お茶の水女子大学自然科学報告　第69巻　第1号、第2号
Natural Science Report, Ochanomizu University, Vol. 69, No. 1, No. 2 (2019)

The Effect of Schmidt Number on the Flow Patterns in Density Instability Model of Bioconvection

Akiko MINAKAWA, Yusaku NAGATA and Tetuya KAWAMURA

(Received : August 31,2018)

Abstract

Patterns of convective flows produced by microorganisms swimming upward (namely, bioconvection) are investigated by two dimensional numerical calculations of a density model. The effect of Schmidt number on the flow patterns is focused in this study. To perform stable computation even at high Schmidt number, third order upwind scheme is used to approximate the nonlinear terms of the density equation. The calculation shows that there is a tendency for the aspect ratio of the convection cell to become smaller with increase in Schmidt number. Since small aspect ratio is observed in experiments, this result suggests that the effective Schmidt number is larger than the usually used value in the simulation of bioconvection.

1. INTRODUCTION

Convective flows driven by heat are well known, but there is another type of convection that is driven by particle motions. One of such convection is bioconvection caused by motion of microorganisms[2]. Bioconvection occurs by density instability of microorganisms having the property of swimming upward (anti-gravitytaxis). Patterns of bioconvection are expected to vary with physical quantities such as swimming velocity of microorganisms, density of microorganism, depth of the vessel and so on. But the detail of the convection is not well known. For example, in the bioconvection due to *Chlamydomonas*, a kind of dinoflagellates, initial convection changes abruptly to new convection where the aspect ratio of the convection cell is smaller than the initial one and this pattern is maintained during long time period[3]. In density model[4], there are three parameters (Rayleigh number: Ra, Prandtl number: Pr and Schmidt number: Sc) that govern the flow as shown in the next section. In the previous study, two dimensional simulations are performed by changing Ra and Pr to see the effect on the flow while Sc is fixed to that suggested by Kessler[5]. As a result, when Prandtl number increases, the aspect ratio of the convection cell increases and it is shown that the time evolution of convection can be classified into “steady”, “unsteady” and “collapse”. However these results are not consistent with the experiments. In this study, we try to change Schmidt number to see its effect on the flow.

2. NUMERICAL METHODS

2.1 Basic equations of the density model

In this study, we use a density model in which microorganisms are expressed by volume fraction[2) 4)]. Basic equations of this model consist of Navier-Stokes equations and advection-diffusion equation for the volume fraction of microorganism. We employ Boussinesq approximation for the momentum equation, i.e. vertical direction of the Navier-Stokes equation has a gravity term that represents the gravity acting against the fluid element composed of the water and suspended microorganisms.

Swimming speed of the microorganism appearing in the advection term in the density equation is assumed to be constant.

Taking the characteristic length to the vessel's depth H, characteristic velocity to the swimming speed w_p and the characteristic number density of microorganism to the average density n_0, the dimensionless basic equations (dimensionless variables are represented in capital letters) become as follows:

$$\begin{aligned} &\frac{\partial U}{\partial X}+\frac{\partial W}{\partial Z}=0 \\ &\frac{\partial U}{\partial T}+U\frac{\partial U}{\partial X}+W\frac{\partial U}{\partial Z}=-\frac{\partial P}{\partial X}+\mathrm{Pr}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Z^2}\right) \\ &\frac{\partial W}{\partial T}+U\frac{\partial W}{\partial X}+W\frac{\partial W}{\partial Z}=-\frac{\partial P}{\partial Z}+\mathrm{Pr}\left(\frac{\partial^2 W}{\partial X^2}+\frac{\partial^2 W}{\partial Z^2}\right)-\mathrm{Ra}\cdot\mathrm{Pr}\cdot N \\ &\frac{\partial N}{\partial T}+U\frac{\partial N}{\partial X}+(W+1)\frac{\partial N}{\partial Z}=\frac{\mathrm{Pr}}{\mathrm{Sc}}\left(\frac{\partial^2 N}{\partial X^2}+\frac{\partial^2 N}{\partial Z^2}\right) \end{aligned} \tag{1}$$

where X and Z coordinates represent horizontal and vertically upward direction, U and W are the fluid velocity of X and Z-direction respectively, P is the fluid pressure and N is the volume fraction of microorganisms.

Rayleigh number: Ra, Prandtl number: Pr and Schmidt number: Sc are

$$\begin{aligned} &\mathrm{Ra}=\frac{\rho_p-\rho_w}{\rho_w}\frac{gH^2 n_0}{w_p \nu} \\ &\mathrm{Pr}=\frac{\nu}{w_p H} \\ &\mathrm{Sc}=\frac{\nu}{k}. \end{aligned} \tag{2}$$

where, w_p is swimming speed of the microorganism, ρ_w and ρ_p are mass densities of the fluid and single microorganism respectively, n is number density of the microorganisms, g is gravitational acceleration, ν is coefficient of kinematic viscosity of the fluid and k is diffusivity of the microorganisms.

Rayleigh number Ra represents the ratio of the generation of torque by buoyancy to the dissipation of torque by viscosity. Prandtl number Pr represents the viscous diffusion time divided by the time during which the microorganisms pass the fluid layer. Prandtl number in this study also corresponds to the reciprocal of the Reynolds number. Schmidt number Sc represents the ratio of the diffusion of the microorganisms to the diffusion of the fluid.

2.2 Numerical method

Stream function-vorticity method is chosen to solve these equations numerically since the continuity equation is satisfied exactly in this method. All spatial derivatives are approximated by the central differences except for nonlinear term of the density equation. This term is approximated by the third order upwind difference which enables us to perform stable computations even for high Schmidt number flow. For the time-integration, first order Euler explicit method is used.

The computational domain contained 48×360 grids and the aspect ratio (= (horizontal length)/(depth)) of the domain is 15. All boundaries of the domain are rigid (no-slip condition).

Initial density of microorganism is set around unity (= average density) at all the grids in the domain. The number of microorganisms in the domain is conserved in the calculations, and the number density N on the boundary is set to zero gradient (Neumann condition).

Rayleigh number is set to 10^{-2} to 10^4, Schmidt number is set to 10^2 to 10^5. Prandtl number is set to 10^0.

We perform a numerical calculation of 1000 seconds in terms of real time (=t) and investigating changes in the convection patterns within that time. The aspect ratio of the convection cell is calculated by the numbers of the cell and domain size.

3. RESULTS AND DISCUSSIONS

3.1 Phase diagram of the aspect ratio of bioconvection cell

Numerical simulations for several combinations of Sc and Ra are performed, and the obtained aspect ratios of the convection cell are summarized in Fig. 1. In Fig. 1, vertical axis represents Rayleigh number, horizontal axis represents Schmidt number and the numbers in the graph is the aspect ratio of the typical convection cell.

The aspect ratio of the convection cell is found to become small as Schmidt number becomes large.

With respect to the time evolution of the bioconvection, it is found that there are four states. In the case of pattern 1, the aspect ratio becomes around 1.0 in the range of Rayleigh number of 10^{-2} to 10^1 and Schmidt number of 10^4. In the case of pattern 2, the aspect ratio becomes smaller than 1.0 in the range of Rayleigh number of 10^{-2} to 10^1 and Schmidt number of 10^4 to 10^6. In the case of pattern 3, the aspect ratio becomes larger than 1.0. This state has two parts, one is mainly in the range of Rayleigh number of 10^{-2} to 10^4 and Schmidt number of 10^1 to 10^3, the other is mainly Rayleigh number of 10^2 to 10^3 and Schmidt number of 10^4 to 10^6. In the case of pattern 4, the convection cell collapses mainly in the range of Rayleigh number of 10^3 to 10^4 and Schmidt number of 10^4 to 10^6.

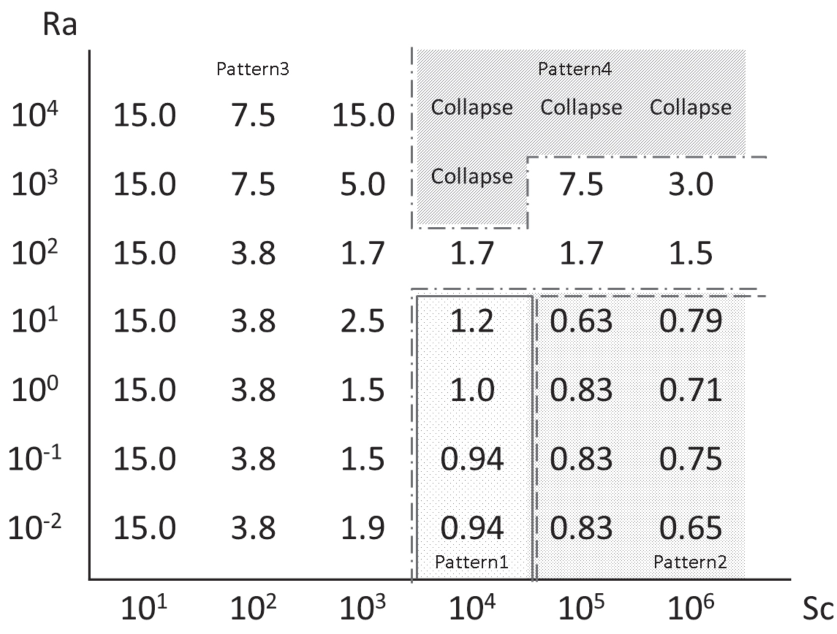

Fig. 1 Phase diagram of aspect ratio of the bioconvection cells as a function of Sc and Ra.

3.2 Pattern 1 : Square (aspect ratio around 1.0)

Streamline and vorticity at t = 1000 in the case of $Ra = 10^0$ and $Sc = 10^4$ are shown in Fig. 2.

In the range of Rayleigh number of 10^{-2} to 10^1 and Schmidt number of 10^4, the aspect ratio of the convection cell becomes around 1.0.

Once the initial convection pattern is formed, the pattern of the convection is maintained without change of the position.

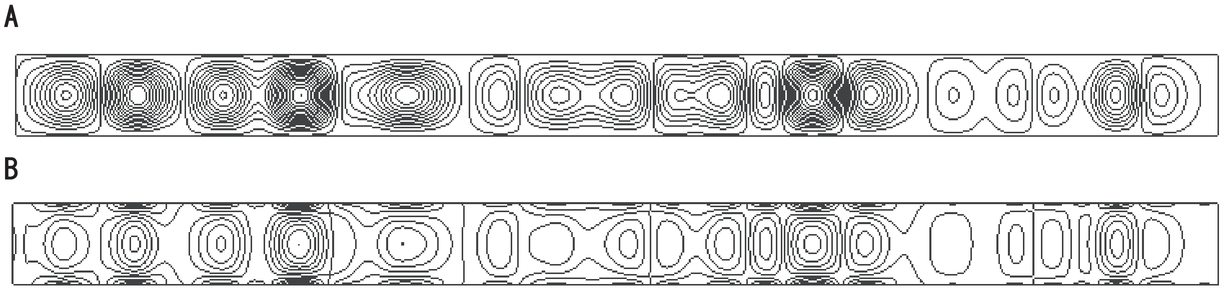

Fig. 2 Pattern 1 ($Ra = 10^0$, $Sc = 10^4$) (A)Stream Lines, (B)Vorticity

3.3 Pattern 2 : Vertical (aspect ratio ≪ 1.0)

Streamline and vorticity at t = 1000 in the case of $Ra = 10^1$, $Sc = 10^5$ are shown in Fig. 3.

In the range of Rayleigh number of 10^{-2} to 10^1 and Schmidt number of 10^5 to 10^6, the aspect ratio of the convection cell is much smaller than 1.0.

Once the initial convection pattern is formed, the pattern of the convection is maintained without change of the position.

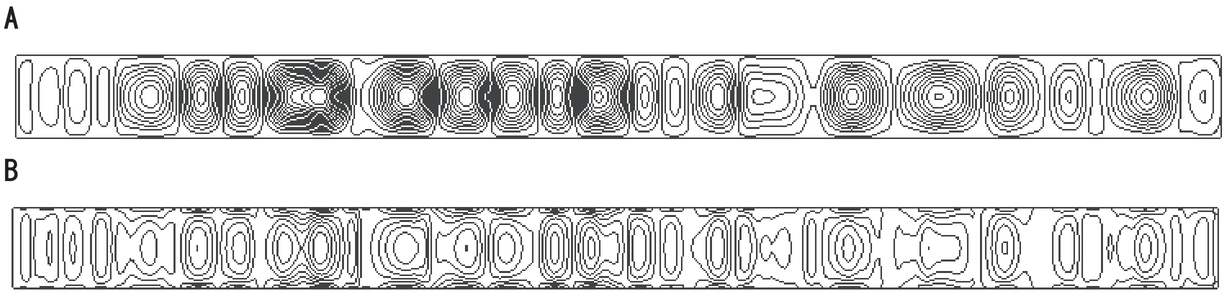

Fig. 3 Pattern 2 ($Ra = 10^1$, $Sc = 10^5$) (A)Stream Lines, (B)Vorticity

3.4 Pattern 3 : Horizontal (aspect ratio ≫ 1.0)

Streamline and vorticity at t=1000 in the case of $Ra = 10^1$ and $Sc = 10^2$, are shown in Fig. 4.

In the range of Rayleigh number of 10^{-2} to 10^4 and Schmidt number of 10^1 to 10^3, and the range of Rayleigh number of 10^2 to 10^3 and Schmidt number of 10^4 to 10^6, the aspect ratio of the convection cell is much larger than 1.0.

The convection cell changes gradually from the early convection pattern to an oblong shape. Although the flow rate decreases with time, convection did not disappear.

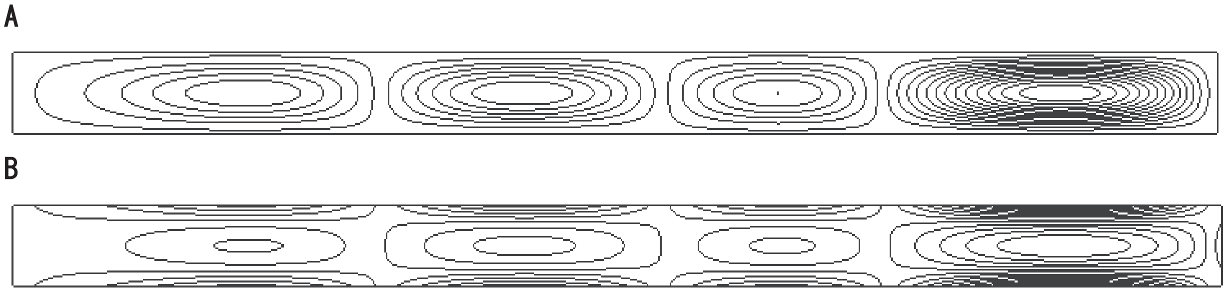

Fig. 4 Pattern 3($Ra = 10^1$, $Sc = 10^2$) (A)Stream Lines, (B)Vorticity

3.5 Pattern 4 : Collapse

In the range of Rayleigh number of 10^3 to 10^4 and Schmidt number of 10^4 to 10^6, the convection cell collapses.

Streamline and vorticity at t=1000 in the case of $Ra = 10^4$ and $Sc = 10^5$ are shown in Fig. 5. Although convection occurs initially, density of the microorganism is increased with time in the vicinity of the bottom, and finally the convection disappears. In the bottom where a microorganism is dense, back and forth flow in the horizontal direction is observed.

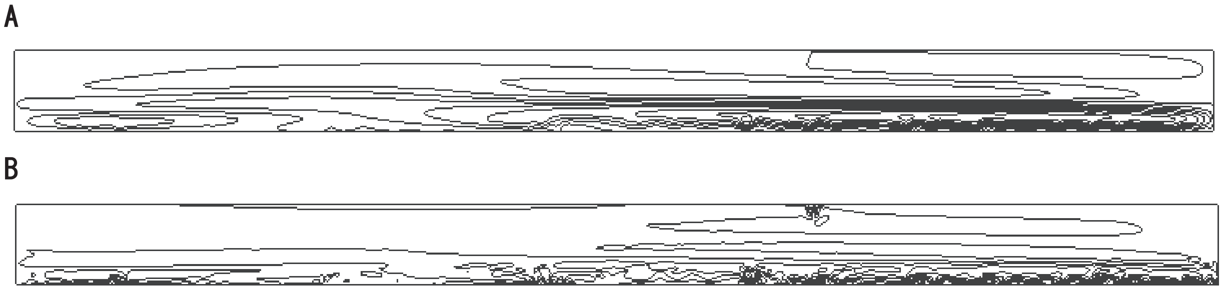

Fig. 5 Pattern 4($Ra = 10^4$ and $Sc = 10^5$) (A)Stream Lines, (B)Vorticity

4 DISCUSSIONS

In this section, we discuss the effect of non-dimensional parameters (Prandtl, Schmidt and Rayleigh numbers) on bioconvection patterns.

In the previous study of Nagata *et al.*[6], it was performed that numerical simulations for several combinations of Rayleigh and Prandtl number with fixed Schmidt number ($Sc=10^3$). As the results, the aspect ratio of the convection cell was mainly changed by the Prandtl number. Stable convection occured when the Rayleigh number was small, while convection disappeared as the Rayleigh number increased. In this study, we performed numerical simulations for several combinations of Rayleigh and Schmidt numbers under a fixed Prandtl number ($Pr=10^0$). Results show that the aspect ratio of the convection cell is mainly changed by the Schmidt number. And, stable convection occurs when the Rayleigh number is small. This is the same result as that obtained by our previous study.

Considering the Schmidt number, bioconvection patterns can be clearly divided between the range of 10^3 and 10^4. In the range smaller than 10^3, the convection cell changes gradually from the initial convection pattern to an oblong shape with the aspect ratio larger than 1.0. We observe different convection patterns that depend on the Rayleigh number in the range larger than 10^4. When the Rayleigh number is smaller than 10^1, the initial convection cell maintains steadily. When the Schmidt number is 10^4, the aspect ratio of bioconvection is around 1.0. Convection cell changes to vertically oblong shape with the aspect ratio smaller than 1.0 as the Schmidt number becomes larger than 10^5. When the Schmidt number is larger than 10^4, the convection cell gradually changes to oblong shape with the aspect ratio larger than 1.0 in the Rayleigh number range of 10^2 to 10^3. On the other hand, convection occurs initially but the convection cells collapse with time in the Rayleigh number range of 10^3 to 10^4.

The Schmidt number is the ratio of the diffusion of the microorganisms to the diffusion of the fluid as shown in equation (2). Since the diffusion of the fluid is constant, this number decides the diffusion of microorganisms. The large Schmidt number indicates that the diffusion of microorganisms is small. This means that the microorganism locally moves vertically so that elongated cells are formed. As Schmidt number increases, the pattern of microorganism becomes finer. This result is close to the experimental one of Kage *et al.*[3] to some extent. As far as the density model is used, this means that results closer to the experiment are obtained when values larger than those of Kessler[5] are used for the Schmidt numbers. Furthermore, since the diffusivity of the microorganisms may be related to the density of microorganisms, it can be changed according to the density of microorganisms. If the diffusion coefficient for bioconvection is appropriately modeled, there is a possibility that a result closer to the experiment is obtained.

5 CONCLUSIONS

From the numerical simulation of a two-dimensional bioconvection by density model, it is shown that

1. Bioconvection cells become smaller as the Schmidt number increases.
2. Convection becomes unstable at large Rayleigh numbers.
3. Bioconvection can be classified into four patterns (Square, Vertical, Horizontal, Collapse).

3-1. Square cell: Sc:10^4, Ra: 10^{-2} to 10^1

3-2. Vertical cell: Sc: 10^5 to 10^6, Ra: 10^{-2} to 10^1

3-3. Horizontal cell: Sc: 10^1 to 10^3, Ra: 10^{-2} to 10^4
Sc: 10^4 to 10^6, Ra: 10^2 to 10^3

3-4. Collapse: Sc: 10^4 to 10^6, Ra: 10^3 to 10^4

4. The effective Schmidt number is larger than the usually used value in the simulation of bioconvection.

REFERENCES

1) Iga, K., and Kimura, R., "Convection driven by collective buoyancy of microbubbles", *Fluid Dyn. Res.* 39 (2007), pp.68-97.
2) Childress, W.S., Levandowsky, M. and Spiegel, E.A., "Pattern formation in a suspension of swimming microorganisms: equations and stability theory", *J. Fluid Mech.,* 69 (1975), pp.591-613.
3) Kage, A., Hosoya, C., Baba, S. A. and Mogami, Y., "Drastic reorganization of the bioconvection pattern of Chlamydomonas : quantitative analysis of the pattern transition response", *J. Exp. Biol.* 216 (2013), pp.4557-4566.
4) Harashima, A., Watanabe, M. and Fujishiro, I., "Evolution of bioconvection patterns in a culture of

motile flagellates", *Phys. Fluids*, 31 (1988), pp.764-775.
5) Kessler, J. O., "Individual and collective fluid dynamics of swimming cells", *J. Fluid Mech.*, 173 (1986), pp.191-205.
6) Nagata, Y., Minakawa, A. and Kawamura, T., "Classification of two-dimensional convection patterns in density instability model of bioconvection", Theoretical and Applied Mechanics Japan 64, (2018), pp.73-77.

Akiko Minakawa
Department of Computer Science, Graduate School of Humanities and Sciences, Ochanomizu University
2-1-1 Otsuka, Bunkyo-ku, Tokyo, 112-8610, Japan
E-mail: minakawa.akiko@is.ocha.ac.jp

Yusaku Nagata
E-mail: nagata.yusaku@ocha.ac.jp

Tetuya Kawamura
E-mail: kawamura@is.ocha.ac.jp