Numerical study of the aspect ratios of the bioconvection cell as a function of Prandtl and Rayleigh number

Yusaku Nagata, Ai Nagasawa, Anna Kuwana and Tetuya Kawamura
(Received July 26, 2016)

Abstract

We performe a numerical calculation of bioconvection to investigate the relationship between the aspect ratio of the convection cell and Prandtl and Rayleigh numbers. The aspect ratio of the convection cell becomes large as Prandtl number becomes large. With respect to temporal change in bioconvection, the flow can be classified into three states (steady, unsteady, collapse). Comparing bioconvection to the bubble convection that is a kind of particle-driven convection, there is a similarity that steady and unsteady-state can be divided by Prandtl number, while the difference is observed in the presence of the collapse pattern.

1. INTRODUCTION

Some kind of the flagellate forms a characteristic fluid motion called "bioconvection"(Childress *et al.* 1975; Kage *et al.* 2013). This phenomenon is caused by density instability of microorganisms having the property of swimming upward (anti-gravitytaxis). Pattern of the bioconvection is expected to vary with physical quantities such as swimming velocity of microorganisms, density of microorganism, depth of the vessel and so on.

On the other hand, there is a similar convection called "bubble convection" which is driven by the particle movement. In bubble convection, it is driven by upward movement of bubbles in the liquid. This convection has been examined minutely in stand point of changes in the convection pattern by these physical quantities (Iga and Kimura 2007). However, the parameters corresponding to the bioconvection have not been investigated adequately. Further, since the boundary conditions in bubble convection is different from those of bioconvection, these difference may affect the result.

In the present study, we investigate the convection pattern of the two-dimensional bioconvection by numerical simulation using density model. We find that aspect ratio of bioconvection cell is determined by both Rayleigh number and Prandtl number. We also find that time variation of the convection patterns can be classified into a few types.

2. NUMERICAL METHODS

2.1 Basic equations of the density model

In this study, we use a density model in which microorganisms are expressed by volume density (Childress *et al.* 1975; Harashima *et al.* 1988). Basic equations of this model consists of Navier-Stokes equations and advection-diffusion equation for the volume density of microorganism. We employ Boussinesq approximation for the momentum equation, i.e. vertical direction of the Navier-Stokes equation has a gravity term that represents the gravity acts against the fluid element composed of the water and suspended microorganisms. Swimming speed of the microorganism appearing in the advection term in density equation is assumed to be constant.

When x and z coordinates represent horizontal and vertically upward direction, the basic equations becomes as follows:

$$
\begin{aligned}
&\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0\\
&\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho_w}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}\right)\\
&\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho_w}\frac{\partial p}{\partial z}+\nu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial z^2}\right)-n\frac{\rho_p-\rho_w}{\rho_w}g\\
&\frac{\partial n}{\partial t}+u\frac{\partial n}{\partial x}+\left(w+w_p\right)\frac{\partial n}{\partial z}=k\left(\frac{\partial^2 n}{\partial x^2}+\frac{\partial^2 n}{\partial z^2}\right)
\end{aligned}
\tag{1}
$$

where u and w are the fluid velocity of x and z-direction respectively, w_p is the swimming speed of the microorganism, ρ_w and ρ_p are density of the fluid and single microorganism respectively, n is the volume density of the microorganisms, p is the fluid pressure, g is the gravitational acceleration, ν is the coefficient of kinematic viscosity of the fluid and k is the diffusivity of the microorganisms.

Taking the characteristic length to the vessel's depth H, characteristic velocity to the swimming speed w_p and the characteristic density of microorganism to the average density n_0, the dimensionless basic equations (dimensionless variables are represented in capital letters) becomes as follows:

$$
\begin{aligned}
&\frac{\partial U}{\partial X}+\frac{\partial W}{\partial Z}=0\\
&\frac{\partial U}{\partial T}+U\frac{\partial U}{\partial X}+W\frac{\partial U}{\partial Z}=-\frac{\partial P}{\partial X}+\mathrm{Pr}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Z^2}\right)\\
&\frac{\partial W}{\partial T}+U\frac{\partial W}{\partial X}+W\frac{\partial W}{\partial Z}=-\frac{\partial P}{\partial Z}+\mathrm{Pr}\left(\frac{\partial^2 W}{\partial X^2}+\frac{\partial^2 W}{\partial Z^2}\right)-\mathrm{Ra}\cdot\mathrm{Pr}\cdot N\\
&\frac{\partial N}{\partial T}+U\frac{\partial N}{\partial X}+\left(W+1\right)\frac{\partial N}{\partial Z}=\frac{\mathrm{Pr}}{\mathrm{Sc}}\left(\frac{\partial^2 N}{\partial X^2}+\frac{\partial^2 N}{\partial Z^2}\right)
\end{aligned}
\tag{2}
$$

where Rayleigh number: Ra, Prandtl number: Pr and Schmidt number: Sc are

$$
\begin{aligned}
&\mathrm{Ra}=\frac{\rho_p-\rho_w}{\rho_w}\frac{gH^2 n_0}{w_p\nu}\\
&\mathrm{Pr}=\frac{\nu}{w_p H}\\
&\mathrm{Sc}=\frac{\nu}{k}
\end{aligned}
\tag{3}
$$

Rayleigh number Ra represents the ratio of the generation of torque by buoyancy to the dissipation of torque by viscosity. Prandtl number Pr represents the viscous diffusion time divided by the time during which the microorganisms pass the fluid layer. Prandtl number in this study also corresponds to the reciprocal of the Reynolds number. Schmidt number Sc represents the ratio of the diffusion of the microorganisms to the diffusion of the fluid.

2.2 Numerical setup

These equations are solved numerically by the finite-differential method (stream function-vorticity method). All spatial derivatives are approximated by the central differences. For time-integration, first order Euler explicit method is used.

The computational domain contained 24×360 grids and the aspect ratio (= (horizontal length)/(depth)) of the domain is 15. All boundaries of the domain are rigid (no-slip condition).

Initial density of microorganism is set around unity (= average density) at all of the grids in the domain. The number of microorganisms in the domain is conserved in a calculation.

Rayleigh number is set to $10^{-8} \sim 10^{2}$, Prandtl number is set to $10^{-3} \sim 10$ and Schmidt number is fixed to 10^{3}. Other parameters are basically the same as those used by Kessler (1986).

We perform a numerical calculation of 1000 second in terms of real time (=*t*) and investigating changes in the convection patterns within that time.

3. RESULTS AND DISCUSSIONS

3.1 Phase diagram of the aspect ratio of bioconvection cell

Numerical simulations for several combinations of Pr and Ra are performed, and aspect ratio of the convection cell is summarized in Fig. 1. In Fig. 1, vertical axis represents Rayleigh number, horizontal axis represents Prandtl number and the numbers in the graph is the aspect ratio of the convection cell.

The aspect ratio of the convection cell is found to becomes large as Prandtl number becomes large.

With respect to temporal change in bioconvection, it is found that there are three states (steady, unsteady, collapse). Steady and unsteady-state can be divided by Prandtl number around unity, steady and collapse-state can be divided by Rayleigh number around 10^{-1} and collapse and unsteady-state can be divided by Prandtl number around 10^{-1}.

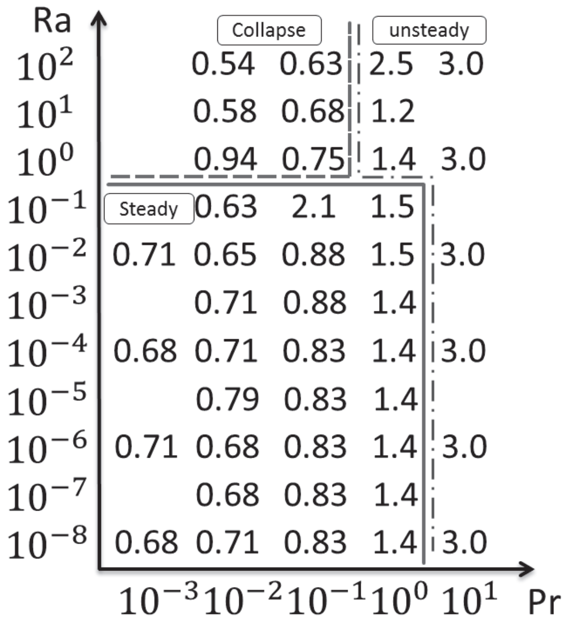

Fig. 1 Phase diagram of aspect ratio of the bioconvection cells as a function of Pr and Ra.

3.2 Steady pattern

In the case of Ra = 10^{-7} and Pr = 10^{-2}, streamline and vorticity at t=1000 are shown in Fig. 2.

Once the initial convection pattern is formed, the pattern of the convection is maintained without change of the position.

If the Rayleigh number is fixed, the aspect ratio of the convection cell is increased as Prandtl number is increased.

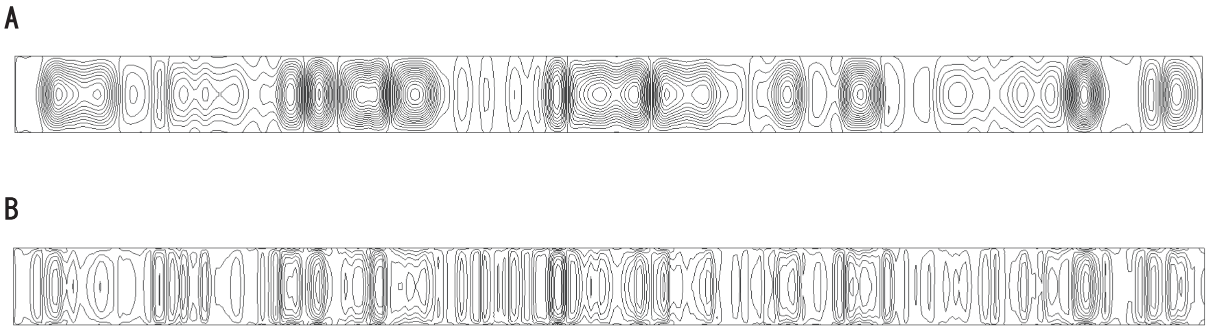

Fig. 2 Steady case(Ra = 10^{-7}, Pr = 10^{-2}) (A)Stream Line, (B)Vorticity

3.3 Unsteady pattern

In the case of Ra = 1 and Pr = 10, streamline and vorticity at t=1000 are shown in Fig. 3. From early convection pattern, convection cell is changed gradually to oblong shape. Although the flow rate decreases with time, convection did not disappear.

If Prandtl number is fixed, the velocity of the convection is increased as Rayleigh number is increased

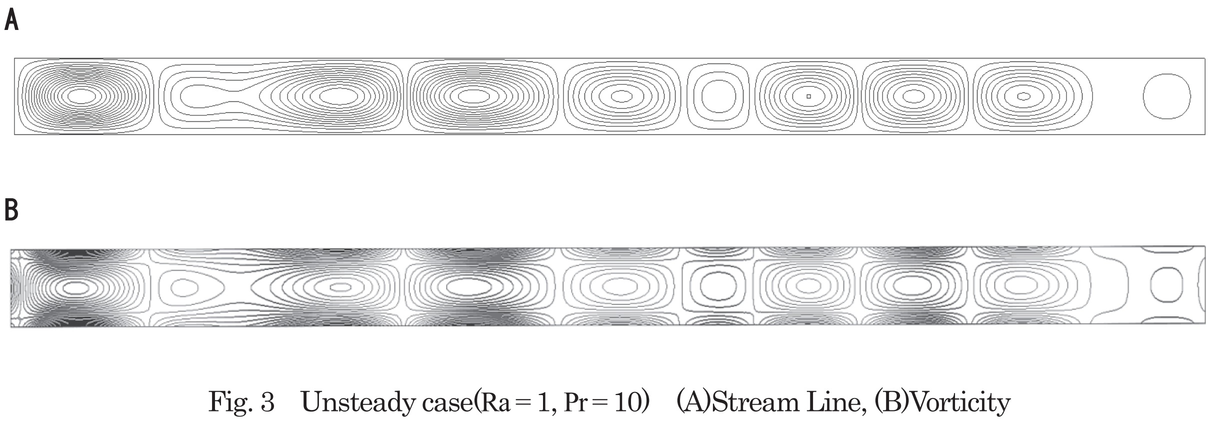

Fig. 3 Unsteady case(Ra = 1, Pr = 10) (A)Stream Line, (B)Vorticity

3.4 Collapse pattern

In the case of Ra = 10 and Pr = 10^{-2}, streamline and vorticity at t=1000 are shown in Fig. 3. Although convection occurs initially, density of the microorganism is increased with time in the vicinity of the bottom, and finally convection disappears. In the bottom where a microorganism is dense, back and forth flow in the horizontal direction is observed.

If Prandtl number is fixed, the collapse of the convection is accelerated and the horizontal flow velocity of the bottom portion after the convection collapsed is increased as Rayleigh number is increased,

A

B

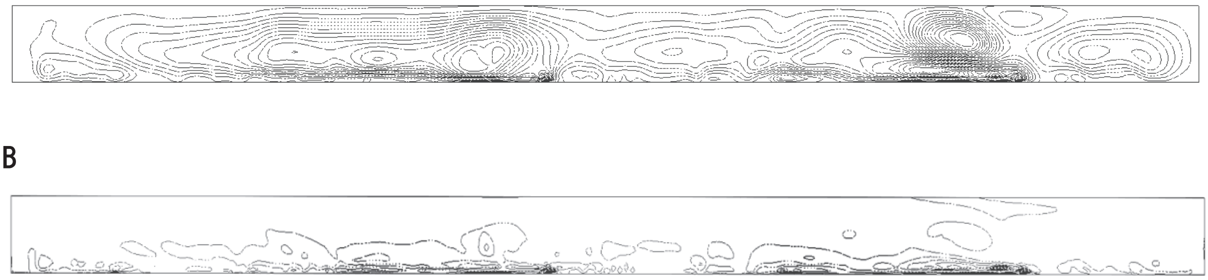

Fig. 4 Collapse case(Ra = 10, Pr = 10^{-2}) (A)Stream Line, (B)Vorticity

3.5 Discussions

In this section, we compare this result (phase diagram in Fig. 1) to the bubble convection (Iga and Kimura 2007: as first case) and apply to the bioconvection due to Chlamydomonas (Kage *et al.* 2013: as second case).

In the first case, bubble convection is some kind of particle-driven-convection as well as bioconvection. Iga and Kimura (2007) report the phase diagram corresponding to Fig.1. Their diagram shows that there are two states, steady convection and unsteady convection, and can be divided by Prandtl number around unity. Comparing our results to the bubble convection, there is a similarity that steady and unsteady-state are divided by the Prandtl number, while the differences are observed in the presence of the collapse pattern and the tendency of the aspect ratio of the convection cell. Bubble convection and bioconvection are common since both convections are driven by the particles. On the other hand, paying attention to behavior of the particles, bubbles are generated in the bottom surface and rise to the top where bubbles vanish in bubble convection, but in bioconvection, microorganisms swim upward and settle by the density difference. Collapse pattern occurs in bioconvection, but did not occur in the bubble convection. Collapse pattern will occur when the effect of gravity is dominant against upward-swimming of microorganisms.

In the second case, Kage *et al.*(2013) report that in bioconvection due to Chlamydomonas, a kind of dinoflagellates, initial convection changes abruptly to new convection where the aspect ratio of the convection cell is smaller than the initial one. The mechanism of this transition is not clear yet. Our study shows that aspect ratio of the convection cell is mainly changed by the Prandtl number. If our results can be applied to the Chlamydomonas' case, effective Prandtl number may decrease rapidly in the transition.

4 CONCLUSIONS

From the numerical simulation of a two-dimensional bioconvection by density model, we can classify convection patterns by the dimensionless parameter, Ra and Pr. When Prandtl number is increased, the aspect ratio of the convection cell is increased. The time evolution of convection can be classified into three types: “steady”, “unsteady” and “collapse”.

REFERENCES

Childress, W.S., Levandowsky, M. and Spiegel, E.A. (1975). “Pattern formation in a suspension of swimming microorganisms: equations and stability theory”, *J. Fluid Mech.,* 69, 591-613.

Harashima, A., Watanabe, M. and Fujishiro, I. (1988). “Evolution of bioconvection patterns in a culture of motile flagellates”, *Phys. Fluids*, 31, 764-775

Iga, K. and Kimura, R. (2007). “Convection driven by collective buoyancy of microbubbles”, *Fluid Dyn. Res.*, 39, 68-97.

Kage, A., Hosoya, C., Baba, S. A. and Mogami, Y. (2013). “Drastic reorganization of the bioconvection pattern of Chlamydomonas : quantitative analysis of the pattern transition response”, *J. Exp. Biol.*, 216, 57-4566.

Kessler, J. O.(1986), “Individual and collective fluid dynamics of swimming cells”, *J. Fluid Mech.*, 173, 191-205.

Yusaku Nagata
Otsuka 2-1-1, Bunkyo-ku, Tokyo 112-8610, Japan
E-mail: nagata.yusaku@ocha.ac.jp

Ai Nagasawa
E-mail: g1220531@is.ocha.ac.jp

Anna Kuwana
E-mail: kuwana.anna@ocha.ac.jp

Tetuya Kawamura
E-mail: kawamura@is.ocha.ac.jp