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Some Considerations on the Mechanism of Atmospheric Diffusion[1)]

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Abstract

From the observations of deformation of smoke lines in the atmosphere, we could conclude that the agents of diffusion are circular vortices existing in the atmosphere. As the data of the occurrence frequencies of vortices are not available at present, we could not obtain the diffusion formulae, so having compared our results with well known formulae of diffusion, we estimated the occurrence frequencies of vortices.

Introduction

In the case of molecular diffusion, the agents of the phenomenon are molecules, and its mechanism is physically a very comprehensive one which is based on the molecular motion itself or Brownian motion occurred by it. The results of theoretical calculations based on these models explain good the phenomenon.

On the other hand, in the case of atmospheric diffusion, the agents of the phenomenon are not so clear than in the former case. Furthermore, 'Verweilzeit' of the agents are not so short compared with the time of observation than in the case of molecular diffusion; and the number of the agents is not so numerous than in the other case. The scale of the agents which takes part in diffusion becomes larger as the time of observation becomes longer.

Therefore, when we apply directly the statistical treatment to the phenomenon, careful considerations seem to be necessary. So we intended to make clear the elementary mechanism of the process in which suspended particles deviate from their mean position.

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Apparatus

a) *Source of smoke* A nichrome wire (0.4 mm in diameter) is formed to make five tiny coils in every 2 cm. The number of winding is three and the diameter of the coil and its length are 2 mm either. This nichrome wire is fixed to a frame and is set vertically. Small amounts of grease are put to the coils and the wire is heated electrically. Then dense smoke lines become visible.

b) *Photographing method* A standard cinecamera is made to be able to adapt a stroboscopic flash light[2)] in order to obtain a very short duration of exposure (2×10^{-5} sec. ca.). We photographed the smoke lines with this cine-camera equipped with a stereographic attachment just as in the same way as we reported formerly (Sakagami[(2)]). Film speed is about 15 pictures per second.

In order to determine the time at which each picture is taken and the wind-speed at that instant, an electric clock whose hand rotates in every 1 sec, and a meter face of a hot-wire anemometer are photographed in the same picture.

Conditions of the observation

The observations were made at night in a clear space near the University. The frame of sources was about 1 m high above the ground. At further leeward distances than 60 cm ca., the deformation of the smoke lines became generally too intense to analyse, so we confined our observations up to 60 cm from the source.

Analysis

Deformations of the smoke lines were at first reproduced three dimensionally by the projecting method described in the former reports. (Sakagami[(3),(4)]; Mochizuki, Sakagami[(5)]). Then we traced the change of the deformed lines of every picture. (A sample of photographs is shown in Fig. 1).

In the ranges of field in our observations, smoke lines did not spread so remarkably, and remained fairly as thin lines. Immediately behind the sources the smoke lines were always shown as straight lines. Then in the course of time, small deformations appeared. If we follow these deformations in every picture, we can see that these deformations grew larger and larger and at last became to be so confused and accordingly to be so diluted that they became unable to

2) The set of the illumination was the same as that used in former experiments. (Mochizuki and Sakagami[(1)]).

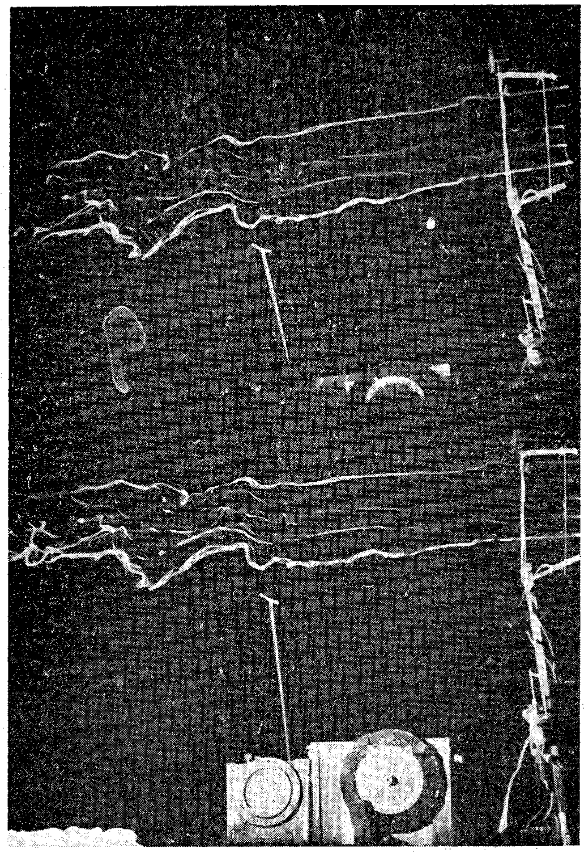

Fig. 1

be distinguished in detail; and that the changes of deformations always showed clearly the rotating nature. (Fig. 2) Therefore, the smoke lines seemed to be deviated from their prescribed positions by eddies existing in the atmosphere. (Fig. 3)

Using a number (700 ca) of very particular wind vanes (e. g. puppus wind vanes), we have reported that the atmospheric turbulence is explained by circular vortices existing in the wind, and measured their characteristic quantities such as diameter, vorticity, three dimensional form, orientation and life time (Sakagami[6],[7],[3],[9])

When we assume circular vortices with given sizes and given vorticities passing a smoke source, we can draw figures of the deformed smoke line caused by them by calculation (Fig. 4). Comparing these figures with those obtained by analysing the observational results, we can conclude that the immediate agents of the diffusion phenomenon are nothing but the circular vortices which have been observed as the constituents of the atmospheric turbulence.

Furthermore, we can estimate the quantities of the vortices which are effective for the deformation of the smoke lines, such as diameter, vorticity, etc. All these quantities are the same to those obtained by the former method.

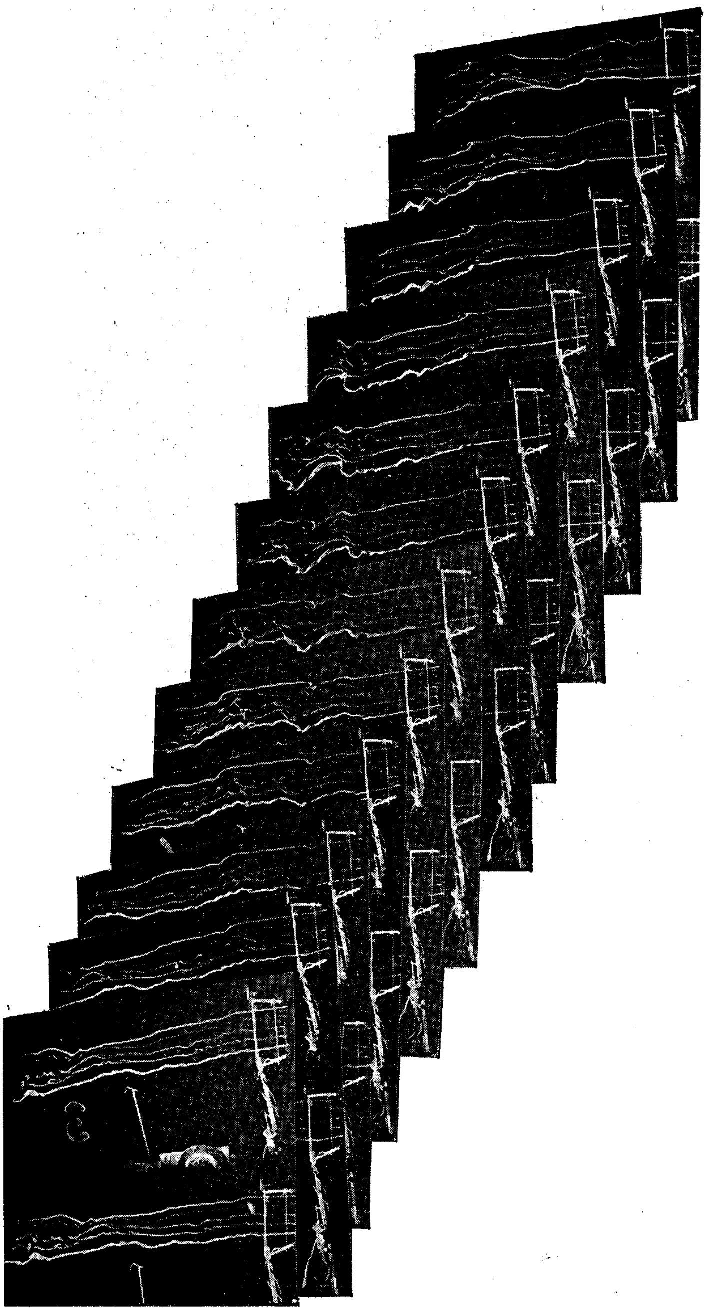

Fig. 2

Theoretical considerations

According to the standpoint that the agents of the atmospheric diffusion are circular vortices and that these deterministical processes are occurring at random, when a particle is released from a source, its position relative to the vortex changes from (x, y or r, θ ; $t=0$) to (x', y' or $r, \theta+\omega t, t=t$) during the time interval of t, where ω is the angular velocity, and the origin is at the center of the vortex and x axis is

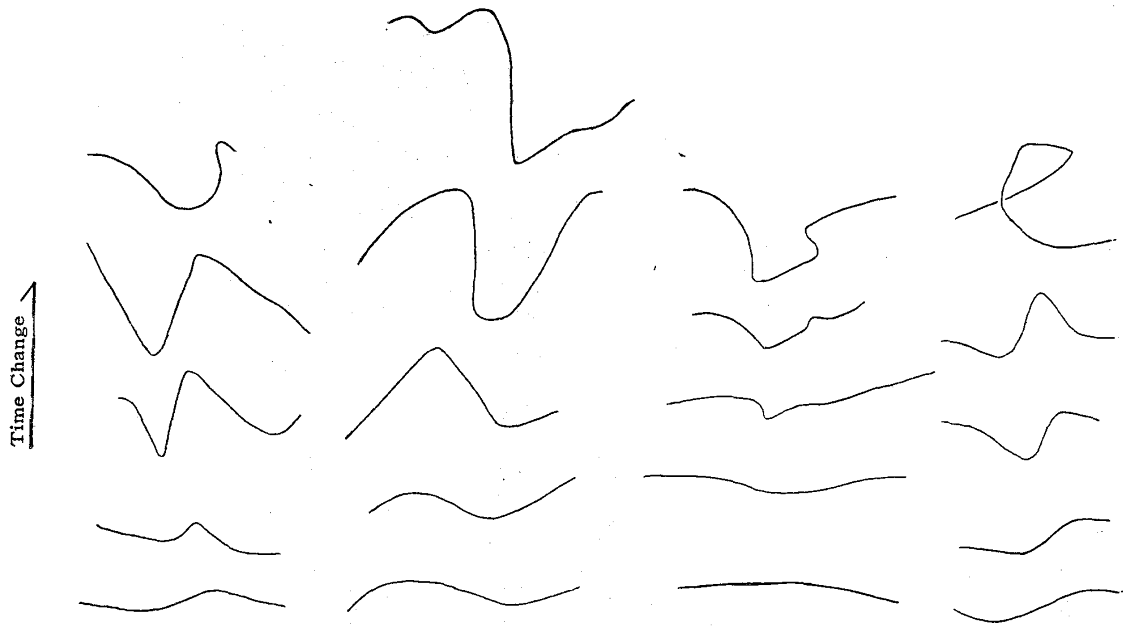

Fig. 3

taken to the direction of mean wind, z axis vertically and y axis perpendicularly to them. Then we get

$$\begin{aligned}\Delta x &= x' - x = r\{\cos(\theta+\omega t) - \cos\theta\} \\ &= -2r\sin\left(\theta+\frac{\omega t}{2}\right)\sin\frac{\omega t}{2} \\ \Delta y &= y' - y = r\{\sin(\theta+\omega t) - \sin\theta\} \\ &= 2r\cos\left(\theta+\frac{\omega t}{2}\right)\sin\frac{\omega t}{2}\end{aligned} \tag{1}$$

As for ω, from the assumption that the vortices are circular ones, following relations hold:

$$\begin{aligned} r &\leqq d/2, & \omega &= \omega_0 \\ r &\geqq d/2, & \omega &= \omega_0 d/2r\,, \end{aligned} \tag{2}$$

where ω_0 is the angular velocity of the part which is in solid rotation and d is the diameter of that part. If we write $r/(d/2)=\rho$, eq. (2) becomes

$$\begin{aligned} \rho &\leqq 1, & \omega &= \omega_0 \\ \rho &\geqq 1, & \omega &= \omega_0/\rho\,. \end{aligned} \tag{3}$$

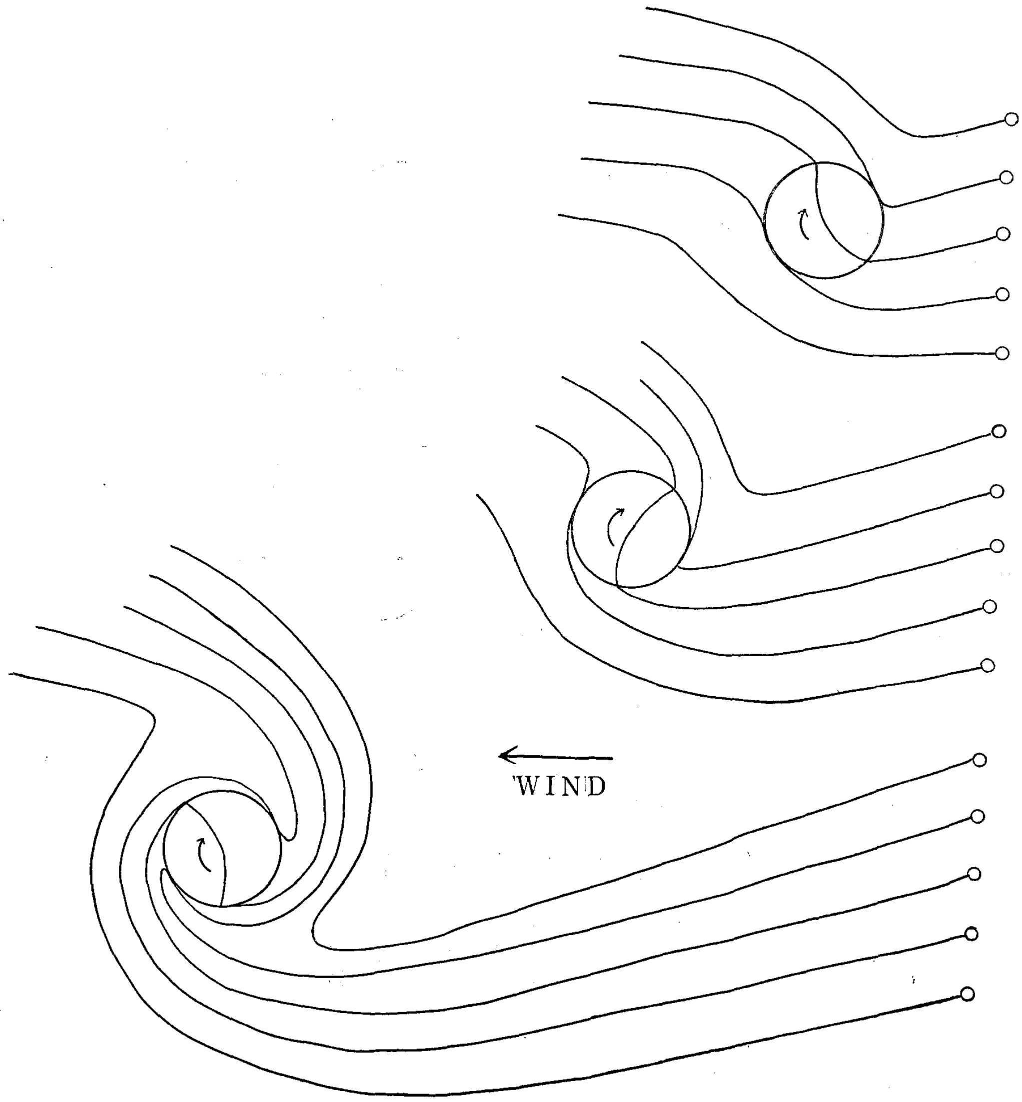

Fig. 4

Putting $\kappa = \Delta x/(d/2)$ and $\lambda = \Delta y/(d/2)$, we get from eq. (4),

$$\kappa = -2\rho \sin\left(\theta + \frac{\omega t}{2}\right) \sin \frac{\omega t}{2}$$

$$\lambda = 2\rho \cos\left(\theta + \frac{\omega t}{2}\right) \sin \frac{\omega t}{2} . \tag{4}$$

We shall hereafter treat only λ. The initial positions (ρ, θ) which change by a same amount λ during a time interval of t, can be determined by the next equations:

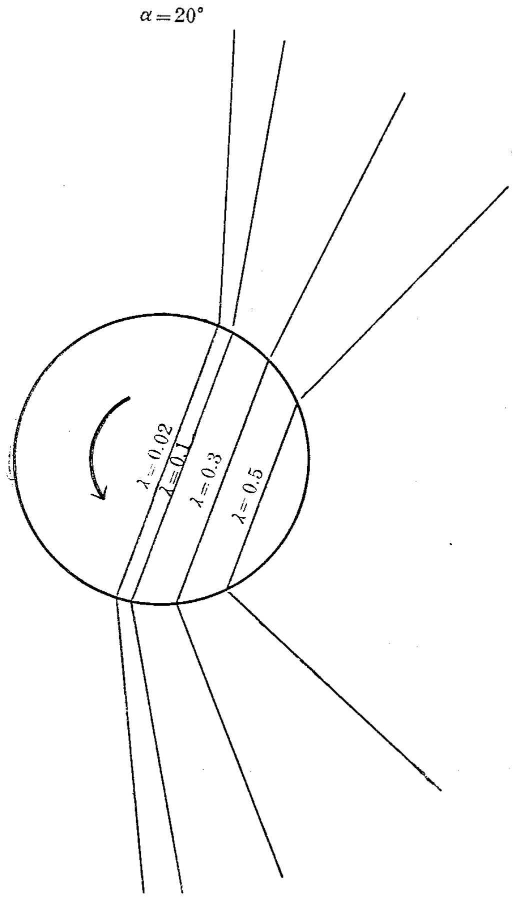

Fig. 5

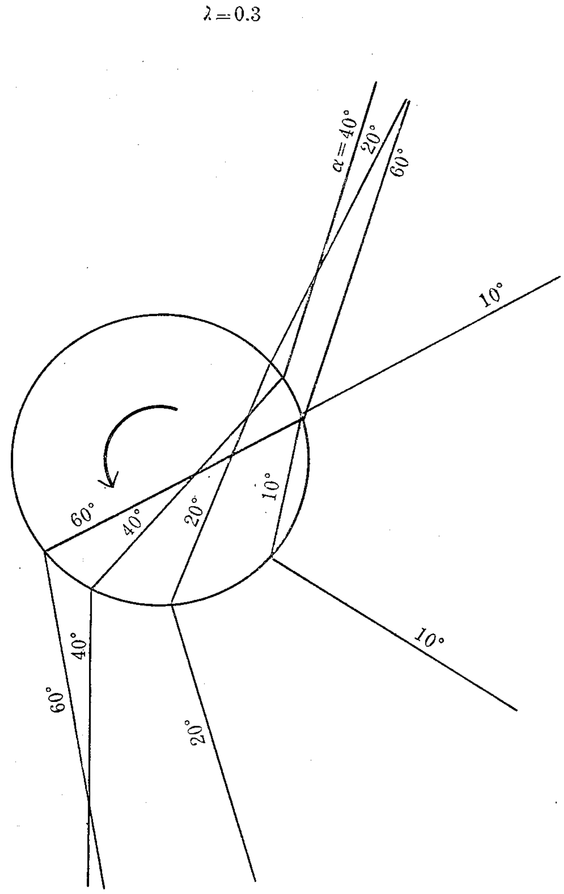

Fig. 6

$$\rho\cos\left(\theta+\frac{\omega_0 t}{2}\right)=\lambda/2\sin\frac{\omega_0 t}{2} \qquad \rho\leqq 1$$

$$\rho\cos\left(\theta+\frac{\omega_0 t}{2\rho}\right)=\lambda/2\sin\frac{\omega_0 t}{2\rho} \qquad \rho\geqq 1 \tag{5}$$

Fig. 5 is obtained from eq. (5) with various λ's and constant value of $\omega_0 t/2$ and Fig. 6 is obtained with various values of $\omega_0 t/2$'s and a constant value of λ[3].

When particles are released from a source at random in position relative to circular vortices with a constant size and vorticity, the length of these loci in Fig. 5 are proportional to the probability $p_i(\lambda)$ by which a particle moves by an amount λ during the time interval of t_i (Fig. 7. Each line corresponds to each value of t_i).

If we put $p_i(\lambda)=kf_i(\lambda)$, where $f_i(\lambda)$ is the length of the locus corresponding λ, and we put

3) In these calculations, values of λ are confined less than 3, because in the range of more larger values of λ, the effect of the vortex becomes considerably weak.

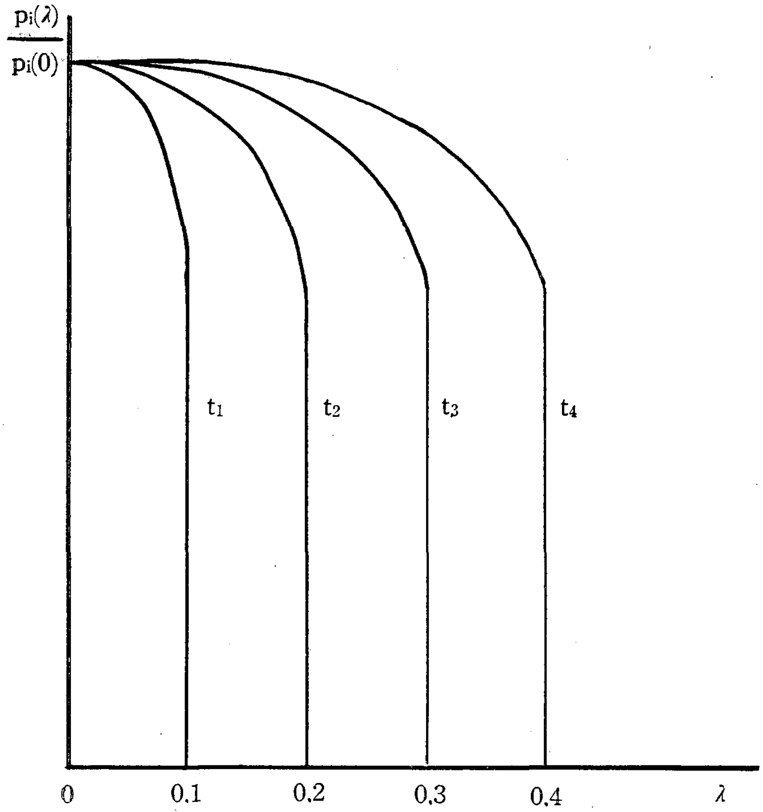

Fig. 7

$$f_i(0)=1\ ,$$

so we get

$$p_i(\lambda)=\frac{kf_i(\lambda)}{k\int_{-\lambda_m}^{\lambda_{m_i}} f_i(\lambda)d\lambda}=\frac{f_i(\lambda)}{2\int_0^{\lambda_{m_i}} f_i(\lambda)d\lambda}\ . \tag{6}$$

It holds in good approximation that all these curves can be expressed by a function $g(\xi)$, where $\xi=\lambda/\lambda_{m_i}$ and λ_{m_i} is equal to $2\sin(\omega_0 t_i/2)$.

$$p_i(\lambda)/p_i(0)=f_i(\lambda)=f_i(\lambda_{m_i}\xi)=g(\xi)\ . \tag{7}$$

Therefore,

$$p_i(\lambda)=p_i(0)g(\xi) \tag{8}$$

$$p_i(0)=f_i(0)/2\int_0^{\lambda_{m_i}} f(\lambda)d\lambda=1/2\int_0^1 \lambda_{m_i}g(\xi)d\xi$$

$$=1/2\lambda_{m_i}\int_0^1 g(\xi)d\xi\ . \tag{9}$$

The standard deviation of particle positions is given by

$$\Delta\overline{y_i^2}=\frac{d^2}{4}\,\overline{\Delta\lambda^2}=\frac{d^2}{4}\int_0^{\lambda_{m_i}} \lambda^2 p_i(\lambda)d\lambda$$

$$=\frac{d^2}{4}\int_0^1 \lambda_m^3\xi^2 p_i(\lambda_{m_i}\xi)d\xi=\frac{d^2}{4}p_i(0)\lambda_{m_i}^3\int_0^1\xi^2 g(\xi)d\xi$$

$$=\frac{d^2}{4}\frac{\lambda_{m_i}^3\int_0^1\xi g(\xi)d\xi}{2\lambda_{m_i}\int_0^1 g(\xi)d\xi}=Ad^2\lambda_{m_i}^2$$

$$=Ad^2\sin^2\left(\frac{\omega_0 t_i}{2}\right), \tag{10}$$

where A is a constant independent of i, and this equation is expressed in good approximation over a fairly wide range of $\omega_0 t_i/2$ by

$$\overline{\Delta y^2}\propto d^2\omega_0^2 t_i^2 . \tag{11}$$

We assume that the diffusion process is occurred deterministically during each process time t_p ($=x/U$, x: drifting distance, U: mean wind velocity), but position of particles, just emitted from the source, relative to the vortices, are at random.

a) *Floating source* When we consider floating sources, the time of observation t is equal to the Lagrangean process time t_p.

There exist various sizes of vortices in the atmosphere and the larger vortices are much more effective in diffusion than the smaller ones.

When t is small, however, the sizes of vortices which take part in the diffusion are not so various and can be considered to be constant. So we get

$$\overline{\Delta y^2}\propto d^2\omega_0^2 t^2\propto t^2$$

When t is larger, diameters of effective vortices become larger and we assume that, for the radius d_l of the largest vortex which takes part in the diffusion,

$$d_l\propto t$$

The relation between ω_0 and d is known (Inoue[10])

$$\omega_0\propto d^{-2/3} . \tag{12}$$

Then we get

$$\overline{\Delta y^2}\propto d^2 d^{-4/3}t^2=d^{2/3}t^2 . \tag{13}$$

The relation between occurrence frequencies $q(d)$ and diameters of vortices is not available at present, so we cannot proceed the calculation. Therefore, we shall consider reversely, and assuming $q(d)\propto d^m$, we shall determine the values of m in some cases from the well known results of diffusion.

Considering the occurrence frequencies, effective diameter d_e becomes, (changing the integrating variable d to δ),

$$d_e = \int_0^{d_l} q(\delta) d\delta = d_l^{m+1} \sim t^{m+1} . \tag{14}$$

So we get

$$\overline{\Delta y^2} \propto d_e^{2/3} t^2 = d_l^{2(m+1)/3} t^2 = t^{2m/3+8/3} . \tag{15}$$

It is known that when t is not small[4)]

$$\overline{\Delta y^2} \propto t^3 . \tag{16}$$

(Batchelor(11), Gifford(12), Inoue(13)(14), Ogura 1953(15)). So we get

$$\frac{2m}{3} + \frac{8}{3} = 3, \quad \text{consequently}$$

$$m = 1/2 \tag{17}$$

b) *Fixed source* In the case of fixed sources, time of observation t is larger than process time t_p, and many elementary processes which occur during t_p are repeated during t. Resultant standard deviation $(\overline{\Delta y^2})_R$ is the sum of the standard deviation in t_p.

$$(\overline{\Delta y^2})_R \propto t_p^2 \int_0^{d_e} d_e^{3/2} d(d_e) = t_p^2 d_e^{5/3}$$

$$= t_p^2 d^{5(m+1)/3} \sim t_p^2 t^{5(m+1)/3} \tag{18}$$

When t is small, $(\overline{\Delta y^2})_R$ should be proportional to t^2, so we get

$$\frac{5}{3}(1+m) = 2, \quad \text{and therefore,}$$

$$m = 1/5 . \tag{19}$$

When t is large, $(\overline{\Delta y^2})_R$ should be proportional to t, so we get

$$\frac{5}{3}(1+m) = 1, \quad \text{and consequently,}$$

$$m = -\frac{2}{5} . \tag{20}$$

From these results, we obtain for the values of m:

$m = \frac{1}{2}$ when t, accordingly the range of d, is very small,

4) But it is naturally very small compared with the time of observation which will be considered in the next case:—Fixed source.

$$m=\frac{1}{5} \quad \text{when } t, \text{ accordingly the range of } d, \text{ is small,}$$
$$m=-\frac{2}{5} \quad \text{when } t, \text{ accordingly the range of } d, \text{ is large.} \tag{21}$$

Conclusion

We have shown that the elementary process of the diffusion is conducted by circular vortices which are the constituents of the atmospheric turbulence. So if we can investigate throughly the characteristic quantities, such as diameter, vorticity, orientation and life time, by some methods, for example, by a number of wind vanes, we should estimate the amount of diffusion.

Especially the vortices orientate nearly vertically near the ground, it is probable to find the relation between horizontal diffusion coefficient and vertical one. We are intending to examine the fundamental quantities of vortices and to built up formulae of diffusion.

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