Tables Useful for the Calculation of the Molecular Integrals III[1)]

Eiichi Ishiguro (石黒英一)
Department of Physics, Faculty of Science,
Ochanomizu University
Tadashi Arai[2)] and **Masataka Mizushima**[2)]

(Received April 30, 1952)

In Part III, the following numeral tables are given:
Table VIII:

$$C_\tau^\nu(k)=\frac{1}{2}\int_{-1}^{1}P_\tau^\nu(\mu)(1-\mu^2)^{\frac{\nu}{2}}\mu^k d\mu \qquad \text{for} \quad \nu=2.$$

Table IX:

$$\overline{C_\tau^\nu}(k)=C_\tau^\nu(k)-C_\tau^\nu(k+2) \qquad \text{for} \quad \nu=2.$$

Table X, XI, XII

$$C_\tau^\nu(j,k)=C_\tau^\nu(k,j)=C_\tau^\nu(j)C_\tau^\nu(k) \qquad \text{for} \quad \nu=0,\ 1,\ 2.$$

Table XIII, XIV, XV:

$$\overline{C_\tau^\nu}(j,k)=\overline{C_\tau^\nu}(k,j)=\overline{C_\tau^\nu}(j)\overline{C_\tau^\nu}(k) \qquad \text{for} \quad \nu=0,\ 1,\ 2.$$

In appendix, the analytical expressions of $C_\tau^\nu(k)$ and $\overline{C_\tau^\nu}(k)$ for $\nu=0$, 1 and 2, are given.

Table VIII.

$C_\tau^2(k)$

τ \ k	0	1	2	3	4	5	6	7
2	$\frac{8}{5}$	0	$\frac{8}{35}$	0	$\frac{8}{105}$	0	$\frac{8}{231}$	0
3	0	$\frac{8}{7}$	0	$\frac{8}{21}$	0	$\frac{40}{231}$	0	$\frac{40}{429}$
4	0	0	$\frac{16}{21}$	0	$\frac{32}{77}$	0	$\frac{240}{1001}$	0
5	0	0	0	$\frac{16}{33}$	0	$\frac{160}{429}$	0	$\frac{112}{429}$
6	0	0	0	0	$\frac{128}{429}$	0	$\frac{128}{429}$	0
7						$\frac{128}{715}$	0	$\frac{2688}{12155}$

1) Contribution from Department of Physics, Faculty of Science, Ochanomizu University, No. 10

2) Department of Physics, Faculty of Science, University of Tokyo

Table IX

$\overline{C_\tau{}^2(k)}$

τ \ k	0	1	2	3	4	5	6
2	$\frac{48}{35}$	0	$\frac{16}{105}$	0	$\frac{16}{385}$	0	$\frac{16}{1001}$
3	0	$\frac{16}{21}$	0	$\frac{16}{77}$	0	$\frac{80}{1001}$	0
4	$-\frac{16}{21}$	0	$\frac{80}{231}$	0	$\frac{16}{91}$	0	$\frac{272}{3003}$
5	0	$-\frac{16}{33}$	0	$\frac{16}{143}$	0	$\frac{16}{143}$	0
6	0	0	$-\frac{128}{429}$	0	0	0	$\frac{128}{2431}$
7	0	0	0	$-\frac{128}{715}$	0	$-\frac{512}{12155}$	0

Table X

$C_0{}^0(j, k)$

j \ k	0	2	4	6	8
0	1	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{7}$	$\frac{1}{9}$
2	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{15}$	$\frac{1}{21}$	$\frac{1}{27}$
4	$\frac{1}{5}$	$\frac{1}{15}$	$\frac{1}{25}$	$\frac{1}{35}$	$\frac{1}{45}$
6	$\frac{1}{7}$	$\frac{1}{21}$	$\frac{1}{35}$	$\frac{1}{49}$	$\frac{1}{63}$
8	$\frac{1}{9}$	$\frac{1}{27}$	$\frac{1}{45}$	$\frac{1}{63}$	$\frac{1}{81}$

$C_1{}^0(j, k)$

j \ k	1	3	5	7	9
1	$\frac{1}{9}$	$\frac{1}{15}$	$\frac{1}{21}$	$\frac{1}{27}$	$\frac{1}{33}$
3	$\frac{1}{15}$	$\frac{1}{25}$	$\frac{1}{35}$	$\frac{1}{45}$	$\frac{1}{55}$
5	$\frac{1}{21}$	$\frac{1}{35}$	$\frac{1}{49}$	$\frac{1}{63}$	$\frac{1}{77}$
7	$\frac{1}{27}$	$\frac{1}{45}$	$\frac{1}{63}$	$\frac{1}{81}$	$\frac{1}{99}$
9	$\frac{1}{33}$	$\frac{1}{55}$	$\frac{1}{77}$	$\frac{1}{99}$	$\frac{1}{121}$

$C_2{}^0(j, k)$

j \ k	2	4	6	8
2	$\frac{4}{225}$	$\frac{8}{525}$	$\frac{4}{315}$	$\frac{16}{1485}$
4	$\frac{8}{525}$	$\frac{16}{1225}$	$\frac{8}{735}$	$\frac{32}{3465}$
6	$\frac{4}{315}$	$\frac{8}{735}$	$\frac{4}{441}$	$\frac{16}{2079}$
8	$\frac{16}{1485}$	$\frac{32}{3465}$	$\frac{16}{2079}$	$\frac{64}{9801}$

$C_3{}^0(j, k)$

j \ k	3	5	7	9
3	$\frac{4}{1225}$	$\frac{8}{2205}$	$\frac{4}{1155}$	$\frac{16}{5005}$
5	$\frac{8}{2205}$	$\frac{16}{3969}$	$\frac{8}{2079}$	$\frac{32}{9009}$
7	$\frac{4}{1155}$	$\frac{8}{2079}$	$\frac{4}{1089}$	$\frac{16}{4719}$
9	$\frac{16}{5005}$	$\frac{32}{9009}$	$\frac{16}{4719}$	$\frac{64}{20449}$

$C_4^0(j,k)$

j \ k	4	6	8
4	64/99225	64/72765	128/1 35135
6	64/72765	64/53361	128/99099
8	128/1 35135	128/99099	256/1 84041

$C_5^0(j,k)$

j \ k	5	7	9
5	64/4 80249	64/2 97297	128/4 95495
7	64/2 97297	64/1 84041	128/3 06735
9	128/4 95495	128/3 06735	256/5 11225

$C_6^0(j,k)$

j \ k	6	8
6	256/90 18009	1024/193 24305
8	1024/193 24305	4096/414 09225

$C_7^0(j,k)$

j \ k	7	9
7	256/414 09225	1024/782 17425
9	1024/782 17425	4096/1477 44025

$C_8^0(j,k)$

j \ k	8
8	16384/1 19672 66025

$C_9^0(j,k)$

j \ k	9
9	16384/5 33355 93025

Table XI

$C_1^1(j,k)$

j \ k	0	2	4	6	8
0	4/9	4/45	4/105	4/189	4/297
2	4/45	4/225	4/525	4/945	4/1485
4	4/105	4/525	4/1225	4/2205	4/3465
6	4/189	4/945	4/2205	4/3969	4/6237
8	4/297	4/1485	4/3465	4/6237	4/9801

$C_2^1(j,k)$

j \ k	1	3	5	7
1	4/25	12/175	4/105	4/165
3	12/175	36/1225	4/245	4/385
5	4/105	4/245	4/441	4/693
7	4/165	4/385	4/693	4/1089

$C_3^1(j,k)$

j \ k	2	4	6	8
2	64/1225	128/3675	64/2695	256/15015
4	128/3675	256/11025	128/8085	512/45045
6	64/2695	128/8085	64/5929	256/33033
8	256/15015	512/45045	256/33033	1024/1 84041

$C_4^1(j,k)$

j \ k	3	5	7
3	64/3969	640/43659	320/27027
5	640/43659	6400/4 80249	3200/2 97297
7	320/27027	3200/2 97297	1600/1 84041

$C_5^1(j,k)$

j \ k	4	6	8
4	256 53361	1280 2 31231	512 99099
6	1280 2 31231	6400 10 02001	2560 4 29429
8	512 99099	2560 4 29429	1024 1 84041

$C_6^1(j,k)$

j \ k	5	7
5	256 1 84041	1792 9 20205
7	1792 9 20205	12544 46 01025

$C_7^1(j,k)$

j \ k	6	8
6	16384 414 09225	4 58752 7039 56825
8	4 58752 7039 56825	128 45056 1 19672 66025

Table XII

$C_2^2(j,k)$

j \ k	0	2	4	6
0	64 25	64 175	64 525	64 1155
2	64 175	64 1225	64 3675	64 8085
4	64 525	64 3675	64 11025	64 24255
6	64 1155	64 8085	64 24255	64 53361

$C_3^2(j,k)$

j \ k	1	3	5
1	64 49	64 147	320 1617
3	64 147	64 441	320 4851
5	320 1617	320 4851	1600 53361

$C_4^2(j,k)$

j \ k	2	4	6
2	256 441	512 1617	1280 7007
4	512 1617	1024 5929	7680 77077
6	1280 7007	7680 77077	57600 10 02001

$C_5^2(j,k)$

j \ k	3	5
3	256 1089	2560 14157
5	2560 14157	25600 1 84041

$C_6^2(j,k)$

j \ k	4	6
4	16384 1 84041	16384 1 84041
6	16384 1 84041	16384 1 84041

Table XIII

$\overline{C_0^0}(j,k)$

j \ k	0	2	4	6
0	$\frac{4}{9}$	$\frac{4}{45}$	$\frac{4}{105}$	$\frac{4}{189}$
2	$\frac{4}{45}$	$\frac{4}{225}$	$\frac{4}{525}$	$\frac{4}{945}$
4	$\frac{4}{105}$	$\frac{4}{525}$	$\frac{4}{1225}$	$\frac{4}{2205}$
6	$\frac{4}{189}$	$\frac{4}{945}$	$\frac{4}{2205}$	$\frac{4}{3969}$

$\overline{C_1^0}(j,k)$

j \ k	1	3	5
1	$\frac{4}{225}$	$\frac{4}{525}$	$\frac{4}{945}$
3	$\frac{4}{525}$	$\frac{4}{1225}$	$\frac{4}{2205}$
5	$\frac{4}{945}$	$\frac{4}{2205}$	$\frac{4}{3969}$

$\overline{C_2^0}(j,k)$

j \ k	0	2	4	6
0	$\frac{4}{225}$	$-\frac{4}{1575}$	$-\frac{4}{1575}$	$-\frac{4}{2079}$
2	$-\frac{4}{1575}$	$\frac{4}{11025}$	$\frac{4}{11025}$	$\frac{4}{14553}$
4	$-\frac{4}{1575}$	$\frac{4}{11025}$	$\frac{4}{11025}$	$\frac{4}{14553}$
6	$-\frac{4}{2079}$	$\frac{4}{14553}$	$\frac{4}{14553}$	$\frac{100}{4\ 80249}$

$\overline{C_3^0}(j,k)$

j \ k	1	3	5
1	$\frac{4}{1225}$	$\frac{4}{11025}$	$-\frac{4}{24255}$
3	$\frac{4}{11025}$	$\frac{4}{99225}$	$-\frac{4}{2\ 18295}$
5	$-\frac{4}{24255}$	$-\frac{4}{2\ 18295}$	$\frac{4}{4\ 80249}$

$\overline{C_4^0}(j,k)$

j \ k	2	4	6
2	$\frac{64}{99225}$	$\frac{256}{10\ 91475}$	$\frac{64}{9\ 45945}$
4	$\frac{256}{10\ 91475}$	$\frac{1024}{120\ 06225}$	$\frac{256}{104\ 05395}$
6	$\frac{64}{9\ 45945}$	$\frac{256}{104\ 05395}$	$\frac{64}{90\ 18009}$

$\overline{C_5^0}(j,k)$

j \ k	3	5
3	$\frac{64}{4\ 80249}$	$\frac{512}{62\ 43237}$
5	$\frac{512}{62\ 43237}$	$\frac{4096}{811\ 62081}$

$\overline{C_6^0}(j,k)$

j \ k	4	6
4	$\frac{256}{90\ 18009}$	$\frac{256}{104\ 05395}$
6	$\frac{256}{104\ 05395}$	$\frac{256}{120\ 06225}$

$\overline{C_7^0}(j,k)$

j \ k	5
5	$\frac{256}{414\ 09225}$

$\overline{C_8^0}(j,k)$

j \ k	6
6	$\frac{16384}{1\ 19672\ 66025}$

Table XIV

$\overline{C_1^1}(j,k)$

j \ k	0	2	4	6
0	$\frac{64}{225}$	$\frac{64}{1575}$	$\frac{64}{4725}$	$\frac{64}{10395}$
2	$\frac{64}{1575}$	$\frac{64}{11025}$	$\frac{64}{33075}$	$\frac{64}{72765}$
4	$\frac{64}{4725}$	$\frac{64}{33075}$	$\frac{64}{99225}$	$\frac{64}{2\ 18295}$
6	$\frac{64}{10395}$	$\frac{64}{72765}$	$\frac{64}{2\ 18295}$	$\frac{64}{4\ 80249}$

$\overline{C_2^1}(j,k)$

j \ k	1	3	5
1	$\frac{64}{1225}$	$\frac{64}{3675}$	$\frac{64}{8085}$
3	$\frac{64}{3675}$	$\frac{64}{11025}$	$\frac{64}{24255}$
5	$\frac{64}{8085}$	$\frac{64}{24255}$	$\frac{64}{53361}$

$\overline{C_3^1}(j,k)$

j \ k	0	2	4	6
0	$\frac{64}{1225}$	$-\frac{64}{3675}$	$-\frac{64}{5775}$	$-\frac{64}{9555}$
2	$-\frac{64}{3675}$	$\frac{64}{11025}$	$\frac{64}{17325}$	$\frac{64}{28665}$
4	$-\frac{64}{5775}$	$\frac{64}{17325}$	$\frac{64}{27225}$	$\frac{64}{45045}$
6	$-\frac{64}{9555}$	$\frac{64}{28665}$	$\frac{64}{45045}$	$\frac{64}{74529}$

$\overline{C_4^1}(j,k)$

j \ k	1	3	5
1	$\frac{64}{3969}$	$-\frac{64}{43659}$	$-\frac{1600}{5\ 67567}$
3	$-\frac{64}{43659}$	$\frac{64}{4\ 80249}$	$\frac{1600}{62\ 43237}$
5	$-\frac{1600}{5\ 67567}$	$\frac{1600}{62\ 43237}$	$\frac{40000}{811\ 62081}$

$\overline{C_5^1}(j,k)$

j \ k	2	4	6
2	$\frac{256}{53361}$	$\frac{512}{6\ 93693}$	$-\frac{256}{6\ 93693}$
4	$\frac{512}{6\ 93693}$	$\frac{1024}{90\ 18009}$	$-\frac{512}{90\ 18009}$
6	$-\frac{256}{6\ 93693}$	$-\frac{512}{90\ 18009}$	$\frac{256}{90\ 18009}$

$\overline{C_6^1}(j,k)$

j \ k	3	5
3	$\frac{256}{1\ 84041}$	$\frac{512}{9\ 20205}$
5	$\frac{512}{9\ 20205}$	$\frac{1024}{46\ 01025}$

$\overline{C_7^1}(j,k)$

j \ k	4	6
4	$\frac{16384}{414\ 09225}$	$\frac{16384}{639\ 96075}$
6	$\frac{16384}{639\ 96075}$	$\frac{16384}{989\ 03025}$

$\overline{C_8^1}(j,k)$

j \ k	5
5	$\frac{16384}{1477\ 44025}$

Table XV

$\overline{C_2^2}(j,k)$

j \ k	0	2	4	6
0	$\frac{2304}{1225}$	$\frac{768}{3675}$	$\frac{768}{13475}$	$\frac{768}{35035}$
2	$\frac{768}{3675}$	$\frac{256}{11025}$	$\frac{256}{40425}$	$\frac{256}{1\ 05105}$
4	$\frac{768}{13475}$	$\frac{256}{40425}$	$\frac{256}{1\ 48225}$	$\frac{256}{3\ 85385}$
6	$\frac{768}{35035}$	$\frac{256}{1\ 05105}$	$\frac{256}{3\ 85385}$	$\frac{256}{10\ 02001}$

$\overline{C_3^2}(j,k)$

j \ k	1	3	5
1	$\frac{256}{441}$	$\frac{256}{1617}$	$\frac{1280}{21012}$
3	$\frac{256}{1617}$	$\frac{256}{5929}$	$\frac{1280}{77077}$
5	$\frac{1280}{21021}$	$\frac{1280}{77077}$	$\frac{6400}{10\ 02001}$

$\overline{C_4^2}(j,k)$

j \ k	0	2	4	6
0	$\frac{256}{441}$	$-\frac{1280}{4851}$	$-\frac{256}{1911}$	$-\frac{4352}{63063}$
2	$-\frac{1280}{4851}$	$\frac{6400}{53361}$	$\frac{1280}{21021}$	$\frac{21760}{6\ 93693}$
4	$-\frac{256}{1911}$	$\frac{1280}{21021}$	$\frac{256}{8281}$	$\frac{4352}{2\ 73273}$
6	$-\frac{4352}{63063}$	$\frac{21760}{6\ 93693}$	$\frac{4352}{2\ 73273}$	$\frac{73984}{90\ 18009}$

$\overline{C_5^2}(j,k)$

j \ k	1	3	5
1	$\frac{256}{1089}$	$-\frac{256}{4719}$	$-\frac{256}{4719}$
3	$-\frac{256}{4719}$	$\frac{256}{20449}$	$\frac{256}{20449}$
5	$-\frac{256}{4719}$	$\frac{256}{20449}$	$\frac{256}{20449}$

$\overline{C_6^2}(j,k)$

j \ k	2	4	6
2	$\frac{16384}{1\ 84041}$	0	$-\frac{16384}{10\ 42899}$
4	0	0	0
6	$-\frac{16384}{10\ 42899}$	0	$\frac{16384}{59\ 09761}$

$\overline{C_7^2}(j,k)$

j \ k	3	5
3	$\frac{16384}{5\ 11225}$	$\frac{65536}{86\ 90825}$
5	$\frac{65536}{86\ 90825}$	$\frac{2\ 62144}{1477\ 44025}$

Appendix I

$C_\tau^0(k)$

for τ=even, k=even

$$C_0^0(k)=\frac{1}{k+1}$$

$$C_2^0(k)=\frac{k}{(k+1)(k+3)}$$

$$C_4^0(k)=\frac{k(k-2)}{(k+1)(k+3)(k+5)}$$

$$C_6^0(k)=\frac{k(k-2)(k-4)}{(k+1)(k+3)(k+5)(k+7)}$$

$$C_8^0(k)=\frac{k(k-2)(k-4)(k-6)}{(k+1)(k+3)(k+5)(k+7)(k+9)}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$C_\tau^0(k)=\frac{k(k-2)(k-4)\cdots\cdots(k-\tau+2)}{(k+1)(k+3)(k+5)\cdots(k+\tau+1)}$$

for τ=odd, k=odd

$$C_1^0(k)=\frac{1}{k+2}$$

$$C_3^0(k)=\frac{k-1}{(k+2)(k+4)}$$

$$C_5^0(k)=\frac{(k-1)(k-3)}{(k+2)(k+4)(k+6)}$$

$$C_7^0(k)=\frac{(k-1)(k-3)(k-5)}{(k+2)(k+4)(k+6)(k+8)}$$

$$C_9^0(k)=\frac{(k-1)(k-3)(k-5)(k-7)}{(k+2)(k+4)(k+6)(k+8)(k+10)}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$C_\tau^0(k)=\frac{(k-1)(k-3)(k-5)\cdots(k-\tau+2)}{(k+2)(k+4)(k+6)\cdots(k+\tau+1)}$$

otherwise $C_\tau^0(k)=0$

$C_\tau^1(k)$

for τ=even, k=odd

$$C_2^1(k)=\frac{6}{(k+2)(k+4)}$$

$$C_4^1(k)=\frac{20(k-1)}{(k+2)(k+4)(k+6)}$$

$$C_6^1(k)=\frac{42(k-1)(k-3)}{(k+2)(k+4)(k+6)(k+8)}$$

$$C_8^1(k)=\frac{72(k-1)(k-3)(k-5)}{(k+2)(k+4)(k+6)(k+8)(k+10)}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$C_\tau^1(k)=\frac{\tau(\tau+1)(k-1)(k-3)\cdots\cdots(k-\tau+3)}{(k+2)(k+4)(k+6)\cdots\cdots(k+\tau+2)}$$

for τ=odd, k=even

$$C_1^1(k)=\frac{2}{(k+1)(k+3)}$$

$$C_3^1(k)=\frac{12k}{(k+1)(k+3)(k+5)}$$

$$C_5^1(k)=\frac{30k(k-2)}{(k+1)(k+3)(k+5)(k+7)}$$

$$C_7^1(k)=\frac{56k(k-2)(k-4)}{(k+1)(k+3)(k+5)(k+7)(k+9)}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$C_\tau^1(k)=\frac{\tau(\tau+1)k(k-2)(k-4)\cdots(k-\tau+3)}{(k+1)(k+3)(k+5)\cdots\cdots(k+\tau+2)}$$

otherwise $C_\tau^1(k)=0$

$C_\tau^2(k)$

for τ=even, k=even

$$C_2^2(k)=\frac{24}{(k+1)(k+3)(k+5)}$$

$$C_4^2(k)=\frac{360k}{(k+1)(k+3)(k+5)(k+7)}$$

$$C_6^2(k)=\frac{1680k(k-2)}{(k+1)(k+3)(k+5)(k+7)(k+9)}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$C_\tau^2(k)=\frac{\tau(\tau^2-1)(\tau+2)k(k-2)(k-4)\cdots(k-\tau+4)}{(k+1)(k+3)(k+5)\cdots\cdots\cdots(k+\tau+3)}$$

for τ=odd, k=odd

$$C_3^2(k)=\frac{120}{(k+2)(k+4)(k+6)}$$

$$C_5^2(k)=\frac{840(k-1)}{(k+2)(k+4)(k+6)(k+8)}$$

$$C_7^2(k)=\frac{3024(k-1)(k-3)}{(k+2)(k+4)(k+6)(k+8)(k+10)}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$C_\tau^2(k)=\frac{\tau(\tau^2-1)(\tau+2)(k-1)(k-3)\cdots(k-\tau+4)}{(k+2)(k+4)(k+6)\cdots\cdots(k+\tau+3)}$$

otherwise $C_\tau^2(k)=0$

Appendix II

$\overline{C_\tau^0}(k)$

for τ=even, k=even

$$\overline{C_0^0}(k)=\frac{2}{(k+1)(k+3)}$$

$$\overline{C_2^0}(k)=\frac{2(k-1)}{(k+1)(k+3)(k+5)}$$

$$\overline{C_4^0}(k)=\frac{2(k-8)k}{(k+1)(k+3)(k+5)(k+7)}$$

$$\overline{C_6^0}(k)=\frac{2(k-19)k(k-2)}{(k+1)(k+3)(k+5)(k+7)(k+9)}$$

$$\overline{C_8^0}(k)=\frac{2(k-34)k(k-2)(k-4)}{(k+1)(k+3)(k+5)(k+7)(k+9)(k+11)}$$

.....

.....

$$\overline{C_\tau^0}(k)=\frac{(2k-\tau^2-\tau+4)k(k-2)(k-4)\cdots(k-\tau+4)}{(k+1)(k+3)(k+5).....(k+\tau+3)}$$

for τ=odd, k=odd

$$\overline{C_1^0}(k)=\frac{2}{(k+2)(k+4)}$$

$$\overline{C_3^0}(k)=\frac{2(k-4)}{(k+2)(k+4)(k+6)}$$

$$\overline{C_5^0}(k)=\frac{2(k-13)(k-1)}{(k+2)(k+4)(k+6)(k+8)}$$

$$\overline{C_7^0}(k)=\frac{2(k-26)(k-1)(k-3)}{(k+2)(k+4)(k+6)(k+8)(k+10)}$$

$$\overline{C_9^0}(k)=\frac{2(k-43)(k-1)(k-3)(k-5)}{(k+2)(k+4)(k+6)(k+8)(k+10)(k+12)}$$

.....

.....

$$\overline{C_\tau^0}(k)=\frac{(2k-\tau^2-\tau+4)(k-1)(k-3)\ldots(k-\tau+4)}{(k+2)(k+4)(k+6).....(k+\tau+3)}$$

otherwise $\overline{C_\tau^0}(k)=0$

$\overline{C_\tau^1}(k)$

for τ=even, k=odd

$$\overline{C_2^1}(k)=\frac{24}{(k+2)(k+4)(k+6)}$$

$$\overline{C_4^1}(k)=\frac{40(2k-5)}{(k+2)(k+4)(k+6)(k+8)}$$

$$\overline{C_6^1}(k)=\frac{168(k-8)(k-1)}{(k+2)(k+4)(k+6)(k+8)(k+10)}$$

$$\overline{C_8^1}(k)=\frac{144(2k-31)(k-1)(k-3)}{(k+2)(k+4)(k+6)(k+8)(k+10)(k+12)}$$

.....

.....

.....

$$\overline{C_\tau^1}(k)$$

$$=\frac{\tau(\tau+1)(4k-\tau^2-\tau+10)(k-1)(k-3)..(k-\tau+5)}{(k+2)(k+4)(k+6).....(k+\tau+4)}$$

for τ=odd, k=even

$$\overline{C_1^1}(k)=\frac{8}{(k+1)(k+3)(k+5)}$$

$$\overline{C_3^1}(k)=\frac{24(2k-1)}{(k+1)(k+3)(k+5)(k+7)}$$

$$\overline{C_5^1}(k)=\frac{120(k-5)k}{(k+1)(k+3)(k+5)(k+7)(k+9)}$$

$$\overline{C_7^1}(k)=\frac{112(2k-23)k(k-2)}{(k+1)(k+3)(k+5)(k+7)(k+9)(k+11)}$$

.....

.....

.....

$$\overline{C_\tau^1}(k)=\frac{\tau(\tau+1)(4k-\tau^2-\tau+10)k(k-2)(k-4)}{....(k-\tau+5)}$$
$$(k+1)(k+3)(k+5)....(k+\tau+4)$$

otherwise $\overline{C_\tau^1}(k)=0$

$\overline{C_\tau^2}(k)$

for τ=even, k=even

$$\overline{C_2^2}(k)=\frac{144}{(k+1)(k+3)(k+5)(k+7)}$$

$$\overline{C_4^2}(k)=\frac{720(3k-1)}{(k+1)(k+3)(k+5)(k+7)(k+9)}$$

$$\overline{C_6^2}(k)=\frac{10080(k-4)k}{(k+1)(k+3)(k+5)(k+7)(k+9)(k+11)}$$

.....

.....

.....

$$\overline{C_\tau^2}(k)=\frac{\tau(\tau^2-1)(\tau+2)(6k-\tau^2-\tau+18)k(k-2)}{\cdot(k-4)....(k-\tau+6)}$$
$$(k+1)(k+3)(k+5)....(k+\tau+5)$$

for τ=odd, k=odd

$$\overline{C_3^2}(k)=\frac{720}{(k+2)(k+4)(k+6)(k+8)}$$

$$\overline{C_5^2}(k=)\frac{5040(k-2)}{(k+2)(k+4)(k+6)(k+8)(k+10)}$$

$$\overline{C_7^2}(k)=\frac{6048(3k-19)(k-1)}{(k+2)(k+4)(k+6)(k+8)(k+10)(k+12)}$$

.....

.....

.....

$$\overline{C_\tau^2}(k)=\frac{\tau(\tau^2-1)(\tau+2)(6k-\tau^2-\tau+18)(k-1)}{\cdot(k-3)....(k-\tau+6)}$$
$$(k+2)(k+4)(k+6)....(k+\tau+5)$$

otherwise $\overline{C_\tau^2}(k)=0$

ERRATA

In "Tables Useful for the Calculation of the Molecular Integrals II"
By Eiichi Ishiguro, Tadashi Arai and Masataka Mizushima
(Natural Science Report of the Ochanomizu University Vol. 2)

Unfortunately our paper indicated above contained the imperfections and the mistakes, which are corrected in the followings:

(1) In Table V, $C_3^1(8)=-\frac{96}{1287}$ can be reduced to $\frac{32}{429}$

(2) Table VI and VII are corrected as followings:

Table VI $\overline{C_\tau^0}(k)$

τ \ k	0	1	2	3	4	5	6
0	$\frac{2}{3}$	0	$\frac{2}{15}$	0	$\frac{2}{35}$	0	$\frac{2}{63}$
1	0	$\frac{2}{15}$	0	$\frac{2}{35}$	0	$\frac{2}{63}$	0
2	$-\frac{2}{15}$	0	$\frac{2}{105}$	0	$\frac{2}{105}$	0	$\frac{10}{693}$
3	0	$-\frac{2}{35}$	0	$-\frac{2}{315}$	0	$\frac{2}{693}$	0
4	0	0	$-\frac{8}{315}$	0	$-\frac{32}{3465}$	0	$-\frac{8}{3003}$
5	0	0	0	$-\frac{8}{693}$	0	$-\frac{64}{9009}$	0
6	0	0	0	0	$-\frac{16}{3003}$	0	$-\frac{16}{3465}$
7	0	0	0	0	0	$-\frac{16}{6435}$	0
8	0	0	0	0	0	0	$-\frac{128}{109395}$

Table VII $\overline{C_\tau^1}(k)$

τ \ k	0	1	2	3	4	5	6
1	$\frac{8}{15}$	0	$\frac{8}{105}$	0	$\frac{8}{315}$	0	$\frac{8}{693}$
2	0	$\frac{8}{35}$	0	$\frac{8}{105}$	0	$\frac{8}{231}$	0
3	$-\frac{8}{35}$	0	$\frac{8}{105}$	0	$\frac{8}{165}$	0	$\frac{8}{273}$
4	0	$-\frac{8}{63}$	0	$\frac{8}{693}$	0	$\frac{200}{9009}$	0
5	0	0	$-\frac{16}{231}$	0	$-\frac{32}{3003}$	0	$\frac{16}{3003}$
6	0	0	0	$-\frac{16}{429}$	0	$-\frac{32}{2145}$	0
7	0	0	0	0	$-\frac{128}{6435}$	0	$-\frac{128}{9945}$
8	0	0	0	0	0	$-\frac{128}{12155}$	0