Natural Science Reports, Ochanomizu University, Vol. 1.
お茶の水女子大學自然科學報告 第1巻 1951.

On the Retardation Effects in the Non-local Field Theory[1]

Tsuneto Shimose (下瀬恒人)

Department of Physics, Faculty of Science,
Ochanomizu University, Tokyo.

Introduction and Summary

Though recently many papers have been published about the non-local field theory of elementary particles such as by Yukawa[1], yet at present these theories are too abstract to obtain definite results. On the other hand Heisenberg's new monistic theory[2] of field is considered to tell us our objects to reach, but it is also far separated from the present concrete theory. Then on our way to reach this theory, there must be many questions to discuss. At present one of the ways to proceed on our theoretical studies about elementary particles, may be to start, firstly, to discuss the relation between the following two theories about the electron, which are founded on the valid experimental evidences, and secondly, to unite these theories into one formulation. This is the idea of the present paper.

The first theory of the electron is Tomonaga[3]-Schwinger's[4] quantum electro-dynamics in a relativistically covariant formulation on a dualistic standpoint, (hereafter designated as T-S theory) which explains experimentally anomalous magnetic moment of the electron and many radiative reaction effects. Contrary to T-S theory there is another important theory of the electron proposed by Bopp[5], which follows the school of Dirac's[6] and Mie[7]-Born's[8] classical theories of the electron founded on a monistic standpoint, and explains the experimental mass spectrum of elementary particles, though it is qualitative. So theoretically the latter theory is rather interesting. Apparently it looks as if these two theories were based on fundamentally different standpoints and do not fuse into one formulation.

In the present report the author introduces in the first place the outline of Bopp's theory, which is necessary to develop our theory. The different points of the above two theories are discussed and we bring to light how the unified formulation of two theories is accomplished. Following these discussions we shall propose a field theory for the above aim, which is nothing but a simple non-local field theory assuming non-locality only in commutation relations of field quantities, and

---

[1] Contributions from Department of Physics, Faculty of Science, Ochanomizu University, No. 1.

considering the expansion by a retardation parameter, but with different interpretation about Lagrange function contrary to T-S theory. In our formulation the 4-dimensional functionals constructed from various dynamical quantities (as electromagnetic potential, spinor fields and especially Lagrange function), vary following generalized Schrödinger equations of motion.

Then we shall show that Dirac's wave equation with matter field and Maxwell's wave equation with electromagnetic field are derived from the 1st order approximation about retardation parameter of our simple Lagrange function, in which case it is derived also that the instantaneous velocity of a particle is equal to light velocity. In the 2nd approximation of retardation parameter the wave equation of matter field takes a generalized form of Dirac's equation, which is slightly different from the equations treated by Hönl-Papapetrou and Bopp. Thus it is shown that Bopp's theory is transformed into a dualistic non-local field theory with a simple formulation. But their full discussions will be postponed till the following paper.

Though we have assumed non-local commutation relations with respect to field quantities, and studied the non-local effects generated from these commutation relations, we did not determine concretely these commutative functions in this paper. As in the generalized Dirac's equation the moments of these quantities with respect to retardation parameter appear, these quantities will be determined by comparing them with experimental results. In this point our theory is based on phenomenological standpoint with respect to commutation relations. Also in the present paper the Heisenberg representation is used to make clear the relation between Bopp's theory and ours.

1. The outline of Bopp's theory

In this section, to make easy to understand our theory, we shall describe the outline of Bopp's theory about elementary particles, but the outline of T-S theory is omitted, because the latter is well known.

At first to describe the electromagnetic field of an electron following the school of Mie-Born, Bopp assumed two field quantities "Erregungstensor" $F_{\alpha\beta}(x)$ and "Feldtensor" $f_{\alpha\beta}(x)$, which are connected by the relation,

$$F_{\alpha\beta} = \frac{\delta L}{\delta f_{\alpha\beta}}, \tag{1.1}$$

where Lagrange function $L(x)$ is a function of field quantities $f_{\alpha\beta}$ only and defined by

$$L(x) = \frac{1}{4}\int f_{\alpha\beta}(x)\varepsilon(x-x')f_{\alpha\beta}(x')dx'. \tag{1.2}$$

In the last equation (1.2) $\varepsilon(x-x')$ means the expanded action of $f_{\alpha\beta}(x)$, in which non-locality is expressed already.

After proposing this idea Bopp[5] treated in detail the motion of an electron in the formalism of classical electrodynamics from monistic standpoint. The path of a moving electron with a charge e is given by $z_\mu(s)$, the magnitude of whose 4-dimensional velocity,

$$\frac{dz_\mu}{ds} = u_\mu(s)\,, \qquad (\mu = 1.2.3.4.) \quad (z_4 = i\,c\,t)\,, \tag{1.3}$$

is light velocity

$$u_\mu{}^2 = c^2, \tag{1.4}$$

The electromagnetic potential at output point x_μ, generated by this charged particle is

$$A_\mu(x) = \frac{2}{c^2}\int g(\tau)u_\mu(x')dx', \tag{1.5}$$

$$\text{where is} \qquad \tau^2 = \frac{1}{c^2}(x_\mu - z_\mu(s))^2,$$

and $g(\tau)$ is spreaded in τ, showing non-local action, too.

The variational principle for field quantities is

$$\delta S = 0\,,$$

where following Born the action integral S is defined as follows,

$$S = \frac{e}{2c}\int A_\mu(x)u_\mu(x)dx\,. \tag{1.6}$$

This form of the action integral is characterized by being contained only interaction term and not the quantities of $u_\mu(x)$ only, or $A_\mu(x)$ only, further the action is direct and does not spread. The Lagrange function $L(x)$ defined by the relation with S, $S = \int L(x)dx$, is derived from (1.5), (1.6) as follows,

$$L(x) = \frac{e^2}{c^3}\int u_\mu(s)u_\mu(s-\tau)g(\tau)d\tau - \frac{e}{c}u_\mu(x)A_\mu^{(e)}(x)\,, \tag{1.7}$$

where an external field $A_\mu^{(e)}(x)$ is separated from $A_\mu(x)$. Then expanding $u_\mu(s-\tau)$ with respect to retardation parameter τ we have,

$$L(x) = -\frac{e}{c}A_\mu^{(e)}(x)u_\mu(x) - \frac{k_0e^2}{c} - \frac{k_2e^2}{c^3}\Big(\frac{3}{8}\dot{u}^2\Big) + \frac{k_4e^2}{c^2}\Big[\frac{5}{124}\ddot{u}_\mu^2 + \frac{85}{1152c^2}(\dot{u}_\mu^2)^2\Big], \tag{1.8}$$

$$\text{where is} \qquad k_\nu = \int g(\tau)\tau^\nu d\tau\,.$$

In the above expression the Lagrange function is only a function of the velocity, higher accelerations of an electron and not its electromagnetic field against Bopp's first formulation (1.2).

The most characteristic property of Bopp's electron theory is that in the form of Lagrange function (1.8) derived from the simple action integral as (1.6), k_2 and k_4 terms are contained in addition to a rest mass term $k_0(m_0c^2)$. As this Lagrange function $L(x)$ has functions of not only a rest mass m_0, but also higher accelerations $\dot{u}_\mu$ and $\ddot{u}_\mu$, in the action of external forces the inertia of an electron is different from the rest case and the motion of an electron does not always return to the equilibrium state after cutting off external forces, falling into the instable state[9]. The cause of this phenomenon is concerned with the form of $g(\tau)$, which expresses the effects of the reaction from another field.

As regards the problem of second quantization, which is necessary as the theory of elementary particles, Bopp selected a different means from usual theory of field. He quantized the coordinate x_μ of a particle as well as its velocity $u_\mu(x)$ separately, from the standpoint of particle dynamics, i.e., two sets of a conjugate quantized one, are x_μ, p_μ and u_μ, $\frac{\partial L}{\partial \dot{u}_\mu}$. In this case the conjugate quantity p_μ of x_μ is distinguished from the instantaneous velocity $\dot{x}_\mu = u_\mu(x)$. This is nothing but the characteristic property of Dirac's free electron, being pointed out by Schrödinger[10]. From this property a particle in rest ($p_\mu = 0$) has another internal degrees of freedom of the motion generated from u_μ, from which a particle has a mass pole and mass dipole moment, proposed by Hönl-Papapetrou[11]. Utilizing the discrete eigenvalues of these quantized internal motions, Bopp explains qualitatively the mass spectrum of the stable elementary particles (as masses of electron, meson and proton, etc.). These characteristic properties of Bopp's theory such as the form of action integral (1.6) and the expression by retardation parameter must be retained in our theory.

2. The relation of T-S theory to Bopp's and the standpoint of our theory

Apparently T-S theory and Bopp's are opposite in their standpoints, one is dualistic and the other monistic, but according to the author's consideration, this opposition is not the principal point of difference between the two theories. In Bopp's theory the motion of an electron is described by two kinds of quantities, the velocity $u_\mu(x)$ and its electromagnetic field $A_\mu(x)$. So Bopp's theory is easily generalized to dualistic theory by replacing $u_\mu(x)$ by $j_\mu(x) = \bar{\psi}_\alpha \gamma_\mu \psi_\alpha$ (refer to 3).

One of the fundamental critical points against the present quantum electrodynamics such as the theory of Heisenberg and Pauli, suffering from the divergent difficulties, is their peculiar standpoint "two free fields and their mutual action," (although in T-S theory these difficulties are eliminated by artificial subtraction method of proper interpretation about two free fields.) This standpoint is evident in the form of Lagrange function, from which Dirac's equation and Maxwell's are derived. The divergent difficulties will be caused by their dualistic standpoint, "two fields *and* mutual action," (not dualism of two fields). Contrary to the above view, Bopp's view is different and assumes that there exists "only mutual action between velocity u_μ and its electromagnetic field of a point electron," as shown in (1.6). So Bopp's theory does not fall into dualistic contradiction. In our theory this Bopp's opinion is adopted, i.e., our theory is based on the idea "there exists only mutual action between two fields." This idea was discussed by Feynman[12] some years ago, but he failed to discuss it further.

Secondly, it seems the principal point of difference between the two theories comes from the difference of the object to be treated. One of the characteristic properties of T-S theory is, as was pointed out by Dyson[13], that by eliminating the reaction process by the mutual action between different fields, the divergent difficulties are omitted. But contrary to this, Bopp made much of these reaction processes between different fields as retardation actions and introduced these effects of retardation into the internal structure of an electron. So the quantization of u_μ and $\frac{\partial L}{\partial \dot{u}_\mu}$ is nothing but the introduction of the effects of the retardation by the electromagnetic fields. These electromagnetic fields are quantized by particle mechanics through the above quantities of a particle.

In our theory the main interest is directed to the retardation effects of one field upon the other, (which is caused by non-local commutation relations,) contrary to the usual theory of field, but the formulation of our theory is constructed in the form of a usual simple non-local field theory, only differing in the interpretation of Lagrange function and the derivation of Dirac's and Maxwell's equations. The Lagrange function of 1st order in our theory, which is transformed from the initial simple form by the generalized equations of motion, is essentially related with the non-local commutative functions. From this Lagrange function of 1st order with expansions by a retardation parameter, generalized Dirac's and Maxwell's equations are derived. In this point our theory also differs from the regulator theory with infinitely many mixed fields, in each of which Dirac's equation with different mass is established.

3. Mathematical Formulation

In this section we shall describe at first relations between field quantities[1] necessary to the formulation of our theory. Then whole

[1] In this paper Schwinger's notations are used so far as not otherwise described.

mathematical formulations will follow. But our discussions are specialized to the mutual action between the spinor and electromagnetic field of an electron[1].

1) Field.

a) 4-dimensional electromagnetic potential field $A_\mu(x)$,[2] $(\mu=1,2,3,4)$

b) Spinor field $\psi_\alpha(x)$, $\overline{\psi}_\beta(x)\,(=\psi^*_\beta\gamma_4)$,

c) 4-Hermite matrix γ_μ, their commutation relations are $\gamma_\mu\gamma_\nu+\gamma_\nu\gamma_\mu=2\delta_{\mu\nu}$,

d) Successive expansion by retardation effects. According to orders of the retarded action from another field, the above fields are expanded successively as follows,[3]

$$A_\mu(x) = A_\mu^{(0)}(x)+A_\mu^{(1)}(x)+\cdots\cdots$$
$$\psi_\alpha(x) = \psi_\alpha^{(0)}(x)+\psi_\alpha^{(1)}(x)+\cdots\cdots.$$

2) Commutation relations of field quantities.

Contrary to T-S theory, we shall assume non-local commutation relations of field quantities as follows,[4]

$$[A_\mu(x),\ A_\nu(x')] = ic\hbar\delta_{\mu\nu}f(x-x'),$$
$$\{\psi_\alpha(x),\ \overline{\psi}_\beta(x)\} = -ig_{\alpha\beta}(x-x'). \tag{3.1}$$

The functions $f(x-x')$ and $g_{\sigma\beta}(x-x')$ in (3.1) correspond to Schwinger's $D(x-x)$ and $S_{\alpha\beta}(x-x')$, $\bar{f}$ to $\bar{D}$, $f^{(1)}$ to $D^{(1)}$ etc., but our $f(x-x')$ and $g_{\alpha\beta}(x-x')$ are considered to spread over certain intervals of $x-x'$, though their concrete forms are not yet defined here.

3) Current density and equation of continuity.

The current density of spinor field is as follows,

$$j_\mu(x) = \frac{iec}{2}[\overline{\psi}(x)\gamma_\mu\psi(x)-\overline{\psi}'(x)\gamma_\mu\psi'(x)], \tag{3.2}$$

equation of continuity is as follows,

$$\frac{\partial j_\mu(x)}{\partial x_\mu} = 0. \tag{3.3}$$

4) Equations of motion.

In this paper we shall describe the variation of field quantities

---

[1] This method will be applied to the elementary particles with other spin than the electron.

[2] As in this paper virtual processes are discussed, supplementary condition of $A_\mu(x)$ needs not be prescribed.

[3] The upper suffix (0) indicates incoming field.

[4] The meaning of bracket symbols is $[A,B]=AB-BA$, $\{A,B\}=AB+BA$. The brackets, in which comma are not contained, are used as usual.

in the Heisenberg representation to retain the close relation to Bopp's theory, contrary to the usual application of the interaction representation as in T-S theory. According to Dirac[14)], generalized equations of motion in the Heisenberg representation, which prescribe the variation of a functional $\xi(\sigma)$ of certain dynamical variables, is expressed by the introduction of Hamilton function $H(x)$ as follows,

$$i\hbar c\frac{\delta\xi(\sigma)}{\delta\sigma} = [\xi(\sigma), H(x)] + i\hbar c\frac{\partial\xi(\sigma)}{\partial\sigma}. \tag{3.4}$$

The relation between Hamilton function $H(x)$ and Lagrange function $L(x)$ is expressed in a simple form, $H(x) = -L(x)$, as far as we are concerned with interaction terms only. Moreover the concrete form of $L(x)$ is assumed as follows like T-S theory,

$$L^{(0)}(x) = -H^{(0)}(x) = \frac{1}{c}j_\mu(x)A_\mu(x). \tag{3.5}$$

5) Exact and approximate solutions of equations of motion.

Exact solution of (3.4) is obtained generally as a form of integral equation,

$$\xi(\sigma) = \xi^{(0)}(-\infty) - \frac{i}{\hbar c}\int_{-\infty}^{\sigma}[\xi(\sigma), H(x')]dx', \tag{3.6}$$

where dx' is the differential 4-dimensional volume.

When fuctional $\xi(\sigma)$ is expressed in a form of 4-dimensional integral about certain dynamical variable $\varphi(x)$ up to the 4-dimensional surface σ,

$$\xi(\sigma) = \int_{-\infty}^{\sigma}\varphi(x')dx', \tag{3.7}$$

solution about dynamical variable $\varphi(x)$ takes the following form,[1]

$$\varphi(x) = \varphi^{(0)}(x) - \frac{i}{\hbar c}\int_{-\infty}^{\sigma}[\varphi(x), H(x')]dx'. \tag{3.8}$$

Further when the approximate form (3.5) is assumed the above solution takes the following form,

$$\varphi(x) = \varphi^{(0)}(x) + \frac{i}{\hbar c^2}\int_{-\infty}^{\sigma}[\varphi(x), j_\mu(x')A_\mu(x')]dx'. \tag{3.9}$$

6) Examples of the solution.

a) When $\varphi(x)$ is electromagnetic potential $A_\mu(x)$, after using commutation relation (3.1) we have in the 1st approximation,

$$A_\mu(x) = A_\mu^{(0)}(x) - \frac{1}{c}\int_{-\infty}^{\sigma} f(x-x')j_\mu^{(0)}(x')dx', \tag{3.10}$$

which is the form of well known formula as the solution of in-

[1] It is assumed the integrability condition is satisfied.

homogeneous Maxwell's equation.

b) When $\varphi(x)$ is spinor field $\psi_\alpha(x)$, we have as above example,

$$\psi_\alpha(x) = \psi_\alpha^{(0)}(x) + \frac{ie}{\hbar c}\int_{-\infty}^{\sigma} g_{\alpha\beta}(x-x')\gamma_\mu \psi_\beta^{(0)}(x') A_\mu^{(0)}(x')dx'. \tag{3.11}$$

c) When $\varphi(x)$ is Lagrange function $L(x)$, this case is very important, so after introducing the expansion by a retardation parameter we shall treat its detail in the next section.

7) Expansion by retardation parameter.

Solution of equations of motion (3.8) about dynamical variable takes generally the following form after introducing non-local commutation relations,

$$\varphi(x) = \varphi^{(0)}(x) - \int_{-\infty}^{\sigma} G(\tau)\Phi(x')dx', \tag{3.12}$$

where is $\quad \tau^2 = \frac{1}{c^2}(x-x')^2,$

and $G(\tau)$, expressing a function of $g_{\alpha\beta}(\tau)$ and $f(\tau)$ of (3.1), is generally large in the neighbourhood of $\tau = 0$, decreasing abruptly according to the distance from $\tau = 0$. In this case the integral in the right hand of (3.12) contributes to its value only in the neighbourhood of $\tau = 0$. Then expanding $\Phi(x')$ into τ's powers,

$$\Phi(x') = \Phi(x) + \tau\dot{x}_\mu \frac{\partial \Phi}{\partial x_\mu} + \frac{\tau^2}{2}\left(\dot{x}_\mu \dot{x}_\nu \frac{\partial^2 \Phi}{\partial x_\mu \partial x_\nu} + \ddot{x}_\mu \frac{\partial \Phi}{\partial x_\mu}\right) + \cdots \tag{3.13}$$

Introducing (3.13) into (3.12) we have expansion formula by retardation effects as follows,

$$\varphi(x) = \varphi^{(0)}(x) - \bar{k}_0\Phi(x) - \bar{k}_1\dot{x}_\mu\frac{\partial \Phi}{\partial x_\mu} - \bar{k}_2\left(\dot{x}_\mu \dot{x}_\nu \frac{\partial^2 \Phi}{\partial x_\mu \partial x_\nu} + \ddot{x}_\mu \frac{\partial \Phi}{\partial x_\mu}\right) + \cdots,$$

where is $\quad \bar{k}_\nu = \frac{1}{\nu!}\int_{-\infty}^{\sigma} G(\tau)\tau^\nu dx'. \qquad (3.14)$

From the property of $G(\tau)$ any term containing higher k_ν than k_2 will not have important physical meaning. This expansion method is peculiar against usual field theories and provides powerful means for our field theory.

8) Variational principle.

When we define the action integral S of the given system of particles, wave equations of spinor field and electromagnetic field are derived from the following variational principle,

$$\delta S = \delta \int L(x)dx = 0$$

as $\quad \delta L = \frac{\delta L}{\delta \psi_\alpha}\delta\psi_\alpha + \frac{\delta L}{\delta \bar{\psi}_\alpha}\delta\bar{\psi}_\alpha + \frac{\delta L}{\delta A_\mu}\delta A_\mu .$

$$\text{Then} \quad \frac{\delta L}{\delta\psi_\alpha} = 0, \quad \frac{\delta L}{\delta\bar{\psi}_\alpha} = 0, \quad \frac{\delta L}{\delta A_\mu} = 0. \tag{3.16}$$

From this variational principle generalizations of Dirac's equation and Maxwell's equation are easily performed by the selection of a suitable Lagrange function, which is discussed in the next section.

4. Discussion of Lagrange Function

For the Lagrange function we shall apply the preceding mathematical formulation and obtain the next results. Applying (3.8) to $L(x)$ we have,

$$L(x) = L^{(0)}(x) - \frac{i}{\hbar c}\int_{-\infty}^{\sigma}[L(x), H(x')]dx', \tag{4.1}$$

replacing $L^{(0)}(x)$ and $H^{(0)}(x)$ of (3.5) into the right hand side of (4.1) we have,

$$L(x) = L^{(0)}(x) + \frac{i}{\hbar c^3}\int_{-\infty}^{\sigma}[j_\mu(x)A_\mu(x), j_\nu(x')A_\nu(x')]dx'. \tag{4.2}$$

By computing the commutation relation in the integral (4.2) following Schwinger[4], we have as expectation values,

$$\begin{aligned} 2<[jA, j'A]> &= <[j, j']\{A, A'\}> + <\{j, j'\}[A, A']> \\ &= <[j, j']>_1 <\{A, A'\}>_1 + \{j, j'\}_0[A, A']_0 \\ &\quad + <[j, j']>_0 <\{A, A'\}>_1 + \{j, j'\}_1[A, A']_0 \\ &\quad + <[j, j']>_1 <\{A, A'\}>_0 + \{j, j'\}_2[A, A']_0 \end{aligned} \tag{4.3}$$

Then the integral (4.2) becomes as follows,

$$L(x) = L^{(0)}(x) + L^{(1)}_A(x) + L^{(1)}_B(x) + L^{(1)}_C(x) + L^{(1)}_D(x) + L^{(1)}_E(x). \tag{4.4}$$

Each term of the right hand is tabulated in the following table.

Notation	Name of term	Integrand
$L^{(1)}_A(x)$	Interaction (1 particle, 1 photon)	$<[j, j']>_1<\{A, A'\}>_1$
$L^{(1)}_B(x)$	Self Energy	$\{j, j'\}_0\,[A, A']_0$
$L^{(1)}_C(x)$	Elmag. Field	$<[j, j']>_0<\{A, A'\}>_1$
$L^{(1)}_D(x)$	Matter Field (1 particle)	$\{j, j'\}_1[A, A']_0 + <[j, j']>_1<\{A, A'\}>_0$
$L^{(1)}_E(x)$	Matter Field (2 particles)	$\{j, j'\}_2[A, A']_0$

One example of the expansion by retardation parameter, which is the most interesting for our theory, is shown by $L^{(1)}_D(x)$ for 1 particle case. After some calculations we have,

$$L^{(1)}_D(x) = -\frac{e^2}{2}\int_{-\infty}^{\sigma}\{\bar{\psi}(x)\gamma_\mu F(x-x')\gamma_\mu\psi(x') + \bar{\psi}(x')\gamma_\mu\overline{F(x-x')}\gamma_\mu\psi(x)\}dx', \tag{4.5}$$

where is $F(x-x') = \{\bar{f}(x-x')g^{(1)}(x-x') + f^{(1)}(x-x')\bar{g}(x-x')\}$.

Expanding $\psi_\alpha(x')$, $\bar{\psi}_\alpha(x')$, $\gamma_\mu(x')$ in the neighbourhood of x about retardation parameter, we have,

$$\psi_\alpha(x') = \psi_\alpha(x) + \tau\dot{x}_\mu\frac{\partial\psi}{\partial x_\mu} + \frac{\tau^2}{2}\left(\ddot{x}_\mu\frac{\partial\psi}{\partial x_\mu} + x_\mu\dot{x}_\nu\frac{\partial^2\psi}{\partial x_\mu\partial x_\nu}\right) + \cdots\cdots, \tag{4.6}$$

$$\gamma_\mu(x') = \gamma_\mu(x) + \tau\dot{\gamma}_\mu + \frac{\tau^2}{2}\ddot{\gamma}_\mu + \cdots\cdots.$$

Further considering the relations,

$$\gamma_\mu^2 = 1, \qquad \gamma_\mu\dot{\gamma}_\mu = 0, \qquad \gamma_\mu\ddot{\gamma}_\mu = -\dot{\gamma}_\mu^2\,, \tag{4.7}$$

$$\text{and} \qquad k_\nu = \frac{e^2}{2\nu!}\int_{-\infty}^{\sigma} F(\tau)\tau^\nu dx',$$

we have as $L_D^{(1)}(x)$

$$L_D^{(1)}(x) = -k_0\bar{\psi}(x)\psi(x) - k_1\bar{\psi}(x)\dot{x}_\mu\frac{\partial}{\partial x_\mu}\psi(x) - k_2\bar{\psi}(x)\left\{-\dot{\gamma}_\mu^2 + \ddot{x}_\mu\frac{\partial}{\partial x_\mu} + \dot{x}_\mu\dot{x}_\nu\frac{\partial^2}{\partial x_\mu\partial x_\nu}\right\}\psi(x) + \textit{conjugate complex}. \tag{4.8}$$

Other Lagrange functions of the 1st order are described only about their results,

1) Interaction term

$$L_A^{(1)}(x) = -k_0'\bar{\psi}(x)\gamma_\mu\gamma_\nu\psi(x)A_\mu(x)A_\nu(x) + \textit{conjugate complex}, \tag{4.9}$$

where is $\quad k_0' \propto \int_{-\infty}^{\sigma} g(x-x')dx'.$

2) Self energy term

$$L_B^{(1)}(x) = const. \tag{4.10}$$

3) Matter field term (2 particles)

$$L_E^{(1)}(x) = -k_0''\sum_{\alpha,\beta,\gamma,\delta}\bar{\psi}_\alpha(x)\bar{\psi}_\beta(x)\psi_\gamma(x)\psi_\delta(x)\,, \tag{4.11}$$

where $\quad k_0'' \propto \int_{-\infty}^{\sigma} f(x-x')dx'.$

4) Electromagnetic field term

$$L_C^{(1)}(x) = -k_0'''\frac{\partial A_\mu(x)}{\partial x_\lambda}\frac{\partial A_\mu(x)}{\partial x_\lambda} - k_1'''\dot{x}_\nu\frac{\partial A_\mu}{\partial x_\lambda}\frac{\partial^2 A_\mu(x)}{\partial x_\lambda\partial x_\nu} - k_2'''\dot{x}_\nu x_\sigma\frac{\partial A_\mu(x)}{\partial x_\lambda}\frac{\partial^3 A_\mu(x)}{\partial x_\lambda\partial x_\nu\partial x_\sigma}\,, \tag{4.12}$$

where is $\quad k_\nu''' = \frac{32\pi\alpha}{\nu!}\int G(x-x')dx'$

$$\left(\frac{\partial}{\partial x_\mu}\frac{\partial}{\partial x_\nu} - \delta_{\mu\nu}\Box^2\right)G(x-x') = \frac{1}{8}Tr.\left[g^{(1)}(x-x')\gamma_\mu\bar{g}(x-x') + \bar{g}(x-x')\gamma_\mu g^{(1)}(x-x')\gamma_\nu\right].$$

5. Derivation of Dirac's and Maxwell's equations and their generalizations.

The Lagrange function of up to the 1st order in the previous section, retaining the expansion by retardation till the 1st order of τ, and neglecting $L_A^{(1)}(x)$, $L_B^{(1)}(x)$, $L_E^{(1)}$ terms, takes the following form,

$$\begin{aligned} L(x) &= L^{(0)}(x)+L_C^{(1)}(x)+L_D^{(1)}(x) \\ &= \frac{1}{c}j(x)A_\mu(x)-k_0'''\frac{\partial A_\mu(x)}{\partial x_\lambda}\frac{\partial A_\mu(x)}{\partial x_\lambda}-k_0\overline{\psi}(x)\psi(x) \\ &\quad -k_1\overline{\psi}(x)\dot{x}_\mu\frac{\partial}{\partial x_\mu}\psi(x)+conjugate\ complex\,, \end{aligned} \tag{5.1}$$

which is equivalent to the formula of Schwinger's I (1.9)[4], if

$$k_0'''=\frac{1}{2},\quad k_0=\frac{m_0c^2}{2},\quad k_1=\frac{\hbar c^2}{2}\quad \text{and}\quad \dot{x}_\mu=c\gamma_\mu \tag{5.2}$$

From the above Lagrange function usual Dirac's and Maxwell's equations are easily derived. The last equation of (5.2) is also required by the equation of continuity (3.3). Its meaning is nothing but that the instantaneous velocity of a particle is light velocity.

The full Lagrange function of the 1st order up to the 2nd order of τ is obtained by summing the formulas (3.5) and (4.8)–(4.12), and by applying them the variational principle (3.16), generalized Dirac's and Maxwell's wave equations will be obtained, which are no more linear in $\psi_\alpha(x)$ or $A_\mu(x)$. But we shall delay their full discussions to the next paper, including their comparison with Bopp's generalized Dirac's equation.

Literature

1) Yukawa, H. Phys. Rev. **77**: 219 (1950)
2) Heisenberg, W. Z. Naturforsch. **5a**: 251 (1950)
3) Tomonaga, S. Progress. theor. Phys. **1**: 27 (1946)
Koba, Tati and Tomonaga, Progress. theor. Phys. **2**: 101, 198 (1947)
Kanesawa and Tomonaga, Progress. theor. Phys. **3**: 1, 101 (1948)
4) Schwinger, J. Phys. Rev. **74**: 1439 (1948); **75**: 651, 790 (1949)
5) Bopp, F. Ann. Phys. (5), **43**: 565 (1943); Z. Naturforsch. **1**: 196 (1946); ibid. **3a**: 564 (1948); Z. Phys. **125**: 615 (1949)
6) Dirac, P. A. M. Proc. Roy. Soc., Lond. (A), **167**: 148 (1938)
7) Mie, G. Ann. Phys. (4), **37**: 511 (1912)
8) Born, M. Proc. Roy. Soc. Lond. (A), **143**: 410 (1933/34)
9) Bopp, F. Z. Naturforsch. **1**: 53 (1946)
10) Schrödinger, E. Sitz. ber. preuss. Akad. Wiss. Berlin, 418 (1930)
11) Hönl and Papapetrou, Z. Phys. **112**: 612 (1939)
12) Feynman, R. P. Phys. Rev. **74**: 939 (1948)
13) Dyson, F. J. Phys. Rev. **75**: 436 (1949)
14) Dirac, P. A. M. Phys. Rev. **73**: 1092 (1948)