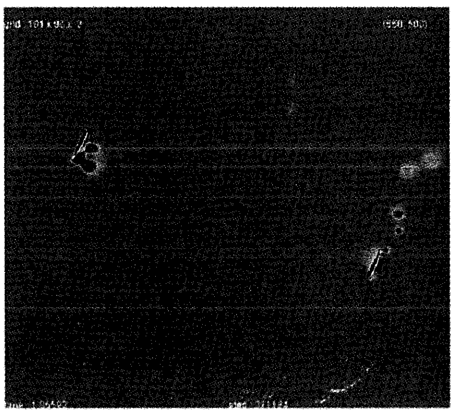


Fig. 14 (λ =4.7)

Fig. 15 (λ =4.7)

5. Conclusion

The flow around the SW-VAWT with two or three blades as well as single blade rotating stationary is simulated numerically by solving the two dimensional incompressible

Navier-Stokes equation. The rotating coordinate system expressed by the boundary fitted coordinate system is employed so that the boundary conditions on the blades of the rotor become simple. Fractional step method is used to solve the basic equations. Following results are obtained:

- The complex flow field around the rotating SW-VAWT with two or three blades as well as single blade is simulated successfully and it is visualized effectively;

- The effect of tip speed ratio on the performance is examined. It is found out that the maximum power coefficient of the single blade SW-VAWT(=0.3) is obtained when the tip speed ratio is around 8. These results agree well with experiments.

- The effect of blade number of the SW-VAWT on the power coefficients is investigated. The peak value is almost the same. On the other hand, the optimum tip-speed ratio decreases from 8 to 6 as the number increases from 1 to 3. This tendency is also verified in experiments. Moreover, the range of tip-speed ratio suitable for the turbine is becomes narrow with increase of the number of the blade.

References

[1] I.Paraschivoiu: Wind Turbine Design With Emphasis on Darrieus Concept, (2002) Presses internationals Polytechnique, Montreal, Canada

[2] A.Allet, S. Halle and I.Paraschivoiu, Unsteady Turbulent Flow Solver for Aerodynamic Analysis of VAWTs , Wind Engineering vol.22, (1998),pp.63-79.

[3] K. Horiuchi, I. Ushiyama and K. Seki : Straight wing vertical axis wind turbines: A flow analysis, Wind Engineering, vol.29,(2005), pp.243-252.

[4] Y.Sato, Y.Mizukami, T.Ito and T. Kawamura: Numerical simulation of flows around a straight-wing-vertical-axis wind turbine using overset grid method, Proc. of 37th Fluid Dynamics Conf.,(2005),pp.207-208.

[5] Y.Mizukami, Y.Sato and T.Kawamura: Numerical Analysis for Steady and Unsteady Aerodynamic Characteristics of SW-VAWT, CFD Journal Vol.15(4),(2007), pp.603-607

[6] T.Hayashi, Y.Hara, T.Morita, K.Tagawa and K. Nozima: The Study on the Solidity Dependency of the Straight Bladed Vertical Axis Wind Turbine, Proc. of 42th The Japan Society Mechanical Engineers Conf., (2004), pp.307-308