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Turbulent Diffusion in Turbulent Boundary Layers*

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Introduction

One of the authors has made some experiments of heat diffusion in a small wind tunnel[1], and now we made a series of experiments of turbulent diffusion in neutral condition in a larger wind tunnel in our University, and used gas as diffusive matter, because we have an intention to make diffusion experiments in thermally stratified layers in future. Furthermore, as there have been scarcely any papers in which the diffusion coefficient is calculated directly from results of concentration measurements, we intended to investigate the mechanism of the diffusion in turbulent boundary layers along this line. So the functional form of the diffusion coefficient, especially in the vertical direction was investigated, directly from the results of the concentration measurements, based only upon the Fick's diffusion equation, and then we examined which factor of the structure of the turbulence should have principal effect to the diffusion coefficient.

Method of Experiments

The two-dimentional Fick's equation in stationary state is given by

$$\overline{u}\frac{\partial C}{\partial x}=\frac{\partial}{\partial z}\left(K(z)\frac{\partial C}{\partial z}\right) \qquad (1),$$

where x is measured leeward, z is measured vertically, the position of the source is $x=x_s$, $z=0$, $\overline{u}$ is the (time) mean wind speed at that position, C is the concentration and $K(z)$ is the diffusion coefficient. By integrating the equation (1) z, we get

$$\int\overline{u}\frac{\partial C}{\partial x}dz=K(z)\frac{\partial C}{\partial z} \qquad (2),$$

so we get for $K(z)$

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$$K(z)=\frac{\int \overline{u}\frac{\partial C}{\partial x}dz}{\frac{\partial C}{\partial z}} \tag{3}$$

From the results of the measurements of $\overline{u}$ and C, we can calculate the quantities of $\int \overline{u}\frac{\partial C}{\partial x}dz$ and $\frac{\partial C}{\partial z}$, then we can obtain the values of $K(z)$.

Instrumentation

The wind tunnel used is of Göttingen type and it has the test section of the dimensions $50\times 50\,\mathrm{cm}^2$ in cross section and about 150 cm in length, and a flat plate is set at the height of 15 cm from the bottom.

In order to make the boundary layers to be turbulent, one of the L-shaped metals (L_{13}, L_{19}) whose side is 13 mm or 19 mm is set at the leading edge of the flat plate.

The wind velocity outside of the boundary layer was 3 m/sec. For the measurements of velocities and velocity fluctuations in x direction ($\overline{u}$ and u'), where ($'$) means root mean square, we used a single hot wire about 2 mm in length and 5μ in diameter, and for those in the vertical direction (w'), we used an X-meter, the length of a side of the probe was 2 mm, and we measured them at the positions of $x=50$, 70, 80 and 100 cm.

The sketch of the instrumentation is shown in Fig. 1.

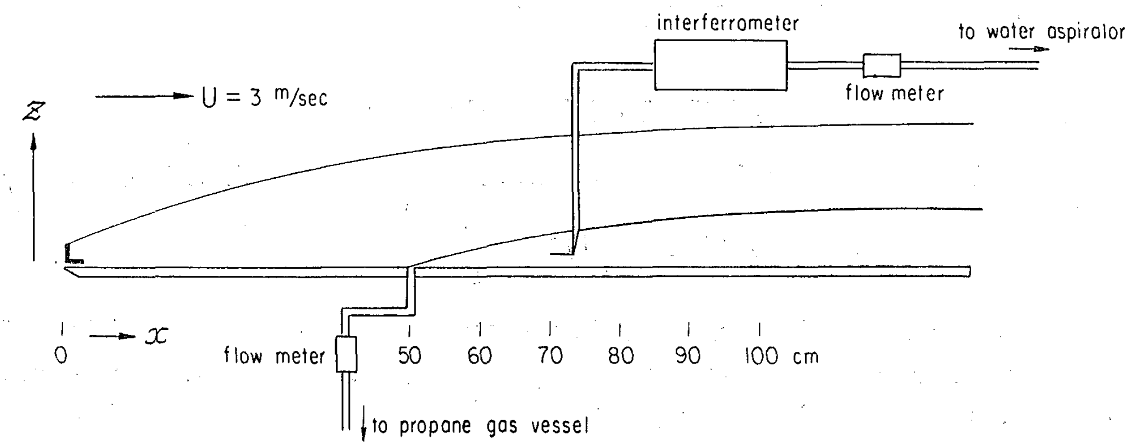

Fig. 1. Sketch of the instrumentation.

The gas used was pure propane gas, and it was emitted from a silt, 6 cm in length and 1 mm wide, which was set at the position of $x=50$ cm, flush with the plate. So we made experiments for a ground level line source.

The flow rate of the gas was 4 cc/sec and the emitting linear velocity of the gas was about 7 cm/sec. The sampling probe is an L-shaped metal tubing whose inside diameter is 1 mm and it has the inlet part of 1.7 mm in width and 0.2 mm in height, and the air contaminated with propane is sucked by a water aspirator. The aspirating rate was about 0.7 cc/sec and it corresponds to the velocity of about 1.6 m/sec. At the positions of $x=60$, 70, 80, 90 and 100 cm, the concentration profiles were measured by an interferrometer, whose measuring range is from 10 ppm to 6000 ppm. Though the wind tunnel is of Göttingen type, back ground concentration was not affected by recirculating gas during the measurement to get one concentration profile, because the amount of emitting gas was little.

Results

The distributions of mean velocity $\bar{u}$ for L_{13} and L_{19} are shown in Fig. 2, and those of the mean velocity fluctuation u' and w' are shown in Figs. 3, 4 and 5, 6. Calculated values of shearing velocities v_* are 12.5 cm/sec for L_{13} and 10.6 cm/sec for L_{19} at every measuring position.

Adopting a parameter $\Lambda(x)$, we get the concentration distributions shown in Figs. 7 and 8, which correspond L_{13} and L_{19} respectively. The relations between $\Lambda(x)$ and x is shown in Table 1. The points of the

Table 1. Values of $\Lambda(x)$ for L_{13} and L_{19}.

x (cm)	60	70	80	90	100
Λ for L_{13}	0.32	0.68	0.98	1.28	1.57
Λ for L_{19}	0.35	0.67	1.08	1.28	1.75

results for L_{19} arrange themselves on a straight line, but those for L_{13} scatter in some extent.

Evaluated values of $K(z)$ after the calculations of $\int \bar{u}\frac{\partial C}{\partial x}dz$ and $\frac{\partial C}{\partial z}$ are shown in Fig. 9.

Considerations

Considering the difference of the results of the diffusion for L_{13} and L_{19}, there is no apparent difference for $\bar{u}$ in both cases, because they follow almost the law of one-seventh power, and by the distributions of u' and w' for L_{13} and L_{19}, we cannot directly explain the difference in shapes of $K(z)$.

The dimension of $K(z)$ is $[L]^2[T]^{-1}$, so the simplest one is $[K]=\left[\frac{L}{T}\right]\times[L]$, and the velocity most closely related to $K(z)$ is considered as w'. So if we adopt z as the characteristic length, we get $w'z$ as shown in Fig. 10. Though we used z tentatively as the characteristic length in this case,

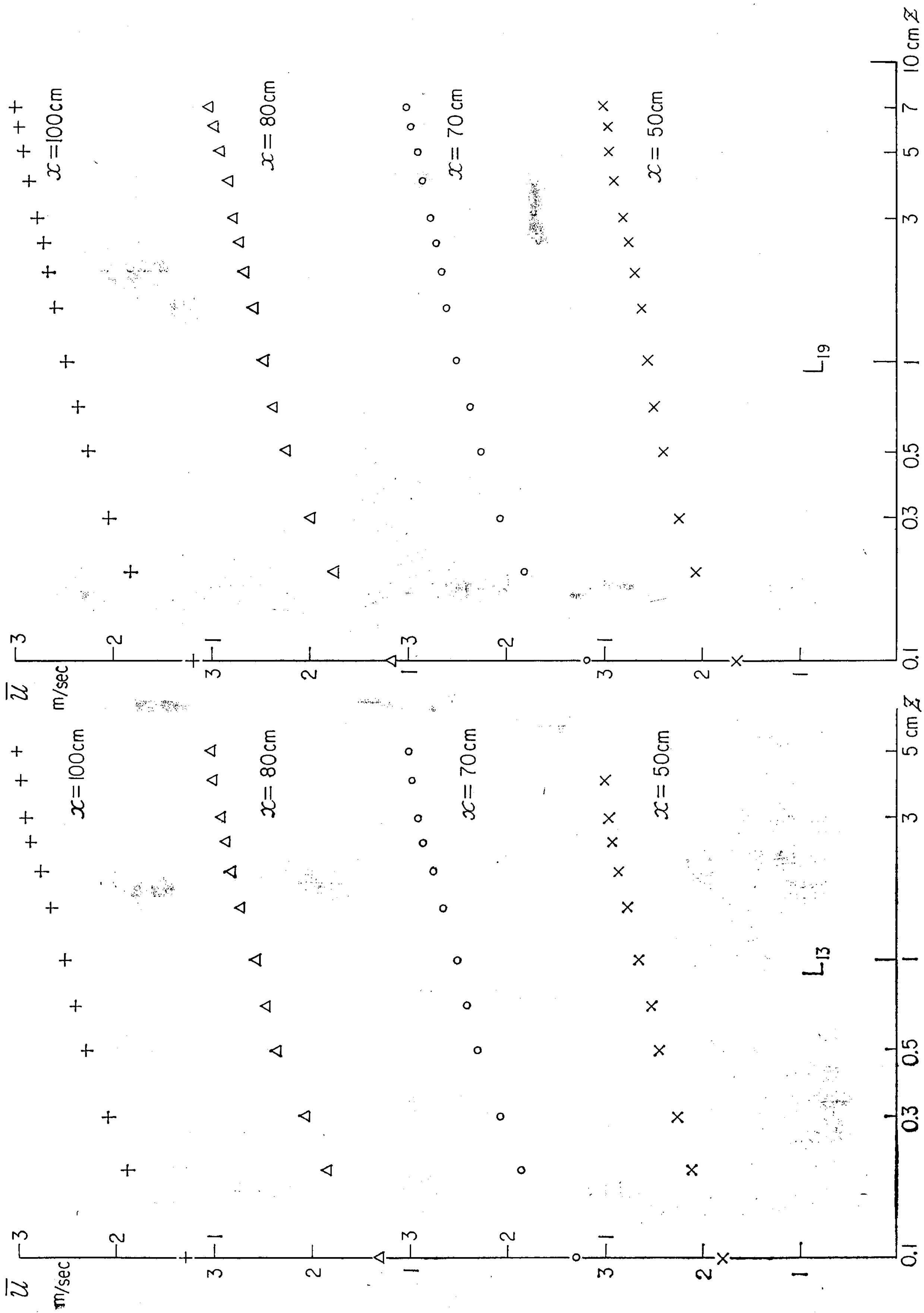

Fig. 2. Profiles of $\bar{u}$ for L_{13} and L_{19}.

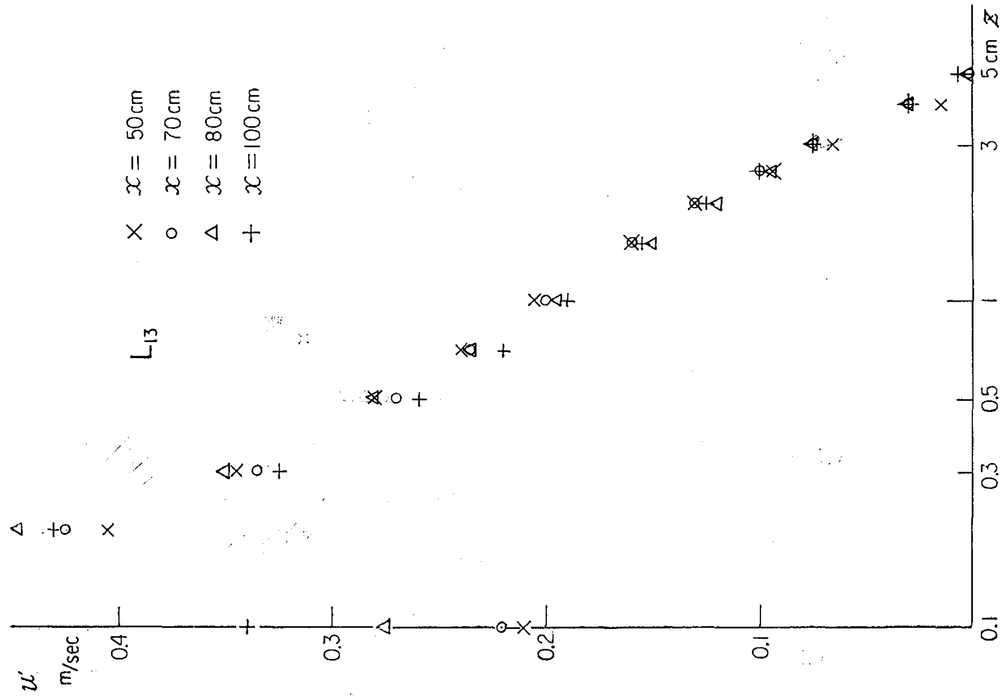

Fig. 3. Profiles of u' for L_{13}.

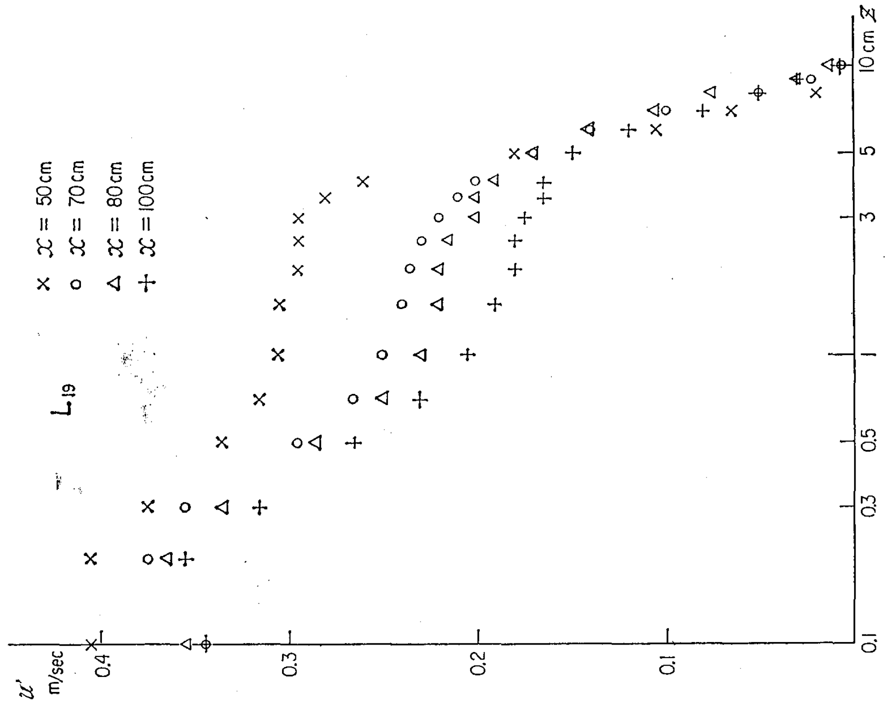

Fig. 4. Profiles of u' for L_{19}.

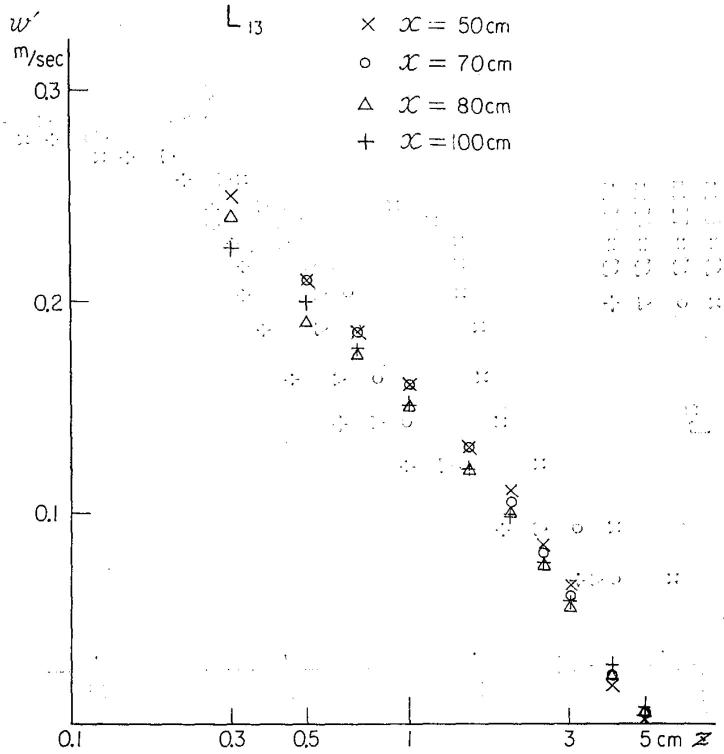

Fig. 5. Profiles of w' for L_{13}.

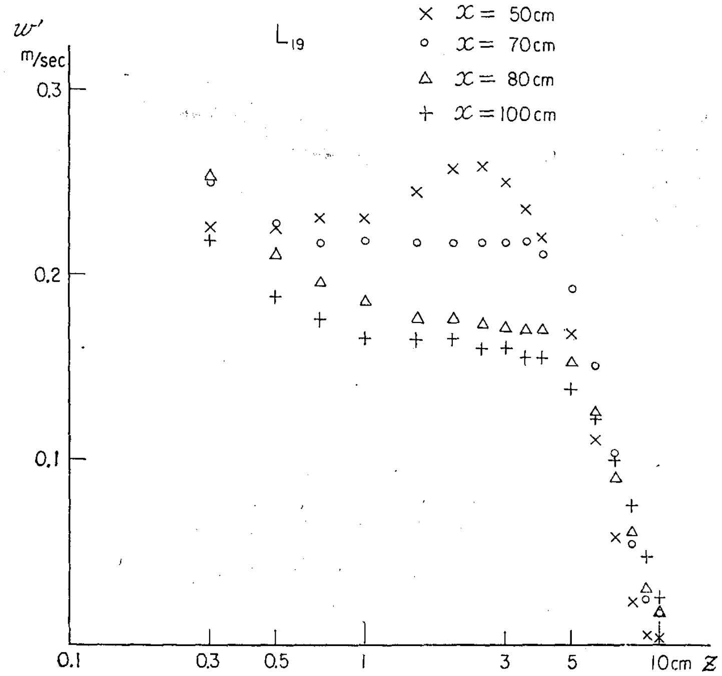

Fig. 6. Profiles of w' for L_{19}.

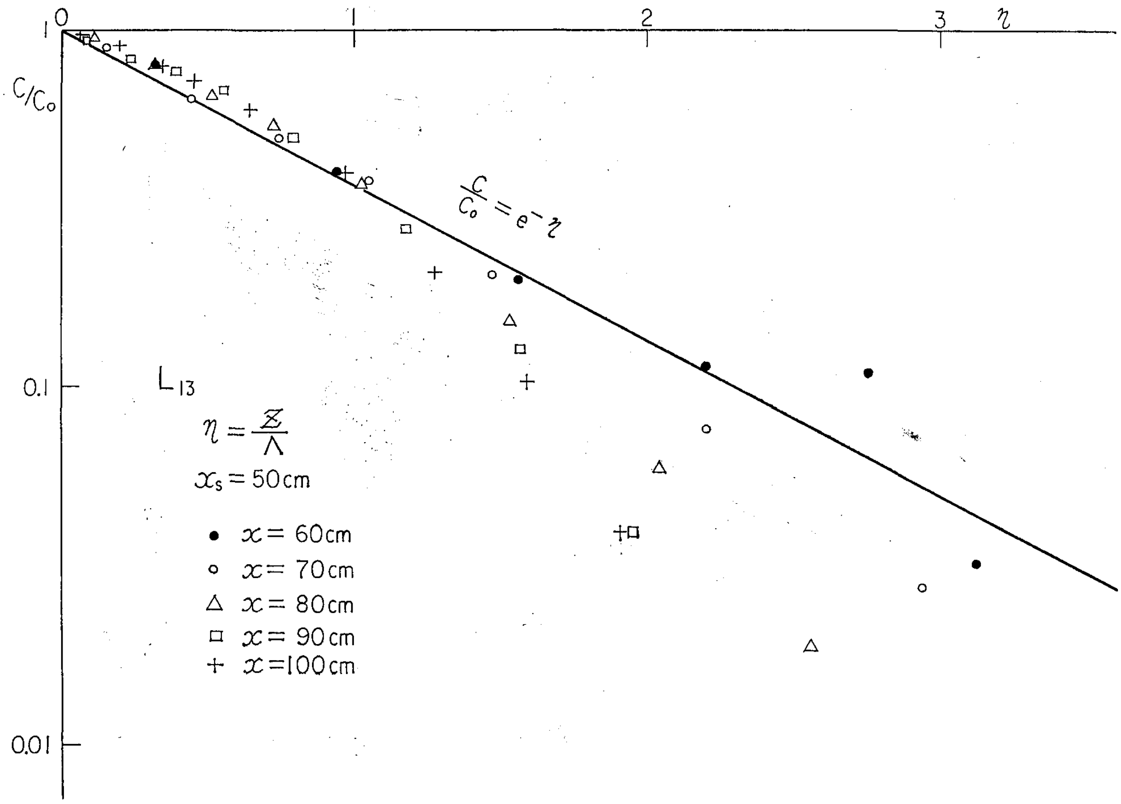

Fig. 7. Concentration distributions for L_{13}.

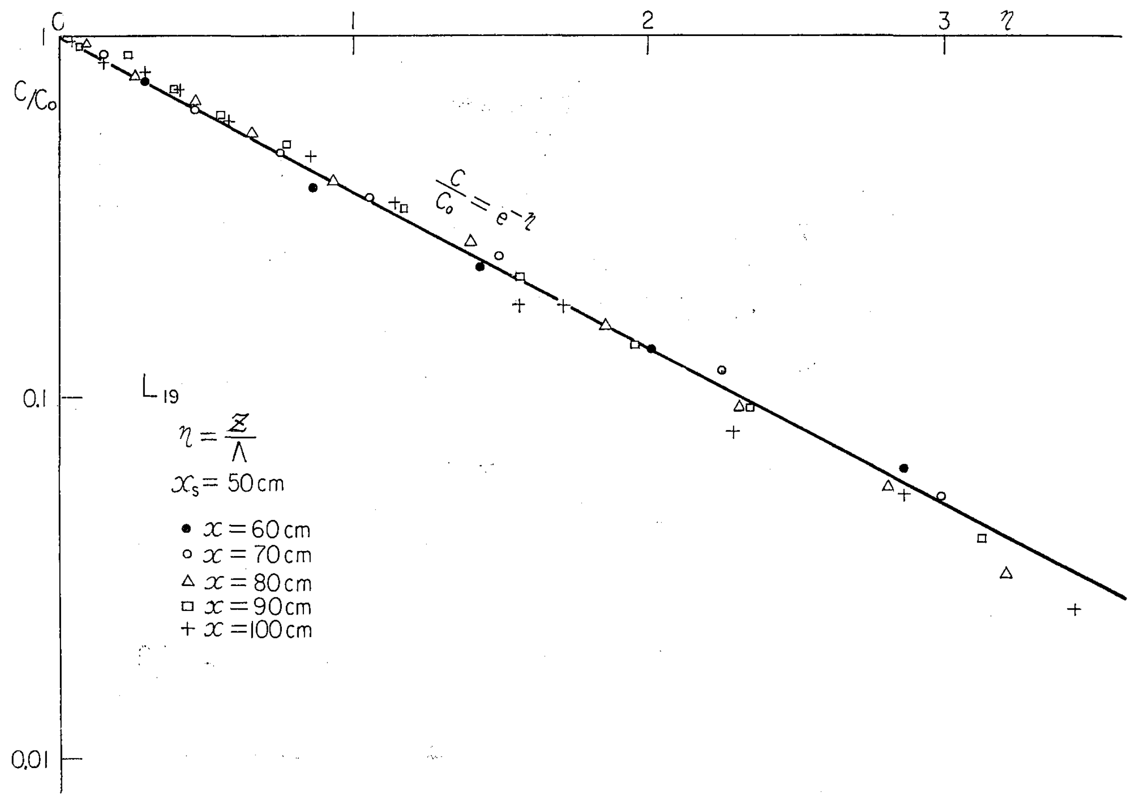

Fig. 8. Concentration distributions for L_{19}.

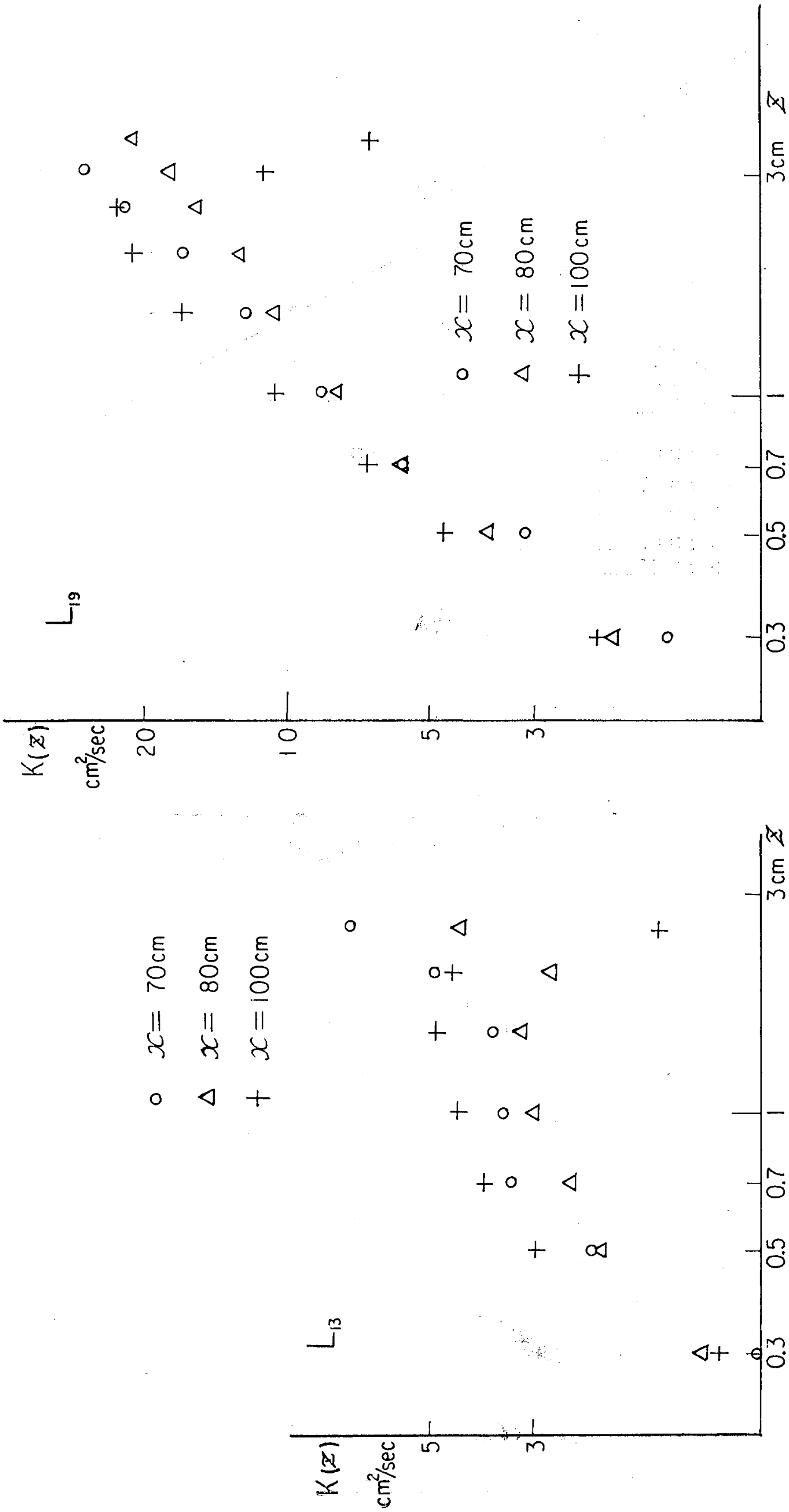

Fig. 9. Values of K(z) for L_{13} and L_{19}.

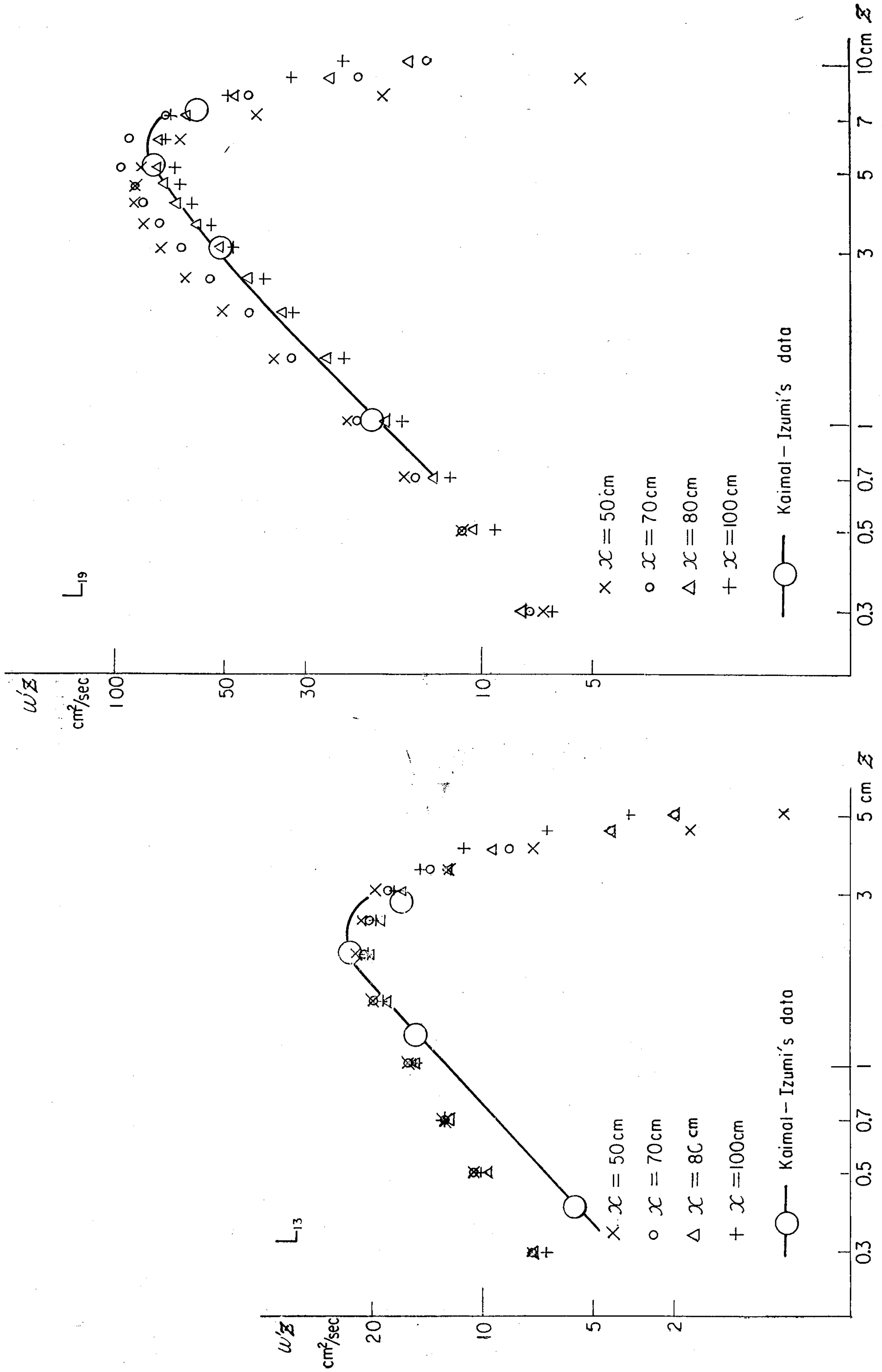

Fig. 10. Profiles of $w'z$ for L_{13} and L_{19}.

essentially we should adopt Lagrangian scale of time $\int L(\tau)d\tau$ or Eulerian scale of time $\int E(\tau)d\tau$, where $L(\tau)$ is Lagrangian correlation function and $E(\tau)$ is Eulerian one.

As it is shown in Figs. 9 and 10, the curves of $K(z)$ and $w'z$ for L_{19} are similar each other and their gradients are nearly 1. The curves for L_{13} are also similar each other and their gradients in the region of smaller z are nearly 0.6. Therefore, we can conclude that the quantity which can dominate $K(z)$ is that like $w'z$.

In the atmospheric diffusion, there are many cases in which the concentration distributions as those in the case of L_{19} occur. From the data of vertical distributions of w' in the atmosphere observed by Kaimal and Izumi[2], we can calculate the distribution of $w'z$, which is shown by circles and a full line in Fig. 10. This curve has close similarity to those for L_{19}, but do not have any similarity to those for L_{13}. So, in the case of diffusion experiments in wind tunnel, it is necessary to use turbulence generators which produce the profiles of $w'z$ similar to those in the atmosphere.

According to the results of Calder[3], Walters[4] and Ide[5], when u is proportional to z^m and $K(z)$ is proportional to z^α, the concentration C is proportional to $e^{-z^{(m-\alpha+2)}}$. m can be evaluated by the velocity distributions and $(m-\alpha+2)$ is evaluated by the concentration distributions, then α can be calculated. Values of α are ranging from 0.69 to 1.06 for L_{13} and from 1.00 to 1.10 for L_{19}. For L_{19} which has the similar turbulent structure as in the atmosphere. $(m-\alpha+2)$ is equal to 1, which corresponds to m equal to 0 and α equal to 1. This results coincide with that for which the velocity is constant along z and $K(z)$ is proportional to z. So, as Pasquill[6] has already remarked, we get the result that m equal to zero and α equal to one is a good approximation at least.

Future Planning

We are intending to observe the Lagrangian or Eulerian scale of time and also to examine the diffusion from elevated sources and then to investigate the diffusion in thermally stratified layers.

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