Generation of spiral shock wave in the simulation of formation of accretion disk in primitive planetary system

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Abstract
Smoothed Particle Hydrodynamics (SPH) simulation is carried out to find the possibility of spiral shock wave in accretion disk in primitive planetary system. In order to avoid uncertainness due to artificial viscous term, we employ SPH method combined with Riemann solver in this study. We have shown that the spiral shock wave can be formed in primitive planetary system under the limiting initial conditions of temperature and angular velocity.

1. Introduction

It is well-known that the accretion disk is generated in binary system where two stars locate within short distance and rotate around the common center of mass. The existence of spiral shock wave is predicted numerically in binary system [1] and is actually verified by the astronomical observation [2]. Similar to the case of binary system, the accretion disk is considered to exist when the planet is formed in primitive planetary system. This suggests the possibility of existence of spiral shock in such system. If this assumption stands, the shock wave must affect the formation of planet itself. However, we cannot find investigations concerning to the spiral shock wave in primitive planetary system.

From above background, we try to find out the possibility of the generation of spiral shock wave in primitive planetary system by means of numerical simulation.

2. Numerical Method

2.1 SPH method

We employ Smoothed Particle Hydrodynamic (SPH) method to simulate the dynamics of interstellar gas [3]. SPH method is the particle method in which particles are moved according to the equation of motion. Each particle is not real molecule but imaginary substance that has physical quantity such as density and velocity. This method is quite effective for astronomical simulation since the density, for example, changes from 1 to 10^{12} in planetary system. In these situations, the method based on the grid such as a finite difference method is difficult to apply.

In SPH method, density ρ is calculated from SPH particles by using the relation

$$\rho(x)=\sum_{j}^{n} m_j W(x-x_j,h) \tag{1}$$

, where n is the number of SPH particles, h is the quantity that represents the width of the particle and W is so-called kernel function. This function is determined by satisfying the relation

$$\int W(x,h)dx^3 = 1 \tag{2}$$

The equation of motion for each particle is

$$\frac{dv}{dt}=-\frac{1}{\rho}\nabla P-\nabla\Phi \tag{3}$$

where the first term and the second term are the pressure gradient and gravitational force respectively. These are represented by the kernel function as follows:

$$-\frac{1}{\rho}\nabla P=-\sum 2m_j\frac{\sqrt{PP_j}}{\rho\,\rho_j}\nabla W(x-x_j,h) \tag{4}$$

$$\nabla\Phi=-\sum_j m_j\int 4\pi r^2 W(x-x_j,h)dr\cdot\frac{\vec{r}}{r^3} \tag{5}$$

Since no viscous term exists in equation (3), unphysical oscillation takes place in the computation. To avoid this oscillation, it is required to add artificial viscous term in relation (4). Therefore it is usually used the relation

$$-\frac{1}{\rho}\nabla P=-\sum m_j\left(2\frac{\sqrt{PP_j}}{\rho\,\rho_j}+\Pi\right)\nabla W(x-x_j,h) \tag{6}$$

instead of equation (4) where

$$\Pi=-\alpha\frac{a_{ij}\mu_{ij}}{\rho_{ij}}+\beta\frac{\mu_{ij}^2}{\rho_{ij}} \tag{7}$$

is the artificial viscous term and α, β are the adjustable constants.

2.2 SPH method combined with Riemann solver

The values of constant α and β in artificial viscous term depend on the strength of the shock wave and it is necessary to determine empirically. In order to overcome this ambiguity, we employ the SPH method combined with Riemann solver in which one-dimensional approximate Riemman problem is solved at every time step [4].

In this formulation, if we look at i-th particle and consider the effect of j-th particle, the intermediate pressure is determined from following relation

$$p^* = \frac{P_i\rho_i c_i + P_j\rho_j c_j - \rho_i c_i \rho_j c_j (v_i - v_j)}{\rho_i c_i + \rho_j c_j} \tag{8}$$

where Ci and Cj are local sound speed at the position of i-th and j-th particle. This relation is obtained by one-dimensional Riemann problem. The equation of motion for i-th particle is obtained by summing up all effects of other particles and becomes

$$\frac{dv}{dt} = -\sum_j 2\frac{m_i p^*}{\rho\,\rho_j}\nabla W(x - x_j, h) - \nabla\Phi \tag{9}$$

As is usual SPH method, we use polytropic equation for the relation between pressure and density.

$$\frac{P}{\rho^\gamma} = const \tag{10}$$

3 Results

3.1 One-dimensional shock tube problem

At first, we solve one-dimensional shock tube problem in order to examine the effectiveness of the SPH method combined with Riemann solver. The flow region is divided into two parts by partition wall; one is high pressure and the other is low pressure. The flow is computed after the breakdown of the wall. In this case, shock wave is formed and propagates in the tube.

Figure 1 is the pressure distribution in one-dimensional tube at typical time step obtained by ordinal SPH method and SPH method combined with Riemann solver. It is clear that the latter can suppress unphysical oscillation near the shock. Figure 2 is the comparison of velocity. The same tendency is observed. From these figures, we can conclude our method works well for the compressible flow with shock.

3.2 Stellar simulation

In the stellar simulation, we use following parameters as stellar clouds of spherical shape.

Radius: $1000AU(AU = 1.5\times10^{11}m)$

Density: 10^4 particles / cm^3

Mass: 10^4 particles / cm^3

This stellar clouds is assumed to have uniform density and to rotate with constant angular velocity ω around z-axis.

We investigate the effect of temperature and angular velocity on the distribution of the particles and the formation of shock wave in this study. We perform computations under the various initial temperature and angular velocity to see the results. In these computations, the number of SPH particles is set to 3000.

Figures 3-6 are time series of bird's eye view of the particles. Four typical results are indicated. As shown in the figures, the distribution can be classified into following four types: (1) shrinking to form primitive star (fig.3), (2) ring type contraction – the shape is complicated (fig. 4), (3) spiral type of two arms (fig.5), (4) accretion disk type (fig.6). Figure 7 shows the location of shock wave corresponding to above four cases. In case (1), the shock wave appears as it surrounds the star,; in case (2) ,similar shaped shock wave appears outside of the ring; in case (3), spiral shock wave are observed; in case (4), no shock wave exists.

Figure 8 is the state diagram that indicates the parameter (temperature and angular velocity) range of the formation of typical four patterns in the planetary system. It is shown that the spiral shock wave can be formed in primitive planetary system under the limiting initial conditions of temperature and angular velocity.

4. Summary

In this study, SPH simulation is carried out to investigate the possibility of the formation of spiral shock wave in primitive planetary system that is observed in binary system. In order to avoid ambiguity caused by artificial viscous term, Riemann solver is combined with ordinal SPH method. After the verification by solving one-dimensional shock wave problem, we perform the main calculations. We choose initial temperature and angular velocity as the governing parameters and change them systematically to see the effect on the formation of planetary system. As the results, four typical pattern are observed. Moreover, we have shown that the spiral shock wave can be formed in primitive planetary system under the limiting initial conditions of temperature and angular velocity. This information is important for elucidation of the origin of the solar system.

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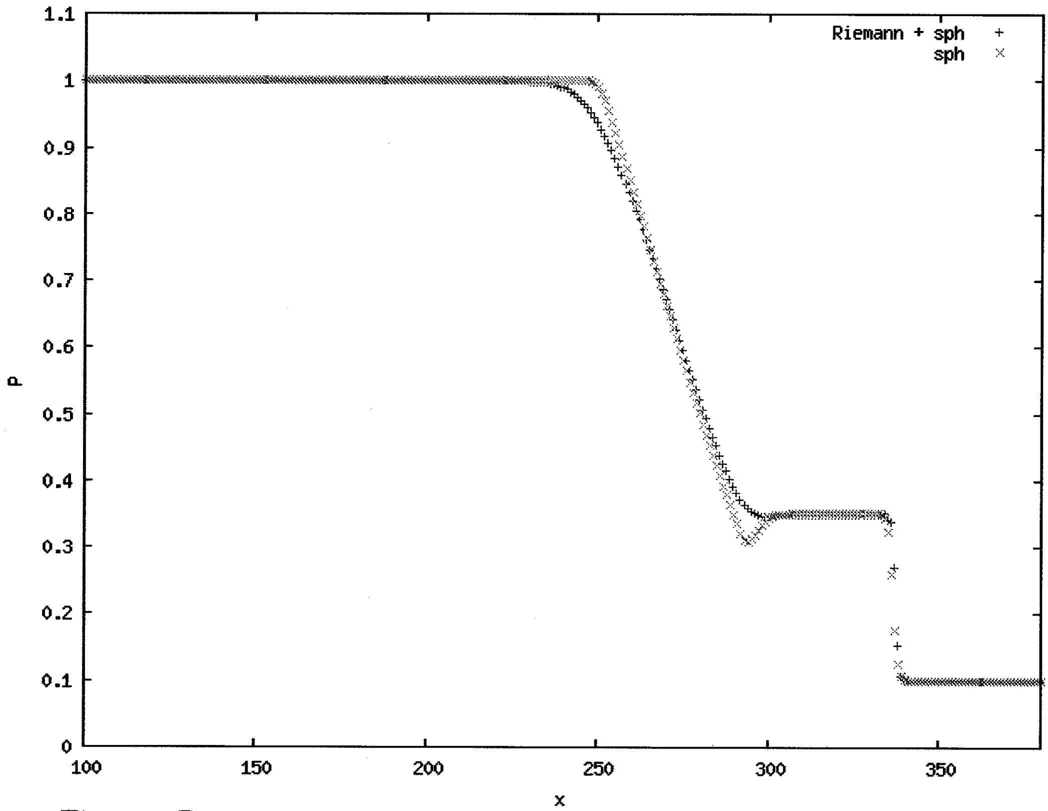

Figure 1 Pressure obtained by ordinal SPH method and present method

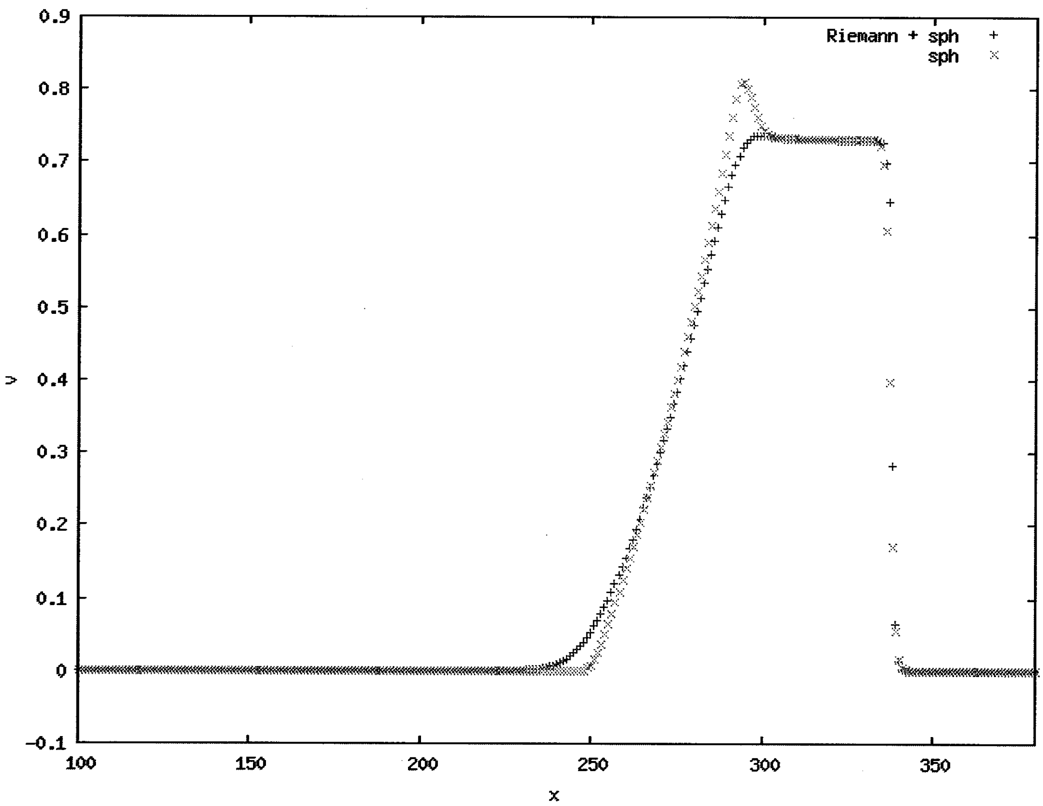

Figure 2 Velocity obtained by ordinal SPH method and present method

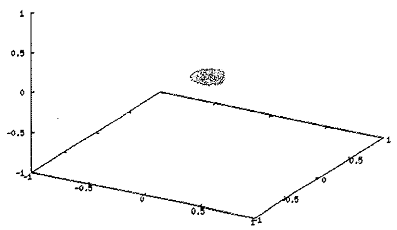

Figure 3 Shrinking to star

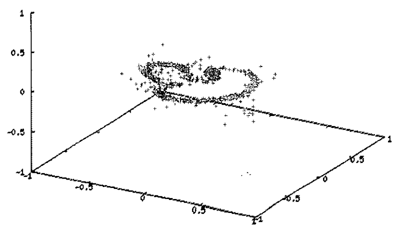

Figure 4 Ring type contraction

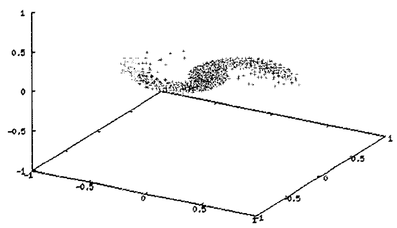

Figure 5 Spiral type of two arms

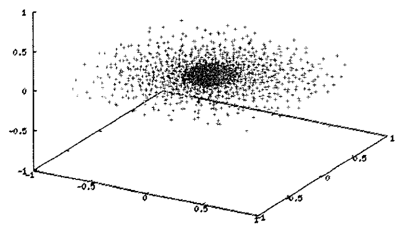

Figure 6 Accretion disk

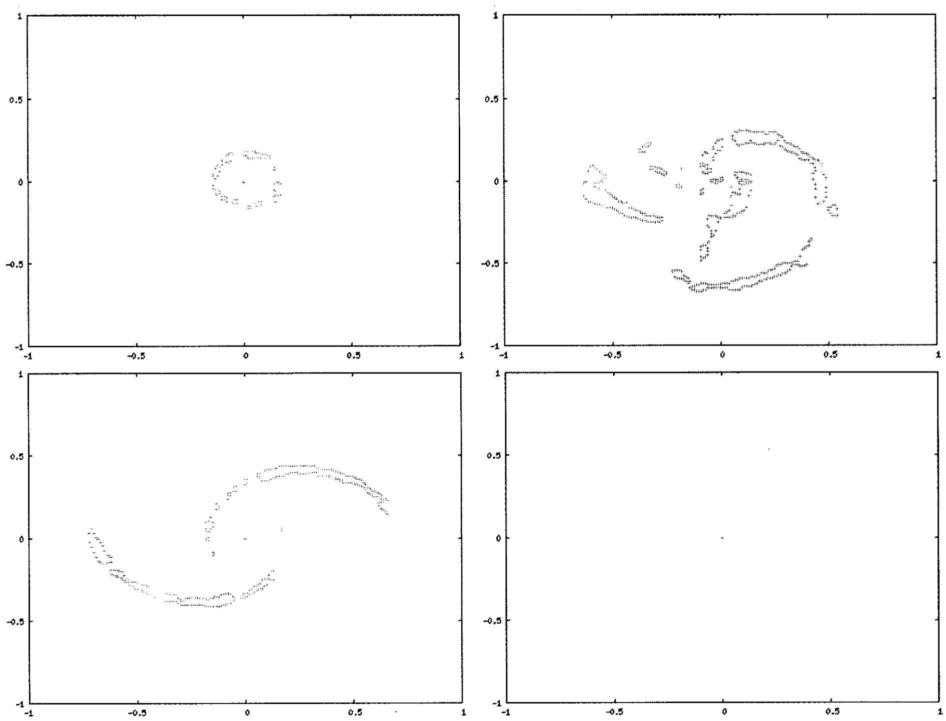

Figure7 Position of shock waves corresponding to fig.3,4,5 and 6.

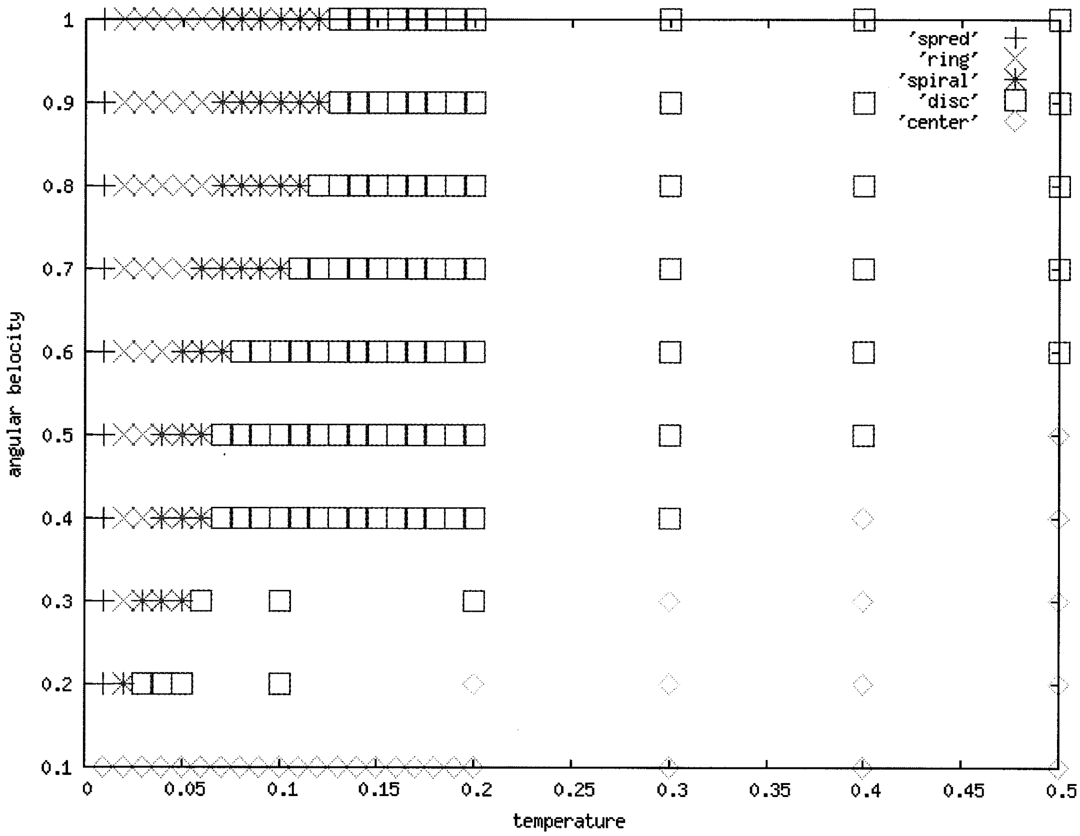

Figure 8 State diagram indicating parameter ranges of temperature and angular velocity