Controlling dark solitons in the one-dimensional bose gas: comparison between quantum and classical solitons

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The aim of this thesis is to reveal the nature of quantum dark soliton states, which are the superpositions of eigenstates in the Lieb-Liniger model. The quantum dark soliton states are associated with soliton solutions of the nonlinear Schrödinger equation with the periodic boundary conditions, which we call classical solitons. Due to the difficulty of studying interacting quantum systems, most of the studies on many-body quantum states use the mean-field approximation. This study of quantum dark soliton states is essential for understanding quantum states.

We begin by reviewing two methods to construct the solutions of the nonlinear Schrödinger equation: the inverse scattering method and the algebro-geometric approach. The review of the inverse scattering method helps readers to understand the algebraic-geometric approach, which is a method to construct gap solitons as classical solitons. This approach solves the inverse scattering problem on hyperelliptic curves. However, physicists rarely apply this method due to the richness of mathematics. In this thesis, we introduce the theory of Riemann surfaces with some simple examples.

Finally, we show the exact time evolution of quantum dark double soliton state and the construction of quantum dark single soliton states with winding number J=1,2. In order to construct the quantum dark soliton state, we use the eigenstates in the Lieb-Liniger model, which are obtained by creating holes in a set of the Bethe quantum numbers. It was found that when we specify the Bethe quantum numbers so that the energy is increased, the winding number J also increases. We construct a quantum dark double soliton state, which is a superposition of eigenstates with two holes in a series of the Bethe quantum numbers. Furthermore, by applying appropriate Gaussian weights to the quantum dark two soliton states, we construct quantum dark two solitons with different narrow notches.