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Numerical Simulation of Formation of a Strange Shaped Rock Due to Erosion by Wind

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Abstract

As a tentative estimate, we made a simple model of simulating erosion by wind using a sand transfer equation and we could reproduce a process of formation of a strange shaped rock. The numerical method is as follows: First, flow field around a rock which is governed by incompressible Navier-Stokes equations is solved numerically by the standard MAC method. Second, we estimate amount of erosion that is proportional to the cube of friction velocity. Third, new shape of an eroded rock is calculated. These three steps are repeated. The flow around a rock and its surface are visualized and the influences of hardness of the rock on erosion by wind have been analyzed. Moreover, we examined several typical cases in order to reproduce strange shaped rocks appeared to Cappadocia in Turkey.

1. INTRODUCTION

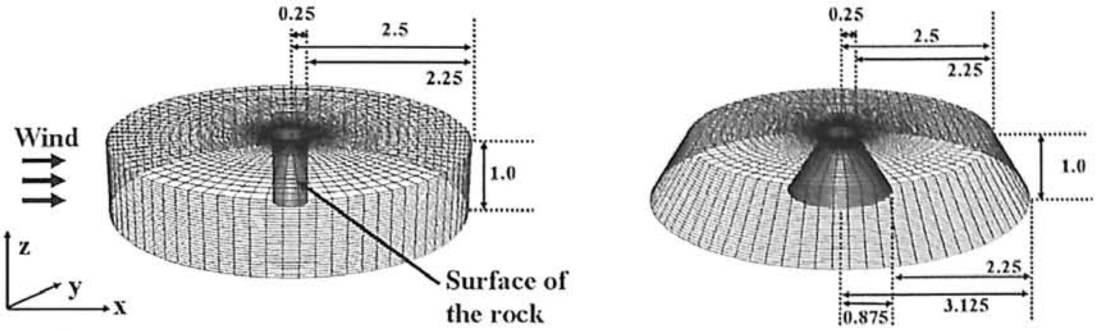
Cappadocia is a world heritage in Turkey, the rock zone extends to nearly 100km^2 on the plateau in central of the Anatolia highlands over the sea level 1,000m (Fig.1 (a)). Soft tuff, made from volcanic ash blown from nearby volcano like Mt. Erciyes and Mt. Hasan, is much eroded by wind as compared with the other kind of rock. Due to difference of eroding speed, strange shaped rocks are formed (Fig. 1 (b)) in the area where hard and soft layers are piled up. The objective of this study is to reproduce a process of erosion by numerical simulation. It is difficult to consider all factors concerned with erosion completely (change of the temperature, erosion by water, effect of plants, etc.). Therefore we make some assumptions on simulating erosion process by wind.

(a) Rock zone　　　　(b) Strange shaped rocks

Fig.1　Pictures of Cappadocia

2. NUMERICAL METHODS

Rocks of a circular cylindrical shape and a truncated-circular-cone shape are assumed to be in the uniform flow. Computational grids are shown in Fig. 2. It is also assumed that only the side surface of the rock is eroded and the upper and the lower ends are not. The no-slip condition is employed to the upper and lower boundary of the computational region.



(a) A rock of circular cylindrical shape (b) A rock of truncated-circular-cone shape

Fig.2 Computational grids

2.1 Calculation Step 1

Flow field around a rock is analyzed, that is governed by equation of continuity (1) and incompressible Navier-Stokes equations (2):

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho_0}\nabla p + \nu \nabla^2 \mathbf{u} \tag{2}$$

where $\mathbf{u} = (u, v, w)$ is the velocity, t : time, ρ_0 : air density (constant), p : pressure, ν : kinematic viscosity of the air (constant). Equations (1) and (2) are non-dimensionalized by using wind speed on upstream boundary and height of cylinder or truncated-cone. In this study, Reynolds number (Re) is set to 10,000. These equations are transformed to the coordinate system fitting to the surface of the eroded rock that is changing to complex shape progressively (Fig.3). These equations are solved numerically by the standard MAC method [1]. All spatial derivatives except nonlinear terms are approximated by the central differences, and nonlinear terms are approximated by the third order upwind difference (3) [2].

$$f\frac{\partial u}{\partial x}\bigg|_{x=x_i} = f\frac{-u_{i+2} + 8(u_{i+1} - u_{i-1}) + u_{i-2}}{12\Delta x} + |f|\frac{u_{i+2} - 4u_{i+1} + 6u_i - 4u_{i-1} + u_{i-2}}{12\Delta x} \tag{3}$$

2.2 Calculation Step 2

We assume that the erosion of the rock by wind is analogous to the sand transfer by wind, i.e. the amount of erosion is proportional to the cube of friction velocity:

$$\mathbf{q} = b_1 \frac{\rho_0}{g} |\mathbf{u}_*|^2 \mathbf{u}_* \tag{4}$$

where b_1 is constant (depending on the shape of the surface of the rock), g : gravitational acceleration, $\mathbf{u}_* = \sqrt{\nu \cdot d\mathbf{u}/dz}$: friction velocity (Fig. 4). This equation is proposed by Bagnold, the first researcher who investigated the sand transfer caused by the wind [3].

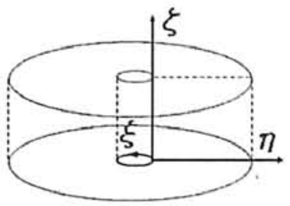


Fig. 3 Coordinate system

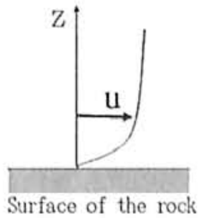


Fig. 4 Friction velocity

2.3 Calculation Step 3

New shape of an eroded rock is determined by the following relation:

$$\rho_s \frac{dh}{dt} = -\frac{dq_1}{dX} - \frac{dq_2}{dZ} \tag{5}$$

where, ρ_s is sand density, (X, Z) : local coordinates on the plane parallel to the surface of the rock shown in Fig. 5, h : height measured from the local coordinate, (q_1, q_2) : X and Z components of the vector $\mathbf{q}$ (Eq. (4)) which express the amount of erosion. Equation (5) is obtained from mass conservation of sand in infinitesimal area on the surface of the rock shown in Fig. 6. Note that dh/dt should be always negative in the process of erosion. Therefore, if it becomes positive, we force it to be zero.

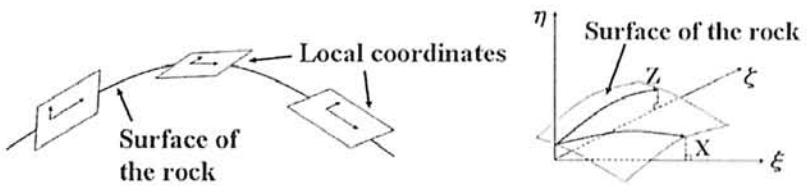


Fig. 5 Local coordinates

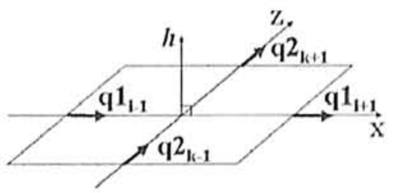


Fig. 6 Mass conservation of sand

Equation (5) is non-dimensionalized by a parameter C expressing hardness of the rock.

$$\frac{d\tilde{h}}{d\tilde{t}} = C\left(\frac{d\tilde{q}_1}{d\tilde{X}} + \frac{d\tilde{q}_2}{d\tilde{Z}}\right), \quad C = -b_1 \frac{\rho_o}{\rho_s} Fr^2 \tag{6}$$

where F_r is Froude number (constant), the symbol ~ means dimensionless quantity. The amount of the sand in a small region is estimated by Eq. (6) and the change of the height Δh is determined at each grid point on the surface of the rock.

The whole grid system that fits the surface of the rock must be reconstructed and the air flow above the rock must be recomputed in every time step due to the change of the surface of the rock. Therefore, these three calculation steps described in 2.1-2.3 are repeated until the time steps exceed the predetermined number.

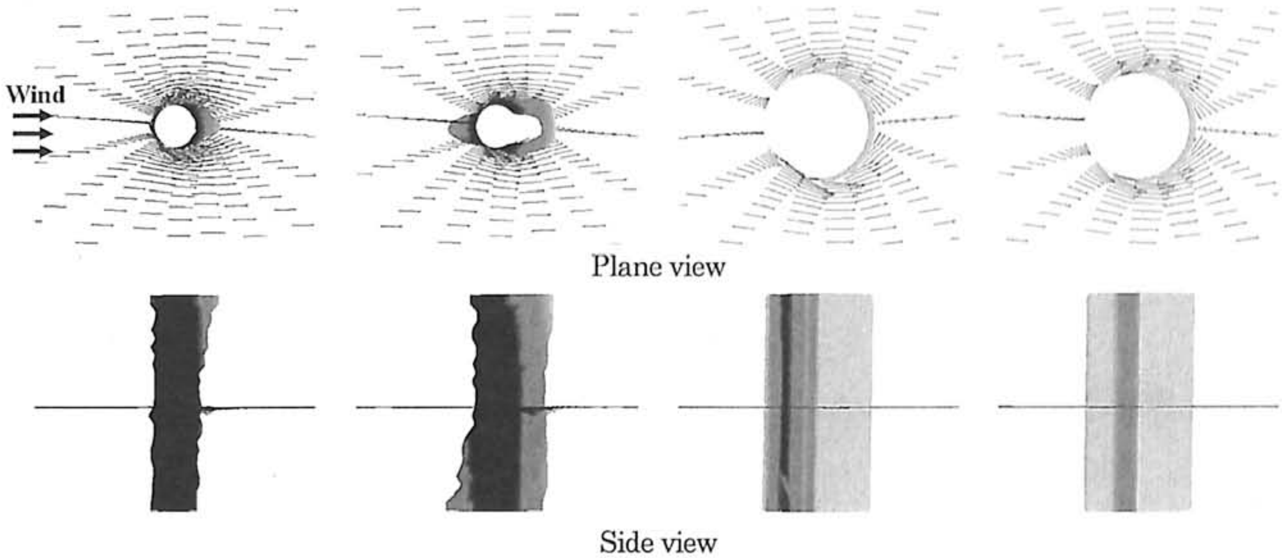
Some smoothing procedures are employed in this study. If the value $(h_{i+1,k}-2h_{i,k}+h_{i-1,k})/2$ that approximates the curvature of the surface along ξ-direction is larger than a certain value (0.005, i.e. 2% of R)), $h_{i,k}$ is replaced by the arithmetic average of $h_{i+1,k}$ and $h_{i-1,k}$. R is the initial value of radius of the rock. If the value $(h_{i,k+1}-2h_{i,k}+h_{i,k-1})/2$ that approximates the curvature of the surface along ζ-direction is larger than 0.025, i.e. 10% of R, $h_{i,k}$ is replaced by the arithmetic average of $h_{i,k+1}$ and $h_{i,k-1}$. Moreover, if the radius of the rock becomes smaller than a certain value (0.1, i.e. 40% of R) at a grid point, erosion stops at that grid point.

3. RESULTS AND DISCUSSIONS

3.1 Simple circular cylindrical rocks

Four kinds of rock which have different value of the parameter C in Eq. (6) (expressing hardness of the rock) are compared. A soft rock with large C is much eroded by wind as compared with the other kind of rock. Because the amount of erosion is proportional to the cube of friction velocity, the side surfaces of the rock are more eroded than the front surface toward the direction of the wind.

Flow fields (velocity vectors) and the surface of the rock are indicated in Fig. 7 at the time step 10,000 when the erosion has almost finished on soft rocks as shown in Figs. 7(a) and (b). The wind blows from left to right side. Eroded area is indicated by black color.

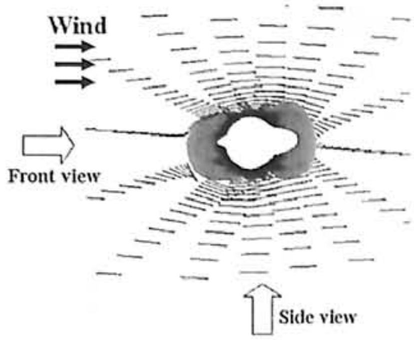


(a) $C=0.5\times10^{-2}$ (A soft rock) (b) $C=0.5\times10^{-3}$ (c) $C=0.5\times10^{-4}$ (d) $C=0.5\times10^{-5}$ (A hard rock)

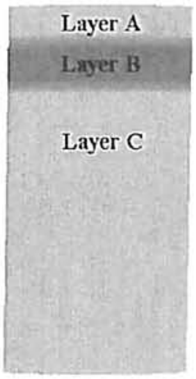
Fig. 7 Flow fields and the surface of the rock of simple circular cylindrical shape

3.2 A strange shaped rock 1

In order to reproduce a rock indicated in Fig. 9, we assume that the rock consists of three layers as shown in Fig. 8 (a). A soft layer between hard layers is much eroded and it became a strange shaped rock. Eroded area is indicated by black color. These are the results obtained after time step 10,000.

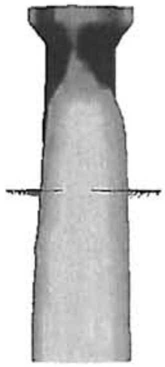


(b) Plane view

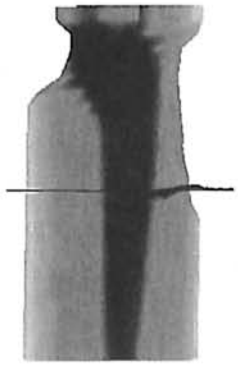


Layer A: $C = 0.5\times10^{-4}$
Layer B: $C = 0.5\times10^{-2}$
Layer C: $C = 0.2\times10^{-4}$
Height of Layers A: B: C = 2: 3: 25

(a) Three layers



(c) Front view



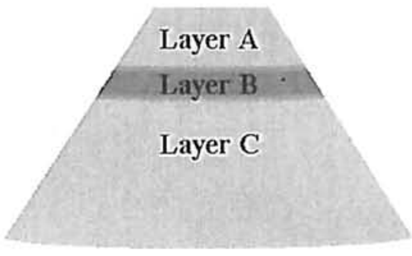
(d) Side view

Fig.8 A rock consisting of three layers corresponding to Fig. 9

Fig.9 A strange shaped rock

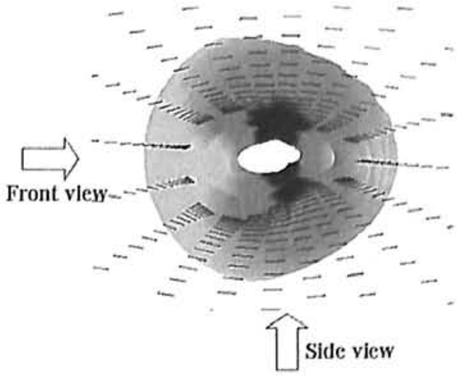
3.3 A strange shaped rock 2

To reproduce a rock indicated in Fig. 11, truncated-circular-cone rock is assumed to consist of three layers. Fig. 10 is the results. From front view in the figure (Fig. 10(c)), it is shown well that the soft layer (Layer B) is eroded flatly.



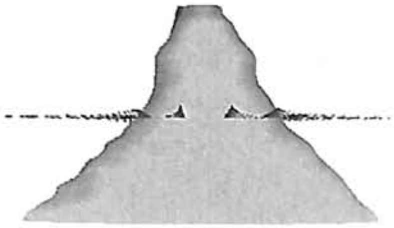
Layer A: $C = 0.5\times10^{-5}$
Layer B: $C = 0.2\times10^{-2}$
Layer C: $C = 0.5\times10^{-4}$
Height of Layers A: B: C = 2: 1: 6

(a) Three layers

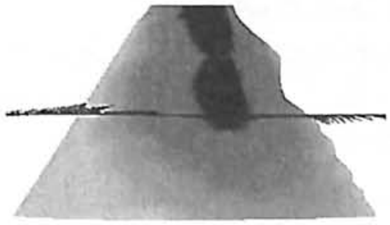


(b) Plane view

(A part of) Fig.10 A rock consisting of three layers corresponding to Fig. 11



(c) Front view



(d) Side view

(A part of) Fig.10 A rock consisting of three layers corresponding to Fig. 11

Fig.11 Strange shaped rocks

4 CONCLUSIONS

A simple model of simulating erosion by wind using a sand transfer equation was proposed. The flow around a rock and its surface are visualized and the influences of hardness of the rock on erosion by wind have been analyzed. As a result, process of formation of a strange shaped rock was reproduced and it confirmed that wind is one of important factors concerned with erosion process. The other factors and more appropriate correction method will be considered in the future.

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