The Correctness Proof of Type-Directed Partial Evaluators in the Continuation-Passing Style

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This thesis presents various methods of extracting a partial evaluator guaranteed to be correct from the formalization of a constructive existence proof utilizing a well-known proof assistant Coq. A partial evaluator reduces a given program to the normal form that cannot be further simplified even in a function body. Since the partial evaluator is able to evaluate all the reducible parts in a given program, the evaluator produces more efficient and fast program than the original program without changing its meaning. The correctness of partial evaluator is important in actual application to guarantee reliability of programs that have been generated by the partial evaluator.

In this thesis, we first attempt to extract an evaluator from formalization in Coq of strong normalization theorem of \(\lambda\)-calculus with delimited control operators shift/reset, paying attention to a well-known property that strong normalization theorem is Curry-Howard isomorphic to an evaluator. On formalizing of our type system, we adopt the Locally Nameless method which leads us to avoid difficulty of \(\alpha\)-renaming problem. Our actual proof is done by fully utilizing logical predicate à la Tait. However, the evaluator obtained at this stage is so complicated that it seems much difficult to apply such an evaluator directly to actual problem.

In order to overcome this difficulty, in this thesis we prove (i) the completeness theorem of logical predicate from which type-directed partial evaluators (TDPEs) are actually derived, and (ii) the correctness property of such evaluators as is derived in (i), and finally, (iii) from the formalization process in Coq of the above, we deduce a concrete program for TDPE that should be correct in application.

Unlike standard partial evaluators, TDPE does not inspect the internal structure of the input program and thus is very fast. For the concept of TDPE, there are a couple of the proceeding studies by Tsushima et al., Ilik and others: Tsushima et al. proposed a TDPE with shift/reset; Ilik proposed a method of extracting a TDPE from the proof process of the completeness theorem for inference rules in the Kripke model. However, Tsushima et al. unfortunately did not succeed to show the correctness property of their TDPE that seems very crucial in application. Also Ilik did not deal with standard shift/reset as is used in Tsushima et al. To fill this gap, we first re-formalize the method proposed by Ilik so as to deal with standard shift/reset. From this Ilik’s method, we extract a continuation-passing style (CPS) TDPE with shift/reset in Coq which can be obtained by transforming direct-style TDPE of Tsushima et al.
However, although this TDPE extracted from completeness theorem is guaranteed to avoid infinite loop, the equivalence of the meaning of programs before and after performing this TDPE leaves still unclear. Therefore, we formalize, in Coq, the proof of Filinski’s correctness theorem of the call-by-name TDPE and call-by-value TDPE, and then by extending this formalization to 2CPS, we prove the correctness theorem of CPS TDPE with shift/reset by which the equivalence of the meaning of the programs before and after the TDPE can be in fact guaranteed. The completeness theorem obtained from the semantics results in a similar TDPE to the above TDPE extracted from the completeness theorem of the Kripke semantics. Both the TDPEs have the same meaning in application, though the forms of semantics are slightly different. The use of parametric higher-order abstract syntax (PHOAS) makes it possible to establish the very simple formalization in Coq of our type system, leading in effect to avoid α-renaming problem. When proving the correctness theorem using PHOAS, we are confronted with a particular property that may be difficult to demonstrate in Coq because of the higher-order nature of PHOAS.