Numerical simulation of the transitional flow in the smooth pipe

Yuka Yoshida and Tetuya Kawamura

(Received December 10, 2012)

Abstract

Numerical simulations of the transitional flow in the smooth pipe are carried out. The governing equations are the incompressible Navier-Stokes equations with cylindrical coordinate systems. The fractional step method is used to obtain the numerical solutions and non-linear terms are approximated by the third-order upwind scheme. From the result of calculation, the transition from the laminar flow to the turbulent flow and the reverse transition from the turbulent flow to the laminar flow can be observed.

1. Introduction

The flow in a pipe is a classical theme in fluid mechanics and is one of the important themes in practical flow. The transition to the turbulent flow in the smooth pipe takes place as Reynolds number of the flow increases. Although the flow is laminar when the Reynolds number is small, the flow changes into the turbulence type when the Reynolds number becomes large. The Reynolds number in the transition point from the laminar flow to turbulent flow is called critical Reynolds number. Both of the laminar flow and the turbulent flow are generated on the critical Reynolds number, and called transitional flow. The laminar flow in the pipe is called Hagen-Poiseuille flow after the works of Hagen and Poiseuille. The flow is fluctuating violently in temporally and spatially in the turbulence flow. The pressure gradient is often treated as the constant in the experiment, and the transition to the turbulence flow is occurred as follows. The turbulent flow is generated on the part of the wall in the pipe, and extends to the radius direction, and finally spreads over the cross-section of the pipe. A complex phenomenon appears in the critical Reynolds number. The flow rate is reduced in the turbulence flow due to hardness of flowing, and this means the Reynolds number of the flow will be also decreased. If it becomes less than the critical Reynolds number, the flow is back to the laminar flow. However, the flow becomes the turbulent flow again by the increase of the Reynolds number the sake of ease of flowing. In other words, the laminar flow and the turbulent flow appear alternately in this condition.

The Navier-Stokes equation, which is the equation of motion of incompressible Newtonian fluid, is solved analytically in the laminar flow. The numerical simulation of the turbulence flow is carried out by using LES.[(1), (2)] However, the studies of the numerical simulation of alternating flow mentioned above are few.[(3)]

This study is focused on such simulation of the transition from the laminar flow to the turbulent flow and vice versa.

2. Preparation of the calculation

2.1 Governing equation and solution

The equations used in this simulation are the incompressible Navier-Stokes equations with cylindrical coordinate systems. The fractional step method is used to obtain the numerical solutions. In this study, spatial derivatives except non-linear terms are approximated by the central differences. Non-linear terms are approximated by the third-order upwind scheme. Note that the velocity u_i is defined by the velocity u at the point of x_i of the grid number i. Function f is assumed to be a function of u.

$$f\ \frac{\partial u}{\partial x} = f\ \frac{2u_{i+1}+3u_i-6u_{i-1}+u_{i-2}}{6\ \Delta x} \qquad f \geq 0$$

$$f\ \frac{\partial u}{\partial x} = f\ \frac{-u_{i+2}+6u_{i+1}-3u_i-2u_{i-1}}{6\ \Delta x} \qquad f < 0$$

that is

$$f\ \frac{\partial u}{\partial x}\ \Big|_{x=x_i} = f\ \frac{-u_{i+2}+8(u_{i+1}-u_{i-1})+u_{i-2}}{12\ \Delta x} + \frac{|f|}{12}\ \Delta x^3\ \frac{u_{i+2}-4u_{i+1}+6u_i-4u_{i-1}+u_{i-2}}{(\Delta x)^4}$$

The initial condition of the flow is given by the uniform flow in the z-axis direction. The boundary condition of the fluid velocity at the pipe wall is non-slip. In z-axis direction, the periodic boundary condition is imposed to the fluid velocity at the inlet and the outlet of the pipe. The pressure gradient is constant. Initial Reynolds number based on the initial flow speed, diameter of the pipe and the kinematic viscosity are 2000, 2500 and 4000 in this study.

2.2 Computational grid of the pipe

Figure 1 shows the computational grid of the pipe. The pipe length is 2.5 times of the pipe diameter. The number of the grid is 50 × 30 × 29 (z × r × θ). A uniform grid is used in axial direction (z) and tangential direction (θ). A non-uniform grid is used in radial direction (r) to yield accurate solutions around the pipe wall.

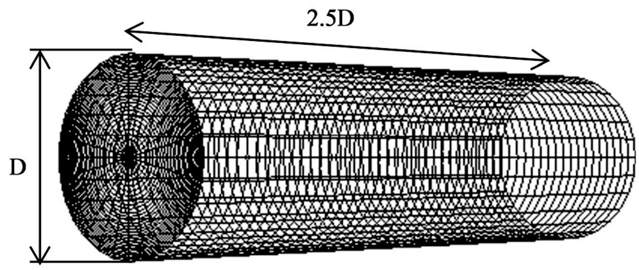


Fig.1 Computational grid of the pipe

3. Results and Discussions

3.1 The instantaneous velocity in the pipe

As the result of the calculation, the flow is always laminar flow at the initial Reynolds number Re = 2000. The transition from the laminar flow to the turbulent flow is observed at the Re = 2500. The turbulent flow is always observed at the Re = 4000. The instantaneous velocities of the flow in the pipe are shown in Fig.2.

The pressure difference between inlet and outlet is assumed to be a constant in this calculation. Therefore, the Reynolds number of the flow (Re_flow) in the pipe is changing due to the variation of flow rate. The Reynolds number of the flow (Re_flow) is defined by the following equation where u_{mean} is the mean velocity in the pipe, ν is the kinematic viscosity.

$$\mathrm{Re\_flow} = \frac{u_{mean} \times D}{\nu}$$

Figure 3 shows the time variation of the Re_flow in the pipe at the initial Reynolds number Re = 2000, 2500 and 4000. T is the dimensionless time. Under the condition of the Re=2000, in which the flow is always laminar, Re_flow is decreased smoothly and then becomes constant. Under the condition of the Re = 2500, the Re_flow is decreased with the velocity drop due to the turbulence, and then Re_flow is increased again by the transition to the laminar flow, and finally it becomes a constant. The flow velocity is decreased by the turbulence flow, and then becomes constant under the condition of the Re=4000.

Fig.2 The instantaneous velocities of the flow

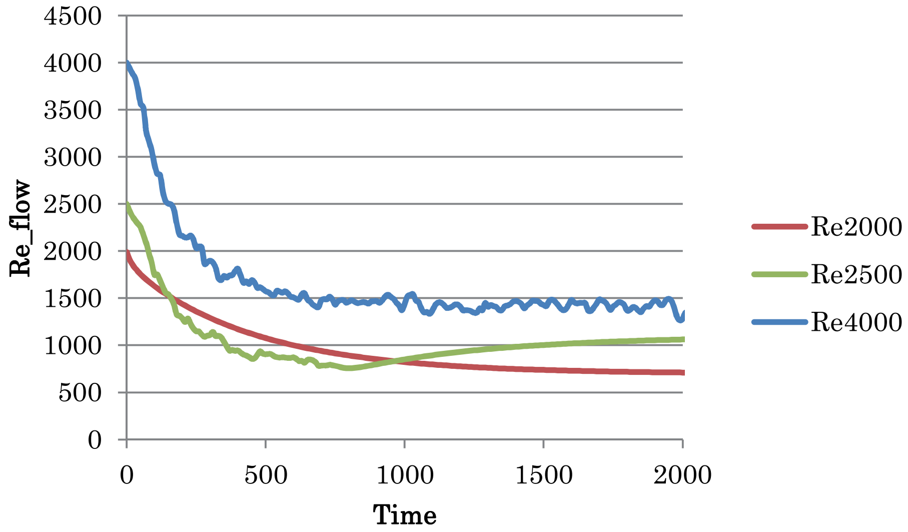


Fig.3 Time variation of the Re_flow

Table 1 Re_flow calculated by the average of the velocity after T=1500

Re	2000	2500	4000
Re_{flow}	700	1100	1420

3.2 The mean velocity in the pipe

J. Nikuradse carried out an experimental investigation of velocity profiles at the several Reynolds numbers in smooth pipes.[(4)] Velocity profile of experiment is plotted in Fig.4 under the condition of Reynolds number of 4000. u is the mean velocity at each time and R is pipe radius (D/2). They are given in dimensionless form in the figure where u/U is plotted against y/R. U is the mean velocity in the y/R = 1 and y is the distance from the pipe wall. The result of calculation under the state of developed turbulence in Reynolds number of Re = 4000 also shows Fig.4. The shape of the velocity distribution of the calculation result is close to the laminar flow than the experimental result. The flow in nearly constant after T = 1500 in all conditions. Re_flow is calculated by the average of the velocity at the period from T = 1500 to T = 2000, and are listed in Table 1.

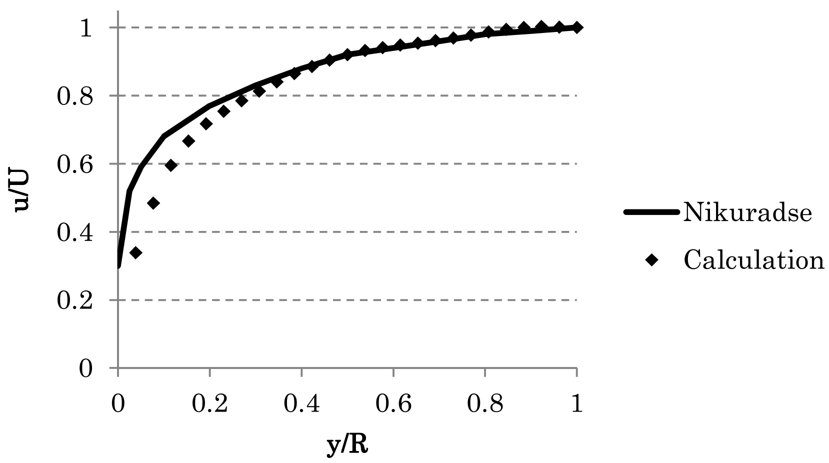
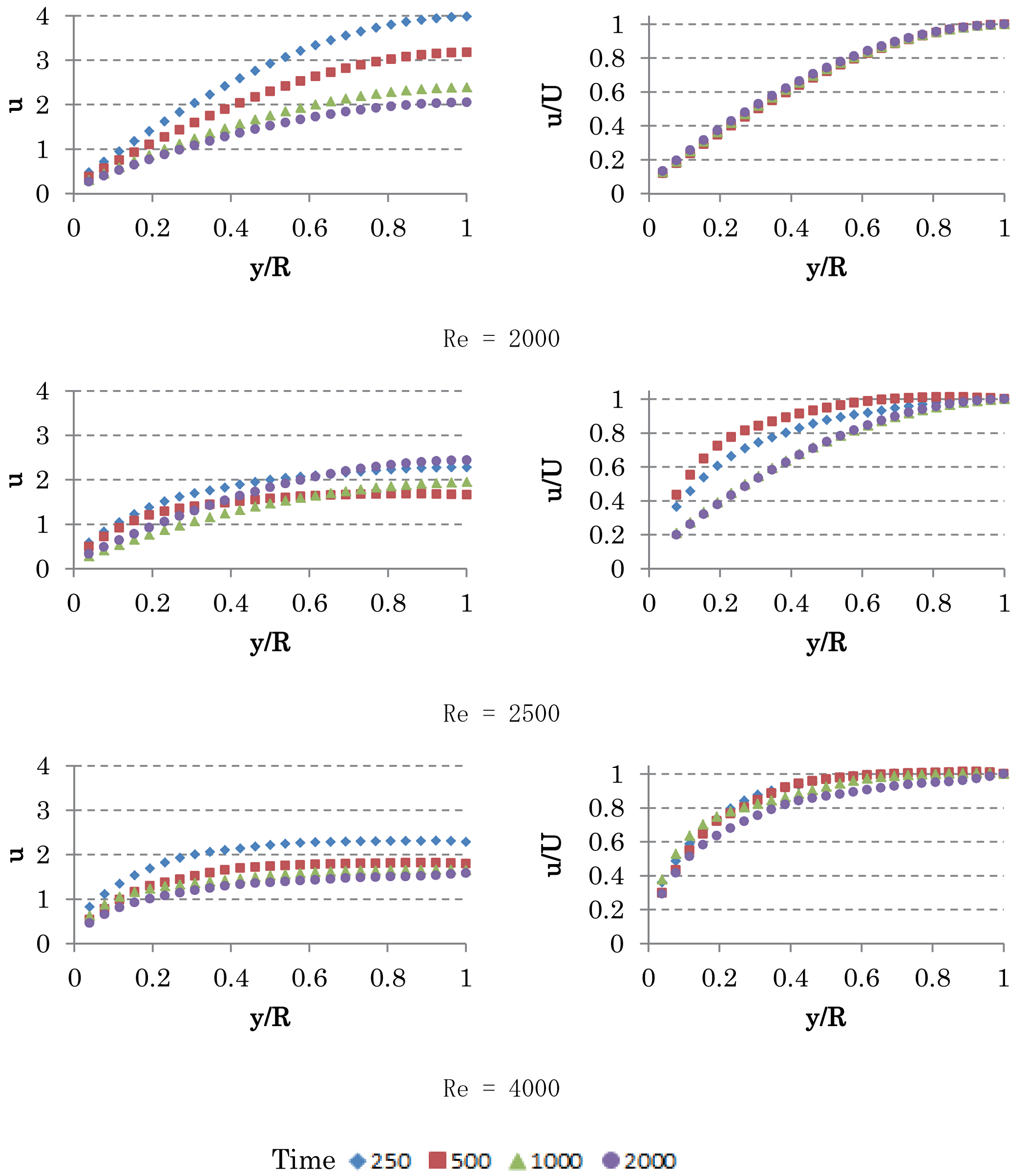


Fig.4 Velocity distribution in the pipe

Figure 5 shows velocity distributions which are axially-averaged in the pipe.

Figure 5 (a) indicates that the flow velocity at the Re = 2000 and 4000 are gradually decreased with time. On the other hand, the velocity at the Re2500 is decreased once in the initial stage and begins to increase in the later stage.

Figure 5 (b) indicates that the flow velocity at the Re = 2000, 2500 and 4000. At the Re = 2000, the flow profile is always laminar, and the tendency of these plots show good agreement in all the time. At T =500 and the Re = 2500, the velocity profile is similar to the turbulent flow. The flow is laminar in the other time. At the Re = 4000, the flow is always turbulent. At the Re = 2500, T = 250 and 500, the velocity profiles resemble those of turbulent flow. While T = 1000 and 2000, those become the laminar flow. The flow has the transition point between these times at the Re = 2500.



(a) Velocity distributions at the several Re (b) Velocity distributions at the several Re (velocity normalization)

Fig.5 Velocity distributions in the pipe

4. Conclusions

In this study, the numerical simulation by using the third-order upwind scheme was carried out for the flow in the smooth pipe. We achieved to simulate the transition from the laminar flow to the turbulent flow. The turbulent flow is observed to turn to the laminar flow with changing the Reynolds number.

References

[1] James W. Deardorff, A numerical study of three-dimensional turbulent channel flow at large Reynolds numbers, J. Fluid Mech, 41,453-480, 1970

[2] Parviz Moin and John Kim, Numerical investigation of turbulent channel flow, J. Fluid Mech, 118,341-377, 1982

[3] Tetuya Kawamura, Solutions of the Navier-Stokes Equations, Publ. RIMS, Kyoto Univ., 539, 53–67, 1984

[4] J. Nikuradse, Gesetzmässigkeiten der turbulenten Strömung in glatten Rohren, Forsch. Arb. Ing.-Wes. No.356, 1932

Yuka yoshida
Otsuka 2-1-1, Bunkyo-ku, Tokyo 112-8610, Japan
Shin-Nakahara-Cho 1, Isogo-ku, Yokohama 235-8501, Japan
e-mail: yuka_yoshida@ihi.co.jp

Tetuya Kawamura
Otsuka 2-1-1, Bunkyo-ku, Tokyo 112-8610, Japan
e-mail: kawamura@is.ocha.ac.jp