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Correctcon to "On Balayaged Measures and Simplexes in Harmonic Spaces"

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The present author would like to make the following corrections to the proof of Proposition 2 and 3 in [2].

PROPOSITION 2. *Let F be a subspace of Hp containing P. Any positive linear form L on F may be extended to a positive linear form L' on Hp. If F is dense in Hp, the extention is unique.*

PROOF. For any $f \in Hp$ we put $p(f) = \inf_{\substack{g \in F \\ g \geqq f}} L(g)$. Then we have $|p(f)| < +\infty$. Since the Mapping: $f \mapsto p(f)$ is a sublinear function on Hp and $p(f) = L(f)$ on F, we may find a linear form L' on $H(p)$ satisfying $L'(f) \leqq p(f)$ for any $f \in Hp$ and $L'(f) = L(f)$ for any $f \in F$. If $f \leqq 0$, $p(f) \leqq 0$ holds. Hence $L'(f) \leqq 0$. Further, any positive linear form on Hp is continuous. If F is dense in Hp, the above extention L' is unique.

LEMMA A. *Let P be an adapted cone in $C^+(X)$. Then for any $f \in Hp$ we may find $u \in P$ such that for any $\varepsilon > 0$, there exists $h \in C_k^+(X)$ satisfying $\|f-h\|_u < \varepsilon$.*

LEMMA B. *Let P be an adapted cone in $C^+(X)$. Assume that a linear subspace B of Hp containing P is linearly separating and min-stable. Then, for any $f \in C_k(X)$, we may find $v \in P$ such that for any $\varepsilon > o$, there exists $g \in C_k(X) \cap B$ satisfying $\|f-g\|_v < \varepsilon$.*
(See, Théorèm 12 in [1])

PROPOSITION 3. *Let C be a min-stable, linearly separating cone satisfying $P \subset C \subset Hp$. If (X, C) is a simplex, the function: $x \mapsto \mu_x(f)$ defined on X is Borel measurable for any $f \in Hp$.*

PROOF. Since (X, C) is a simplex, $\mu_x(f) = Q_x(f)$ holds for any $f \in C$. This implies that $x \mapsto \mu_x(f)$ is Borel measurable for any $f \in C - C$.

Let $g \in C_k(X)$. By Lemma B we may find $v \in P$ such that there exist $f_n \in C - C$ satisfying

$$|g-f_n| \leqq (1/n)v$$

for all $n \in \boldsymbol{N}$. Since μ_x is positive,

$$|\mu_x(g)-\mu_x(f)| \leqq (1/n)\mu_x(v)$$

holds. Hence $\lim_{n\to\infty} \mu_x(f_n)=\mu_x(g)$ for any $x \in X$. This implies that $x \mapsto \mu_x(g)$ is Borel measurable for any $g \in C_k(X)$.

Similarly we may show that $x \mapsto \mu_x(\varphi)$ is Borel measurable for any $\varphi \in Hp$ since we may find $v \in P$ such that there exist $g_n \in C_k(X)$ satisfying

$$|g_n-\varphi| \leqq (1/n)v \qquad \text{for all } n \in \boldsymbol{N}$$

by Lemma A.

References

[1] G. Mokobozki and D. Sibony: Cônes de fonctions et théorie du potentiel II, Séminair, Brelot-Choquet-Deny, 11, (1966/67).

[2] H. Watanabe: On balayaged measures and simplexes in harmonic spaces, Nat. Sci. Rep. Ochanomizu Univ. 23 (1972) 61-68.