On Some Torsion-free Classes

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In [2], we investigated the properties of T-groups. In this paper, we will show some other results on T-groups and on some torsion-free classes containing torsion-free T-groups of rank 1.

For a prime p, let Z_p be the localization of Z at p (i. e., $Z_p=\{m/n\in Q\,|\,\text{g.c.d.}(n,p)=1\}$). Let G be a torsion-free group of rank 1. The type of G has no ∞ (i. e., type(G) is separable) if and only if $Z_p\otimes G\cong Z_p$ for every prime p. Since we can replace Z_p by p-adic integers in this proposition, type(G) is separable if and only if G is locally equivalent to Z (cf. [6]). Hence it follows from proposition 3 in [2] that a rank 1 torsion-free group G is a T-group if and only if G is locally equivalent to Z.

PROPOSITION 1. *The family of all torsion-free groups that are not only T-groups but also R-groups is a smaller torsion-free class than that in proposition 5 in* [2].

PROOF. This follows immediately from Propositions 6 and 7 in [2].

PROPOSITION 2. *The family of all torsion-free T-groups is the unigue maximal torsion-free class whose rank 1 part consists of all the groups that are locally equivalent to Z.*

PROOF. Let $\boldsymbol{C}$ be any torsion-free class which satisfies the same condition as mentioned above and suppose $G\in\boldsymbol{C}$. we can embed G in a finite direct sum of Q's. Consider G's projections into ench Q. They are rank 1 groups belonging to $\boldsymbol{C}$. Hence G is a subgroup of a finite direct sum of rank 1 T-groups. By Proposition 4 in [2], G is a T-group.

COROLLARY. *A torsion-free group G is a T-group if and only if there is an integer n such that* $Z_p\otimes G\cong Z_p^n$ *for every prime p.*

PROOF. Necessity follows from Proposition 4 in [2] and from the fact that any submodule of free Z_p-module is free.

If we prove the fact that the fnmily of all torsion-free groups G satisfying the condition $Z_p \otimes G \cong Z_p^n$ is a torsion-free class, sufficiency follows immediately from Proposition 2. Suppose that H is a pure subgroup of G and $Z_p \otimes G \cong Z_p^n$. Since any pure submodule of free Z_p-module is a direct summand, H and the torsion-free homomorphic image of G also belong to that family.

Finally, we give a characterization of general T-groups.

PROPOSITION 3. *A group G is a T-group if and only if there is an integer n such that $Z_p \otimes G \cong Z_p^n \oplus G_p$ for every prime p where G_p is a bounded p-groups.*

References

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