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On the Application of Diffusion Theory and Stokes' law to the Mode of Volcanic ash fall Deposition

Makiko Watanabe

Department of Geography, Ochanomizu University, Tokyo
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Abstract

An attempt is made to obtain a model for the mode of volcanic ash fall deposition in the present paper. Sakurajima volcano is examined as a case, which offers useful data such as height eruption column, upper wind velocity, grain size distribution and isopach maps of the volcanic ash. Though there are still many problems in the application of the diffusion theory, it is available to apply the Stokes' law to the mode of volcanic ash fall deposition. On the basis of this result, two empirical formulae are obtained. Using a model formula applied in general cases, it is expected to assume palaeo-climate and past volcanic activities.

§ 1. Range of application of the Gaussian Plume Model

When applying diffusion theory to volcanic ash fall, the falling terminal velocity of the ash particles must be considered. For point source problems, we have the diffusion formula, proposed by Chamberlain (Chamberlain, 1953), in which the falling terminal velocity is taken into consideration. Yokoyama (1971) dealt with the diffusion of heavy particles by extending Taylor's theorem (Taylor, 1921). There is, however, some orders of magnitude of difference between volcanoes and chimney stacks for source height and discharge duration and it may be difficult to strictly deal with ash fall by the diffusion formula. However, there are few methods to determine the parameters used in the diffusion formula. In reality, standard deviations of crosswind concentration distribution (σ_y and σ_z) vary with the duration of the eruption. The discharge of volcanic ash usually occurs continuously for a long period, and we cannot simply apply a time dilution coefficient such as that proposed by Meade (1958). It is therefore desirable to estimate the standard deviations of σ_y and σ_z using observed data, which can then be applied to the general theory on eruption of volcanoes. This paper deals with the dispersion problem by applying the diffusion formula as a first degree approximation, neglecting the influence of gravity on ash fall, in spite of the controversial points mentioned above.

The concentration profile of fine particles of volcanic ash in the surface layer was obtained using the Gaussian Plume Model, which provides a basic and general formula for calculation of diffusion. The Gaussian Model gives a diffusion formula based on the research by Sutton (1953), Pasquill (1974), and Gifford (1968). The concentration, C, (mg/m^3) is given as follows, (Hanna, Briggs, Hosker, 1981):

$$C/Q(y,z)=\frac{1}{2\pi\sigma_y\sigma_z u}\exp\left(-y^2/2\sigma_y^2\right)$$
$$\times\left[\exp\left(-(z-h^2/2\sigma_z^2\right)+\exp\left(-(z+h)^2/2\sigma_z^2\right)\right] \qquad (1)$$

where:

Q; the strength of a continuous source (mg/sec)
h; the effective source height (m)
u; the mean wind speed (m/sec)
y; the lateral distance from the main axis of the plume (m)
z; the distance in the vertical direction (m)
σ_y, σ_z; the diffusion width of the plume in the horizontal and vertical direction, respectively. (m)

The distribution of volcanic ash fall deposition, shown in Fig. 1, from the Minamidake Volcano, Sakurajima, which erupted in August, 1978, was

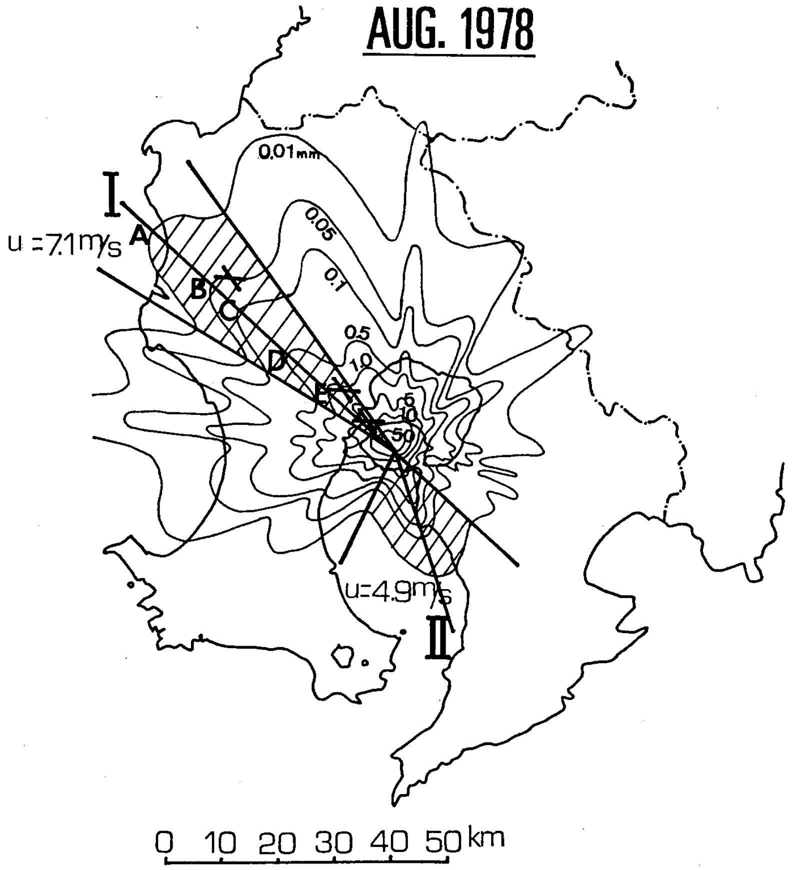

Fig. 1. Isopach area map of the volcano ash erupted in August 1978. (data: Eto and Ishihara, 1982).

used for first-hand data analysis (Egashira, Ishihara, 1982).

Kamo (1977), states that the height of the upper wind, affecting the distribution of volcanic ash fall deposition, from the Sakurajima volcano was about 1500 m. Accordingly, the wind at the standard isobaric surface of 850 mb (corresponding to a height of 1500 m), obtained by radiosonde observations at the Kagoshima meteorological observatory, was applied as the mean wind velocity (u) contained in formula (1). Figure 1 shows a southeast wind, with velocity (u) of 7.1 m/sec, over the main axis (I). The height of the eruption column, h, was assumed to be 1500 m. To estimate the standard deviations, σ_y and σ_z, the BNL (Brookhaven National Laboratory) formula, shown in Table 1, was used, and the coefficients were selected under the assumption that the layer was conventionally of neutral stability, C.

An emagram was drawn using aerological data (daily observation times at 08 : 30 and 20 : 30) during the period 1 Aug. to 31 Aug., 1978, to obtain the stratification of the atmosphere affecting the diffusion of volcanic ash. Table 2 shows the mode of diffusion, classified, according to Hewson (1945), on the relationship between the vertical profile of temperature and the shape of the volcano smoke. Strong stable layers may occur below the 850 mb surface where the eruption source lies and the

Table 1. Brookhaven National Laboratory Parameter Values in the formula $\sigma_y = ax^b$ and $\sigma_z = cx^d$. (Hanna S. R. et al., 1981)

Type	Parameter a	 b	 c	 d
B2	0.40	0.91	0.41	0.91
B	0.36	0.86	0.33	0.86
C	0.32	0.78	0.22	0.78
D	0.31	0.71	0.06	0.71

Table 2. Shapes of the volcanic smoke assumed by the stratification of the atmosphere. (aerological data at the Kagoshima meteorological observatory during the period 1 Aug. to 31 Aug., 1978).

stratification of the atmosphere	days	shapes of the volcanic smoke (as classified by Bierly and Hewson, 1962)
strong stable layer appeared from ground surface to 900 mb layer	0	
strong stable layer appeared from 900 to 850 mb layer	5	inversion in lower layer, lapse in upper layer (lofting),
strong stable layer appeared from 850 to 700 mb layer	10	lapse in lower layer, inversion in upper layer (fumigation)
no stable layer from ground surface to 700 mb layer	16	weak lapse (coning)

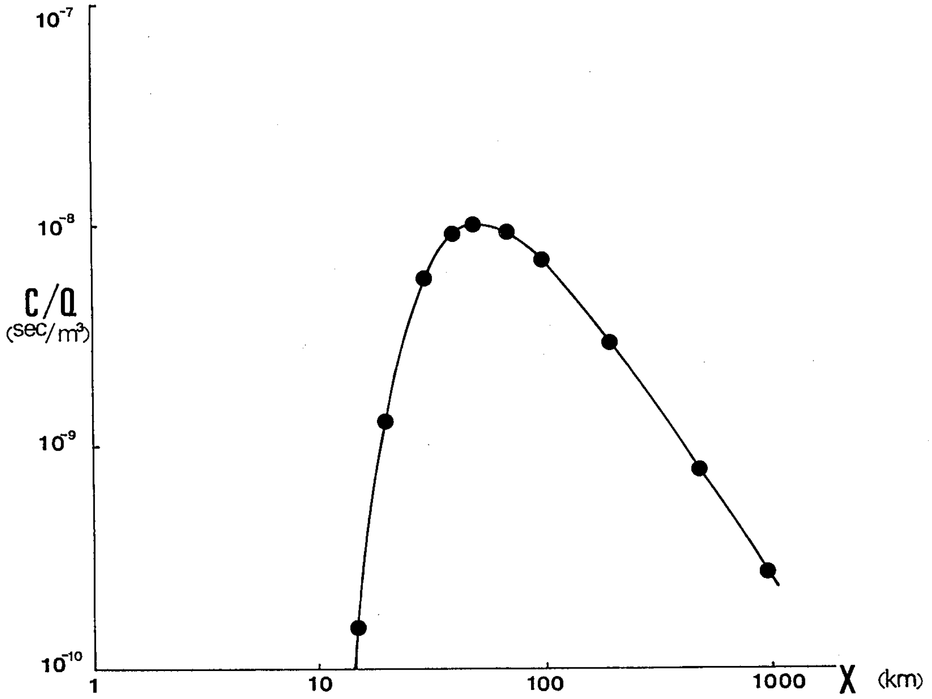

Fig. 2. The relationship between the distance from the eruption source in down wind direction and the concentration at ground surface.

dispersion of the plume may not influence the ground surface. Such cases appeared on only 5 out of 31 days during the observation period, according to the table. Therefore, the effect of stability was neglected for the analysis.

Taking into account the procedures mentioned above, the surface concentration C/Q on the main axis, where $y=0$ and $z=0$ in the formula (1), is shown in Fig. 2. The maximum concentration at the ground surface appeared about 50 km from the eruption source when the mean wind speed (u) was 7.1 m/sec and the height of eruption source (h) was 1500 m, assuming that fine particles of volcanic ash disperse in the same way as the diffusion of gas.

The surface layer sedimentation effect must be considered in the distribution of volcanic ash deposition more than 50 km distance from the eruption source, especially for very small amounts of deposition, but the effect made little contribution to the actual deposition thickness.

§ 2. Estimate of falling terminal velocity of volcanic ash particles from Stokes' Law

The theoretical and observed deposits of volcanic ash were compared on the assumption that volcanic ash particles fall while being drifted horizontally as well as being diffused. Stokes' law was applied for estimation of the falling terminal velocity. In general, when Reynolds number $(R)=\frac{2aW_0}{\nu}\leqq 1$, then Stokes' law:

$$mg=6\pi a\mu W_0 \qquad (2)$$

holds. However, we assumed that Stokes' law was valid up to $R\leqq 2$. On this assumption, the maximum radius of volcanic ash was first examined, where:

m; the mass of a particle (g)
a; the radius of the particle (μm)
W_0; the falling terminal velocity (cm/sec)
μ; the coefficient of molecular viscosity of air (in this paper, 1.78×10^{-4} g/cm·sec at 15°C)
ν; the coefficient of molecular kinetic viscosity of air (0.145 cm^2/sec, at 15°C)

The density of volcanic ash was assumed to be 1.3 g/cm^3 (Egashira and Ishihara, 1982).

Falling terminal velocity of volcanic ash particles was first determined using the data shown in Fig. 1. The time required, for a particle to fall from 1500 m altitude to point A (the horizontal distance from the eruption source $x=58.1$ km), on the northwest main axis, was $t=x/u=8183$ sec (2.27 hours), and the falling terminal velocity (W_0) was 18 cm/sec, obtained from the relation $W_0=h/t$. If Reynolds number $(R)=\frac{2aW_0}{\nu}$ is equal to 2, then the diameter of the particles ($2a$) equals 161 μm when $W_0=18$ cm/sec and $\nu=0.145$ cm^2/sec. Therefore, Stokes' law is assumed to be applicable in the range $2a\leqq 161$ μm.

Using Stokes' law (2), the theoretical value of the particle diameter $\left(2a=\sqrt{\frac{18\mu uh}{\rho gx}}\right)$ was determined in terms of the maximum horizontal distance (x) of particles, and compared with the observed grain size distribution. Results are shown in Fig. 3. The figure shows grain size distribution at the four observation points (cross marks in Fig. 1): HARUTAYAMA $=2.7$ km, TAKE$=5.2$ km, MARUOKA$=14.8$ km, and TOGO$=43.8$ km, as well as the theoretical value of $2a$. The fraction corresponding to $2a$ occupies about 40% (by % weight) of the observed value, and the theoretical value calculated from Stokes' law approximately agreed with the center of distribution of observed data, adequately effective use of Stokes' law.

Fig. 3 shows a radius peak belows 37 μm at distances of 2.7 km and 5.2 km. This effect may be caused by rain washout and the downdraft effect of wind caused by the volcano foot slope acting as a topographical

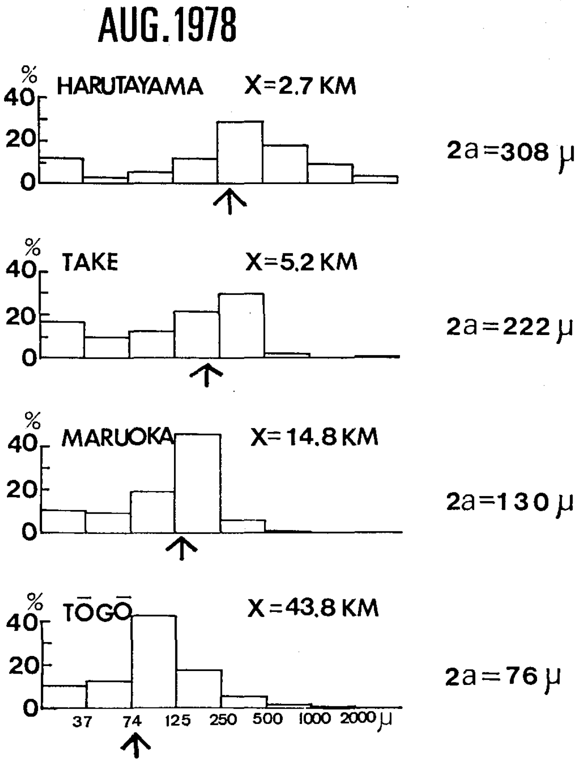

Fig. 3. Observed grain size distribution (Eto and Ishihara 1982) and theoretical value of the particles diameter (2a).

factor, which may decrease the effective height of the eruption and cause fine particles to fall nearer. The effective height should be estimated using wind tunnel surveys because theoretical treatment is very difficult.

Temperature correction for altitude is needed for applying the coefficient of molecular viscosity of air to Stokes' law. Wind speed correction for altitude is also desirable, but was neglected in this paper.

§ 3. Model of Mode of Volcanic Ash Fall Deposition

Fig. 4 shows the relationship between the ratio, $d/2a$, and the ratio, x/H, where d(mm) is the deposition thickness, observed at distance x(km), and $2a$, is the theoretical value of particle diameter, at distance x(km), from the eruption source, obtained by Stokes' law. $H(=1.04\,\text{km})$ is the mountain height. Both these ratios have zero-dimension. When limited by Reynolds number, ($2a \leqq 161\,\mu$), there is a high correlation between the two parameters, $d/2a$ and x/H, ($0.96<|r|<1.00$), at a distance more than 10 km from the eruption source. Empirical formulae for these parameters were determined for 17 sets of data, including the two sets, AUG I and AUG II, shown in Fig. 1 from:

$$d/2a = B(x/H)^A \qquad (3)$$

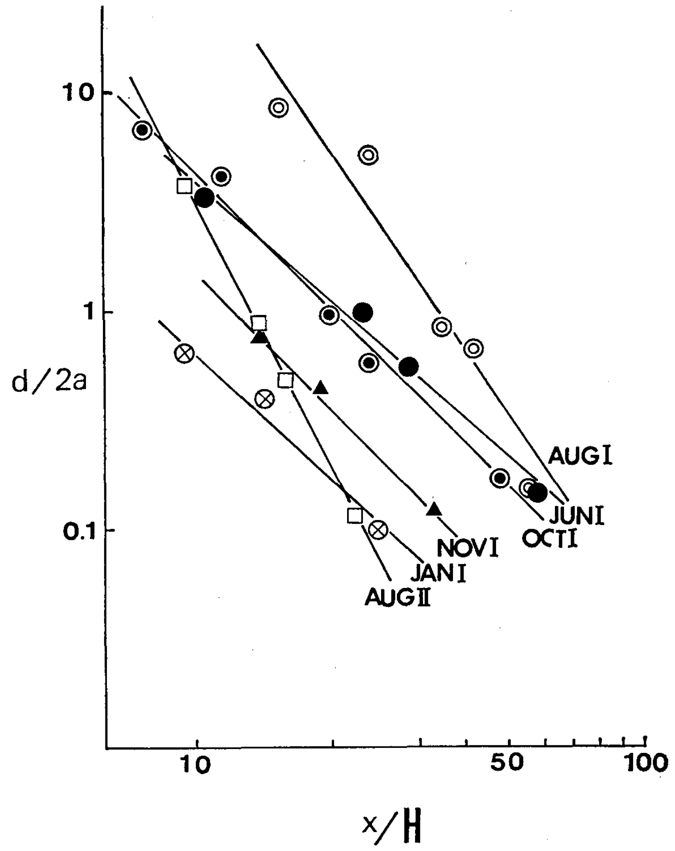

Fig. 4. Relationship between x/H and d/2a.
x : distance from the eruption source (km)
H : mountain height (km)
d : deposition thickness (mm)
2a : theoretical value of the particle diameter (mm)

Coefficients A and B are considered functions of the total eruption volume, V(km^3), where :

$$A=\phi(V), \qquad B=\phi(V)$$

and the respective relationship between the coefficients A and B, and the total eruption volume V, shown in Table 3, were obtained.

The range of volcanic ash distribution (hatched area in Fig. 1), due to an eruption, was conventionally assumed from the isopach maps of volcanic ash, and the area was measured using a planimeter. Area was multiplied by mean thickness, to give volume, V. To obtain a non-dimensional, empirical formula, the ratio V/dm was used, where V is the total eruption volume and d_m is the amount of volcanic ash, deposited per unit area (1 cm^2), in an arbitrary zone on the grain size distribution map. dm(cm^3) was determined as the mean diameter of volcanic ash particles ×1 cm^2, calulated as an arithmetical mean, for the observed grain size distribution.

Table 3. Coefficients A, B, volume V(km^3) and average deposition thickness of volcanic ash d′m(mm).

	SAMPLE	A	B	V(10^{-6}km^3)	dm′(mm)
1	AUG I	−3.06	4.73	189.41	1.38
2	AUG II	−4.04	4.57	127.82	0.58
3	SEP I	−2.28	2.26	13.50	1.08
4	SEP II	−3.13	4.87	28.38	2.05
5	OCT I	−2.10	2.74	7.91	1.32
6	OCT II	−1.54	1.70	7.41	1.04
7	NOV I	−2.07	2.26	26.34	0.90
8	NOV II	−2.34	3.10	23.82	1.46
9	DEC I	−3.38	5.40	50.91	2.38
10	DEC II	−1.59	1.78	21.55	1.26
11	JAN I	−1.89	1.70	15.99	0.74
12	JAN II	−2.27	3.59	24.89	3.00
13	FEB	−1.62	2.00	39.84	1.20
14	JUN I	−1.81	2.41	16.78	1.38
15	JUN II	−1.86	2.17	24.75	1.00
16	JUL I	−3.23	5.17	47.24	6.42
17	JUL II	−1.86	2.22	21.77	1.30

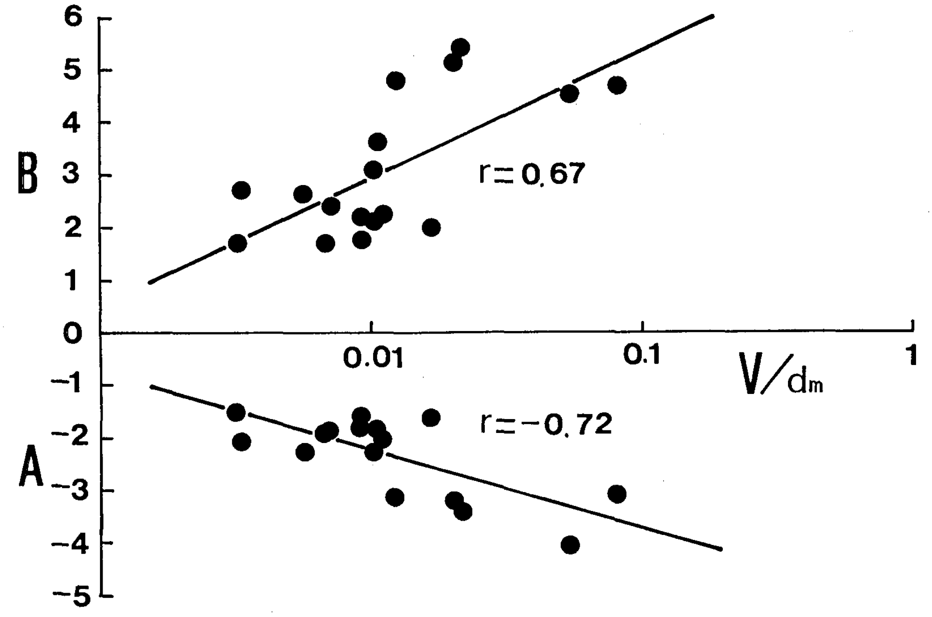

Fig. 5. Relationship between coefficients A, B and V/d_m.
V : eruption volume of volcanic ash (km^3)
d_m : amount of volcanic ash deposited per unit area (cm^3)
A, B : coefficients indicated in the formula $d/2a=B(x/H)^A$

The following empirical formulae were obtained as the result (Fig. 5).

$$\phi\left(\frac{V}{d_m}\right)=e^{-5.17}\times\left(\frac{V}{d_m}\right)^{-1.45} \qquad (4)$$

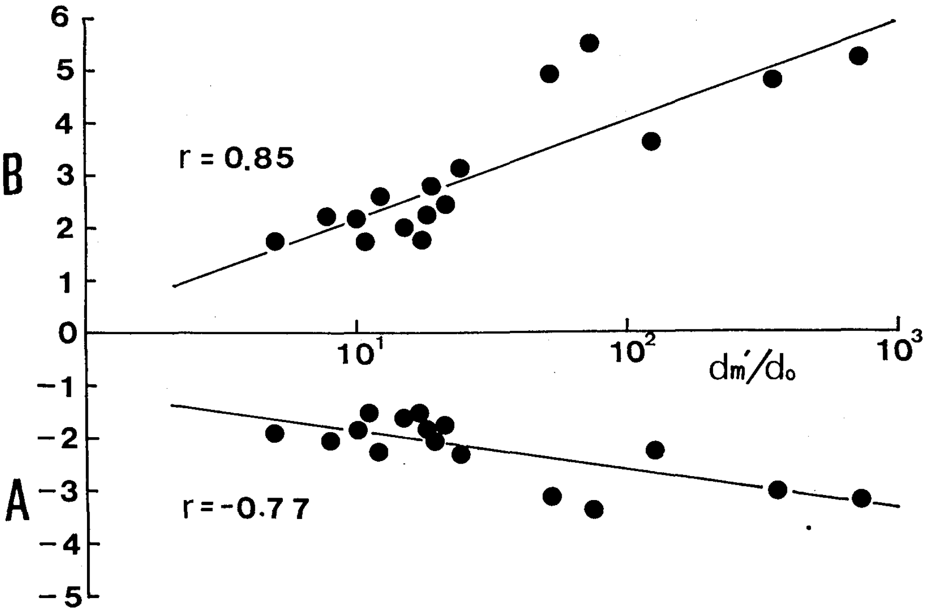

Fig. 6. Relationship between coefficients A, B and d_m'/d_0.
d_m' : average deposition thickness of volcanic ash (cm)
d_0 : observed thickness at an arbitrary point (cm).

$$\phi\left(\frac{V}{d_m}\right)=e^{7.82}\times\left(\frac{V}{d_m}\right)^{2.42} \tag{5}$$

Using the formulae (3), (4) and (5), the model formula:

$$\frac{d}{2a}=e^{7.82}\times\left(\frac{V}{d_m}\right)^{2.42}\times\left(\frac{x}{H}\right)\exp\left[-5.17\times\left(\frac{V}{d_m}\right)^{-1.45}\right] \tag{6}$$

was obtained, and denoted as empirical formula I.

Fig. 6 shows the relationship between $A=\phi(d'_m/d_0)$ and $B=\phi(d'_m/d_0)$, where the coefficients A and B, indicated in the formula (3), are considered as functions of the follows:

d'_m(cm): The average deposition thickness of volcanic ash calculated by the integral mean of the thickness of the volcanic ash, d, with the distance x, using the functional form $d=ax^n$. The lower and upper boundaries of the integral section were assumed to be 10 km and an arbitrary point, x_2, respectively, under the restriction of Reynolds number: [10 km $\leqq x \leqq x_2$ km].

d_0(cm): The observed thickness at the upper boundary (x_2) of the integral section. The empirical formula in this case was obtained as follows.

$$\phi\left(\frac{d'_m}{d_0}\right)=e^{-1.09}\times\left(\frac{d_m}{d_0}\right)^{-0.79} \tag{7}$$

$$\phi\left(\frac{d'_m}{d_0}\right)=e^{0.32}\times\left(\frac{d'_m}{d_0}\right)^{1.85} \tag{8}$$

Table 4. Comparison of the observed values (Sakurajima Volcano by Eto and Ishihara, 1982, and Asama Volcano by Yoshikawa et al., 1982) with the calculated values from empirical formulae I and II.

	sample source height: H(km) wind speed: u(m/sec)	distance from source: x(km)	thickness (cm)			remarks
			calculated from empirical formula I	calculated from empirical formula II	observed	
SAKURAJIMA VOLCANO 1978	AUG III	12.9	1.00×10^{-1}	1.90×10^{-1}	1.00×10^{-1}	$V=62.43\times10^{-5}(km^3)$ $d_m'=1.09(mm)$
	H=1.04	18.1	0.32	0.65	0.50	
	u=7.1	32.9	0.04	0.10	0.10	
		48.1	0.01	0.03	0.05	
		55.4	0.008	0.02	0.01	
	OCT III	10.7	0.29×10^{-1}	0.42×10^{-1}	0.50×10^{-1}	$V=10.64\times10^{-5}(km^3)$ $d_m'=0.17(mm)$
	H=1.04	17.3	0.10	0.12	0.10	
	u=4.9	23.0	0.05	0.06	0.05	
		45.2	0.01	0.01	0.01	
	JUN III	17.1	0.20×10^{-1}	0.07×10^{-1}	0.10×10^{-1}	$V=21.93\times10^{-5}(km^3)$ $d_m'=0.07(mm)$
	H=1.04	23.3	0.09	0.04	0.05	
	u=8.1	49.0	0.01	0.01	0.01	
ASAMA VOLCANO 1982	APR 26 I	(4.64)	$(4.38)\times10^{-3}$		$(8.30)\times10^{-3}$	$V=5.87\times10^{-6}(km^3)$
	H=0.5	(7.54)	(2.35)		(5.16)	
	u=5.0	10.14	1.62		3.65	
		23.91	0.54		0.76	
	APR 26 II	(2.20)	$(55.2)\times10^{-3}$		$(10.8)\times10^{-3}$	$V=2.53\times10^{-5}(km^3)$
	H=4.3	22.2	11.2		4.28	
	u=7.0	46.7	2.99		1.01	
		76.7	1.46		0.25	

According to formulae (3), (7) and (8),

$$\frac{d}{2a}=e^{0.32}\times\left(\frac{d'_m}{d_0}\right)^{1.85}\times\left(\frac{x}{H}\right)\exp\left[-1.09\times\left(\frac{d'_m}{d_0}\right)^{-0.79}\right] \tag{9}$$

was obtained, and denoted as empirical formula (II).

§ 4. Verification of the Empirical formulae and Discussion

To verify errors in two formulae I and II, the observed deposition thickness was compared with the theoretical value calculated from the empirical formula (Table 4). Examples for verification were the Asama volcano, using data provided by Yoshikawa et al. (1982), and the Sakurajima volcano. The theoretical value agreed with observed data in order of magnitude, with an error of less than 20 to 30% on average. Formula II was slightly more precise than formula I.

The strength of the eruption source was taken into consideration in formula I, by introducing the total eruption volume V, but wind velocity (u) has no direct relation to this formula. However, two big problems of formula I are the definition of the distribution range of volcanic ash by an eruption and the method for measuring volume V(km^3). The quantity d_m(cm^3) is significant, because it contains the mean diameter of falling ash but it is difficult to obtain grain size distribution data.

Meanwhile, the mean thickness (d'_m), obtained as a function of distance and thickness, is a function of wind velocity (u) which has physical meaning.

The data used to apply the model formulae, in this report, were : the height of eruption columns, the isopach maps of the volcanic ash deposition, grain size distribution and upper layer wind velocity. The model of volcanic ash fall deposition, can be generally applied to various cases but there are practical problems. Activities of different volcanoes and topography differ and meteorological data and records of activity are often restricted in number. Accordingly, if model formula are to be applied generally, they should be simplified with the least number of parameters.

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References

[1] Bierly, E.W. and Hewson, E.W. (1962): Some restrictive meteorological conditions to be considered in the design of stacks. Jour. Appl. Meteorol., I, 383-390.

[2] Chamberlain, A.C. (1953): Aspects of travel and deposition of aerosol and vapour clouds. A.E.R.E., HP/R 1261, H.M.S.O.

[3] 江頭庸夫・石原和弘 (1982): 火山灰の降下堆積状態に対する上層風の影響, 国土庁編火山ガス等に関する調査報告書. 264 p., p. 115-135.

[4] Gifford, F.A. J. (1968): An outline of theories of diffusion in the lower layers of the atmosphere. Slade D.H. (ed.) Meteorology and Atomic Energy. U.S. Atomic Energy Commission. 445 p., p. 66-116.

[5] Hanna, S.R., Briggs, G.A. and Hosker, Jr. R.P. (1981): Handbook on Atmospheric Diffusion. Technical Information Center, U.S. Dept of Energy 102 p., p. 25-35.

[6] 加茂幸介・江頭庸夫・石原和弘・河原田礼次郎 (1977): 桜島における降下火山灰の堆積について, 昭和51年6月豪雨による鹿児島県の土砂および土石流災害に関する調査研究報告, 文部省自然災害特別研究 (昭和51年度) p. 77-86.

[7] Meade, P.J. and Pasquill, F. (1958): A study of the average distribution of pollution around Staythorpe. Int. J. Air Poll., 1, 60.

[8] Pasquill, F. (1974): Atmospheric Diffusion 2nd edition. Elliss Horwood Publisher, London. 423 p.

[9] Sutton, O.G. (1953): Micrometeorology. A Study of Physical Processes in the Lowest Layers of the Earth's Atmosphere. 333 p.

[10] Taylor, G.I. (1921): Diffusion by continuous movements. Proc. London Math. Soc., Ser. 2, **20**, 196.

[11] 横山長之・井野光秋 (1971): ばいじんや重い粒子の拡散, 公害, **6**, p. 25-31.

[12] 吉川克己・峰岸佳典・宮地直道・遠藤邦彦・倉川　博 (1982): 1982年4月浅間火山噴火の降灰について. 第四紀学会講演会要旨集 **12**, 98.