Simple simulation model of the generation of clouds by the flow over an isolated mountain

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Abstract

A simple model to simulate the formation of clouds by the wind blowing over a mountain is proposed in this study. The flow is assumed to be stably stratified. The incompressible Navier-Stokes equations together with the advection equation of density are chosen for the basic equations. These equations are solved numerically by the standard MAC method. Third order upwind difference is employed for the approximation of the nonlinear terms of the Navier-Stokes equation. Finite number of particles is used to express the clouds. At first, each particle has its own temperature and humidity and is moved according to the velocity of the flow field. When the humidity becomes 100%, the particle is assumed to become a small cloud and is displayed. The effects of the humidity as well as the wind speed etc. on the formation of the clouds are investigated and we can obtain reasonable results which are consistent with our everyday experience.

1 INTRODUCTION

The atmosphere is greatly affected by the temperature distribution. Even if the atmosphere is stable (i.e. the density decrease with height), we can observe various interesting phenomena occurring in the flow around an obstacle such as a mountain[1]. In this case, vertical motion tends to carry heavier fluid upwards and lighter fluid downwards, and is suppressed. This suppression may take the form of modifying the pattern of the laminar motion or of preventing or modifying its instability.

One example is that the wind blowing against a mountain tends to go round within a horizontal plane instead of going over a mountain. If the stratification is not very stable, a fluid particle can go over the mountain. However the particle displaced vertically tends to be restored to its original level; it may then overshoot by inertia and oscillate about its level. Although each fluid particle oscillates up and down as it travels downstream, the overall flow pattern is steady in the frame of reference of mountain. This means upward stream and downward stream exist at fixed position behind the mountain. This wavy flow is called lee wave. If the air is moist at the same time, we may observe clouds at the position where the downward stream exists.

Although we can find various numerical studies for stable stratified flow around an obstacle [2][3][4], there are few researches concerned with the generation of clouds by such flow. In this study, we compute the stably stratified flow around an isolated mountain and try to simulate the generation of the typical clouds around a mountain.

2 NUMERICAL METHOD

2.1 Basic equation

In this study, we take z vertically upwards and suppose that the basic stratification consists of a uniform density gradient ($d\rho_B(z)/dz = -1$). The pressure $p'(= p - p_B(z))$ and the density $\rho'(= \rho - \rho_B(z))$ represent the modification ones, i.e. the difference from the base pressure $p_B(z)$ and base density $\rho_B(z)$. The basic equations in non-dimensional form with u, v, w, ρ' and p' become as follows[2]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{\partial p'}{\partial x}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{\partial p'}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right) \tag{3}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{\partial p'}{\partial z}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)-\frac{\rho'}{Fr^2} \tag{4}$$

$$\frac{\partial \rho'}{\partial t}+u\frac{\partial \rho'}{\partial x}+v\frac{\partial \rho'}{\partial y}+w\frac{\partial \rho'}{\partial z}=w \tag{5}$$

Equation (1) is the continuity equation, equations (2)-(4) are momentum equations of x, y and z directions and equation (5) is the advection equation of density which represents the condition of incompressibility.

2.2 Representation of clouds

In order to represent clouds, we consider small particles consisting of the air. Each particle has its own temperature and humidity and is moved by the flow at its position. These particles are put into flow field at various positions on the upstream boundary. If the temperature of the particle becomes smaller than its dew point, the particle is considered to become cloud and is displayed as cloud. The dew point of the particle is computed beforehand from its initial temperature and humidity by the empirical formula of Tetens:

$$e = 6.11 \times 10^{7.5T/(237.3+T)} \times \frac{h}{100} \qquad (6)$$

where e is vapor pressure, T is temperature and h is humidity.

Dew point(T_d) is defined by the temperature at which the humidity becomes 100%. Therefore, T_d is obtained by substituting equation (7) into h=100 , solving it in terms of T and setting $T = T_d$. Strictly speaking, the equation is obtained for water and it is not accurate for ice. However, we assume the water is still liquid under the freezing point for simplicity. We also assume that the cloud disappears when the temperature exceeds drew point.

In general, the position of the particle does not coincide with the grid points of the calculation. We use the following formula of interpolation

$$v' = \frac{\sum_{i=1}^{8} v_i / r_i}{\sum_{i=1}^{8} 1/r_i} \qquad (7)$$

to compute the velocity at the position as is shown in Fig. 1.

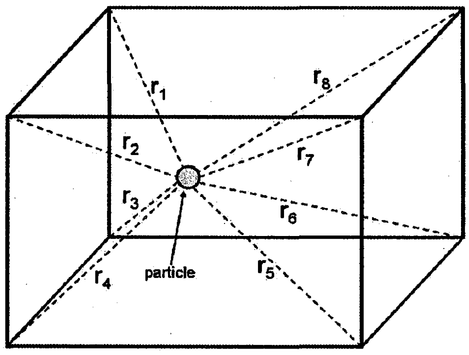


Fig. 1 Interpolation.

For the realistic simulation, it is desired to use huge number of particles. However, the computation time becomes large at the same time. In order to save the computation time, the function F which represents the cloud is defined as follows:

$$F(x,y,z) = \sum_{i=1}^{N} \exp\left(-Ad_i^2\right) \qquad (8)$$

where N is the number of the particles, d_i is the distance between the point (x,y,z) and i-th particle and A is numeric constant.

Taking the following fact into account that the value of the function becomes large when the distance from the particle becomes small and where many particles exist its vicinity, this formula is considered to be reasonable. Note that the function F is used only for the visualization of the cloud and does not affect the calculation of the flow field.

2.3 Relation between Froude number and lapse rate

The flow is affected by the Froude number (Fr) and Reynolds number (Re). On the other hand, the initial temperature of the particle is determined by using the lapse rate (Γ) according to their height. In the computation, it is possible to change Fr, Re and Γ independently. However, in the real situation, since both Fr and Γ affect the stability of the atmosphere, there should be a relation between them.

Froude number is defined by

$$Fr = \frac{U}{NH} \qquad (9)$$

where U is velocity scale(uniform wind velocity), H is length scale (height of mountain) and N is the Brunt-Vaisala frequency. On the other hand, as

$$\frac{\partial T}{\partial z} = -\Gamma \quad \text{and} \quad N^2 = \frac{g}{T}\left(\frac{dT}{dz} + \frac{g}{C_p}\right)$$

we can write

$$Fr = \frac{U}{H\sqrt{(g/T)\left((g/C_p) - \Gamma\right)}}$$

When $Fr = \infty$ (neutral), the lapse rate is equal to the gradient of adiabatic.

Therefore,

$$Fr = B\frac{U}{\sqrt{9.76 - \Gamma}}$$

Parameter B depends on the lapse rate and the gradient of adiabatic. In this study, we choose B=0.24 after performing several test calculations.

2.4 Numerical Method

The basic equation is solved by the standard MAC method[6]. Since the Reynolds number is high, the advection terms in the momentum and the density equations are approximated by the third order upwind scheme[5]. The rest of spatial terms are approximated by the second order central difference. Euler explicit method is employed for the time integration.

The overall computational domain is rectangular region of $80km \times 40km \times 12km$ in windward, span-wise and vertical directions respectively as is shown in Fig.2.

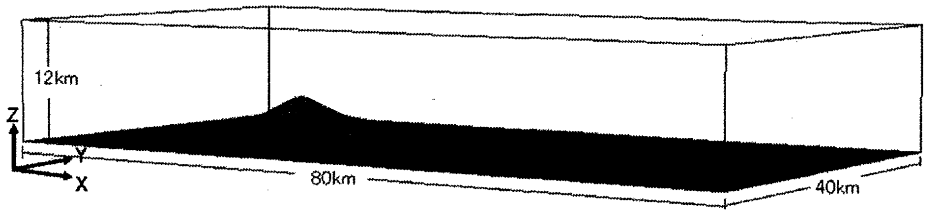


Fig. 2 Computational domain

We assume Mt. Fuji as an isolated mountain; i.e. the shape of the mountain is a cone and its height is 4000m. Since we are interested in investigating the effect of an isolated mountain, the mountains surrounding Mt.Fuji are removed. The particles are put on the inflow boundary of x=0km, 10km<y<30km, 0km<z<7km at the mesh points of 0.5km interval. We put them every interval of unit non-dimensional time.

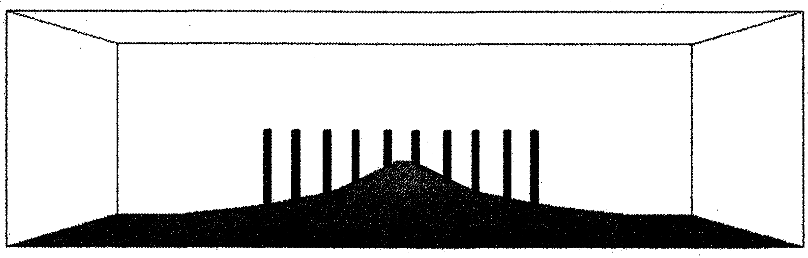


Fig. 3 Initial position of particles putting into the domain.

Fig.3 is the view from downstream boundary indicating the position of initial particles. Since computation of clouds is time consuming, we cannot use enough grids points,i.e. the total grid points are 61(stream-wise)×31(span-wise)×31(vertical). In order to perform the computation as accurate as possible, we employed boundary fitted coordinate system. Also the grid is concentrated near the ground and the mountain.

Fig.4 is a grid system in the vertical plane passing the mountain. Initially, the flow is at rest except on the inflow boundary.

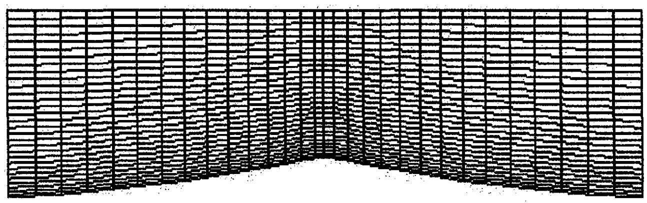


Fig. 4 Grid in y-z plane across a mountain.

Boundary conditions and the parameters used in this study are summarized in Tables 1 and 2.

Table 1 Boundary conditions.

	Velocity	*Pressure*	*Density*
Upwind	Uniform flow u=1, v=w=0	Extrapolated $\frac{\partial p'}{\partial x}=0$	Fixed $\rho'=0$
Downstream	Extrapolated $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}=\frac{\partial w}{\partial z}=0$	Extrapolated $\frac{\partial p'}{\partial x}=0$	Extrapolated $\frac{\partial \rho'}{\partial x}=0$
Top	Slip condition $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}=0$,w=0	Extrapolated $\frac{\partial p'}{\partial z}=0$	Extrapolated $\frac{\partial \rho'}{\partial z}=0$
Bottom	Non-slip condition u=v=w=0	Extrapolated $\frac{\partial p'}{\partial z}=0$	Extrapolated $\frac{\partial \rho'}{\partial z}=0$
Lateral	Extrapolated $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}=\frac{\partial w}{\partial z}=0$	Extrapolated $\frac{\partial p'}{\partial x}=0$	Extrapolated $\frac{\partial \rho'}{\partial x}=0$

Table 2 Parameters used in this study.

Parameter	*Value*
Grid number	61(x)×31(y)×31(y)
Time step	8000
ΔT	Depends on wind speed (0.015, 0.01, 0.075, 0.005)
Iteration of Poisson equation	20
Error of Poisson equation	0.001
Re	2000
Fr	0.5-2.0
Particle number	10(x)×15(z)
Humidity	50%-90%
Lapse rate	5℃/km-8℃/km
Temperature on the ground	15℃

3 RESULTS AND DISCUSSION

We will show typical results obtained by the method mentioned above.

At first, we show the flow around an isolated mountain at Fr=0.5 and Fr=2.0 in order to show the effect of the Froude number on the flow field. The Reynolds number is 2000. In this case, the computation of the clouds is not performed.

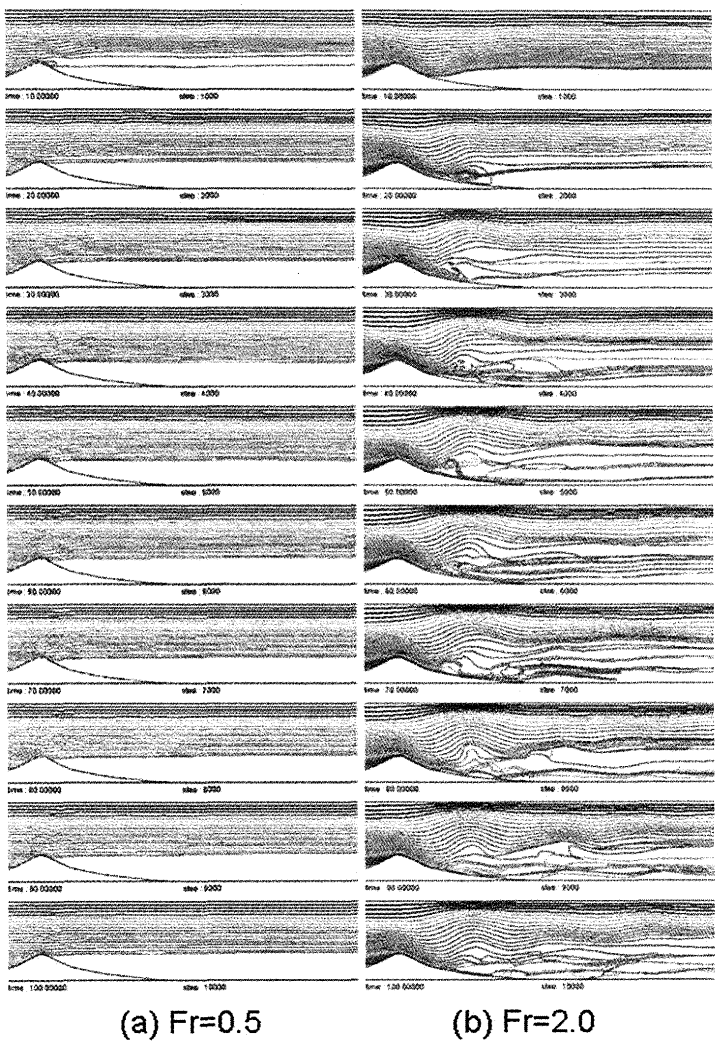


Fig. 5 Stream line in x-z plane across a mountain at various times.

Fig.5 shows the flow in the vertical cross section passing the center of the mountain. Various particle paths started at the inflow boundary are shown. When Fr=0.5 in Fig.5(a), each path is almost straight and we cannot see the particles behind the mountain. This is the effect of strong stratification of the flow. Fig.5(b) shows the result of Fr=2.0. Unlike Fig.5(a), each particle path goes over the mountain and is disturbed strongly showing the flow is not very stable.

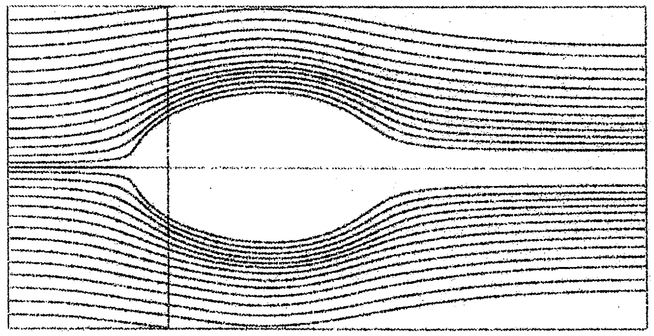


Fig. 6 Stream line in x-y plane at z=2km.

Fig.6 is another view of the same results. This is the particle path in the horizontal plane locating 2km above the ground. Each particle path is largely curved by the mountain. The crossing point of two straight lines in the figure indicates the position of the top of the mountain.

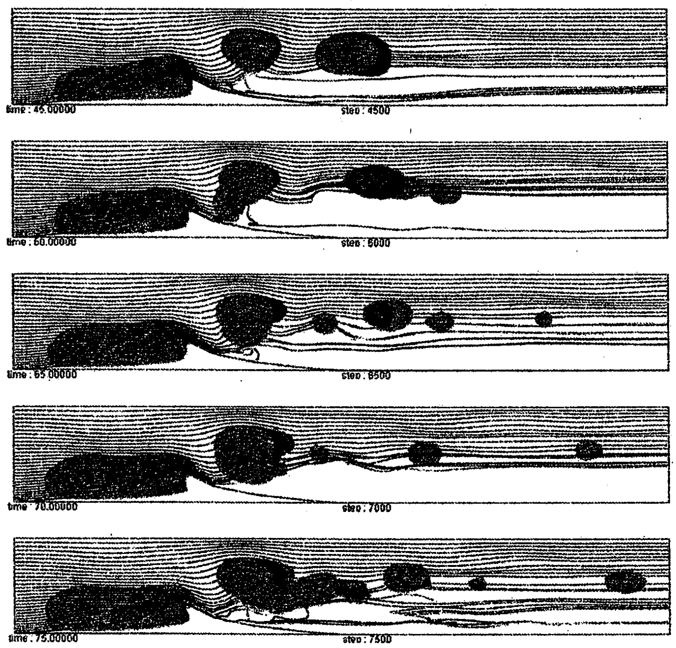


Fig. 7 Stream lines and the position of clouds in x-z plane across a mountain at various times (humidity 70%, lapse rate 8℃/km, wind speed 10m/s, Fr=1.73)

Hereafter we will show the results including the generation of clouds. Fig.7 shows the particle paths and clouds at various times. The humidity is 70%, the lapse rate is 8℃/km and the wind speed is 10m/s(Fr=1.73). From this figure, the clouds are generated at the position where the particle paths are curved convexly. Especially, we can observe large clouds right behind the mountain where the flow goes upward after it is curved downward by the mountain. On the other hand, we cannot see clouds where the downwash exists.

Fig.8 shows the effect of humidity on the shape of the clouds. The Froude number and Reynolds number are fixed (Fr=1.73, Re=2000). Fig.8 (a),(b) and (c) are the results of h=50%,70% and 90% respectively. As is expected, the clouds become larger as the humidity increases. On the other hand, the position of the cloud firstly appears behind the mountain is almost the same in size for each case.

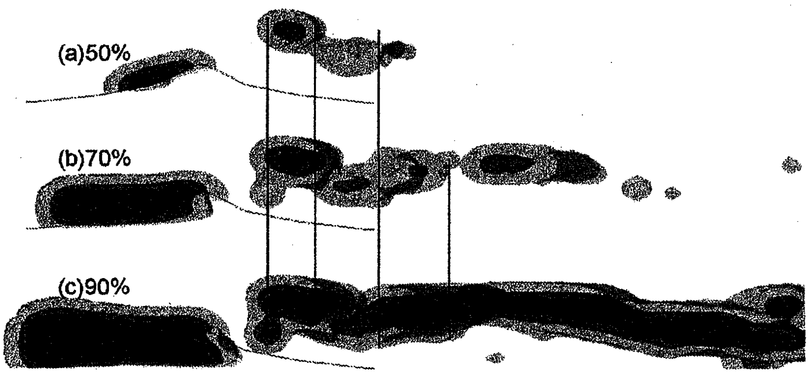


Fig. 8 The effect of humidity on the formation of clouds.

Fig.9 shows the comparison among the top and bottom of the clouds for three cases. Although the position of the bottom of the cloud becomes closer to the ground as the humidity increases, the height of the top position is not affected by the humidity.

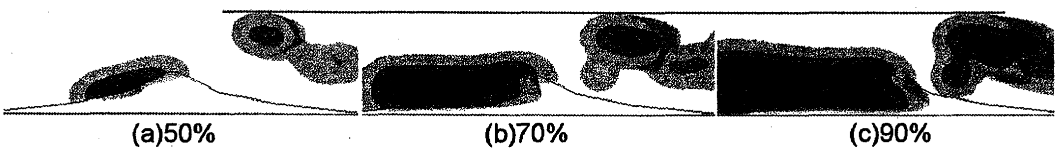
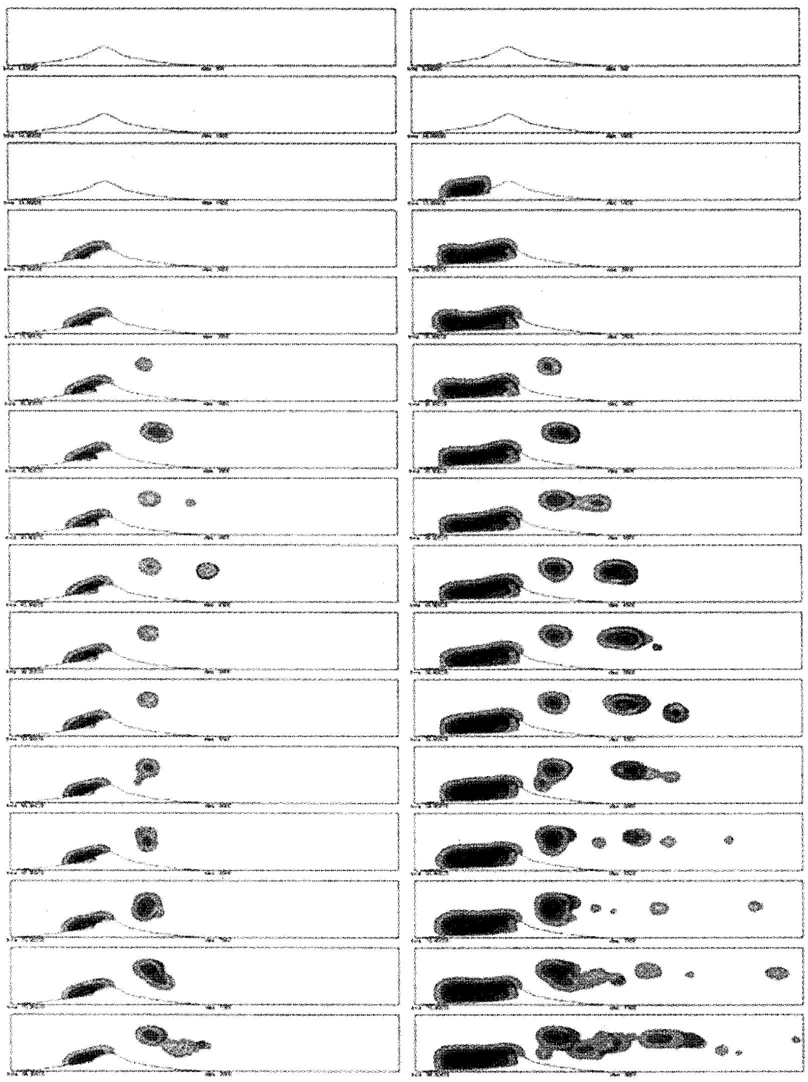


Fig. 9 The effect of humidity on the formation of clouds.

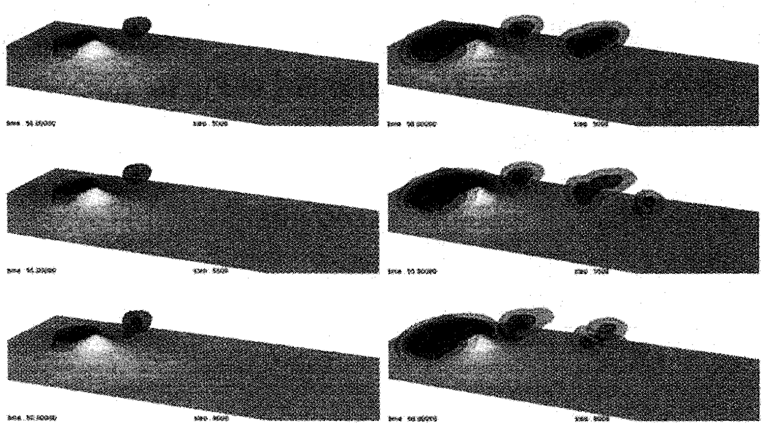
Figure 10 is the side view of the time history of the cloud generation for humidity of 50% and 90%. Figure 11 is the bird's eye view corresponding to Fig.10.

Next, we show the effect of the lapse rate on the generation of the clouds.



Humidity:50% Humidity:70%

Fig. 10 Formation of clouds in x-z plane across a mountain at various times.



Humidity:50% Humidity:70%

Fig. 11 Bird's eye view of formation of clouds corresponding to Fig. 10.

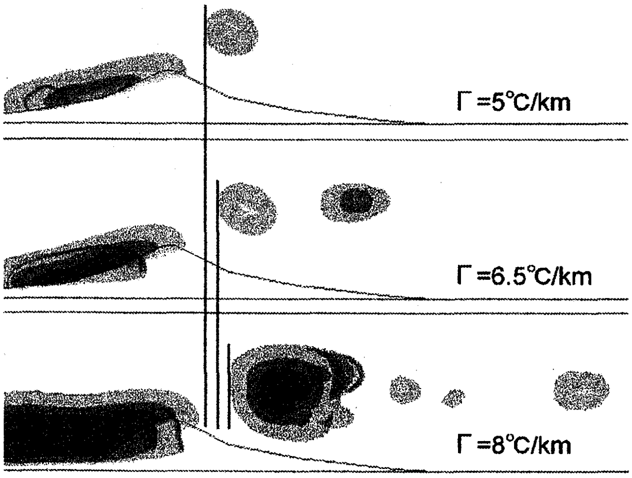


Fig. 12 The effect of lapse rate on the location of clouds behind a mountain.

Fig.12 (a), (b) and (c) are results of $\Gamma = 5$, 6.5 and 8°C/km respectively. From these figures, we can see larger clouds as the lapse rate increases. When the lapse rate becomes large, the Froude number decreases. As the result, the flow tends to become unstable and the upward stream which makes the clouds is formed. Fig.13 shows the comparison of the position of top and bottom of the cloud for various cases. As the lapse rate increase, the position of bottom becomes lower. When the lapse rate is large, the temperature of the air quickly decreases, so the temperature of the particle reaches the dew point at lower position.

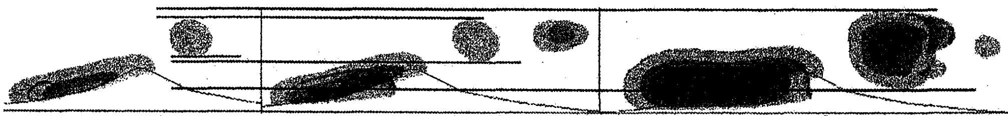


Fig. 13 The effect of lapse rate on the height of clouds behind a mountain.

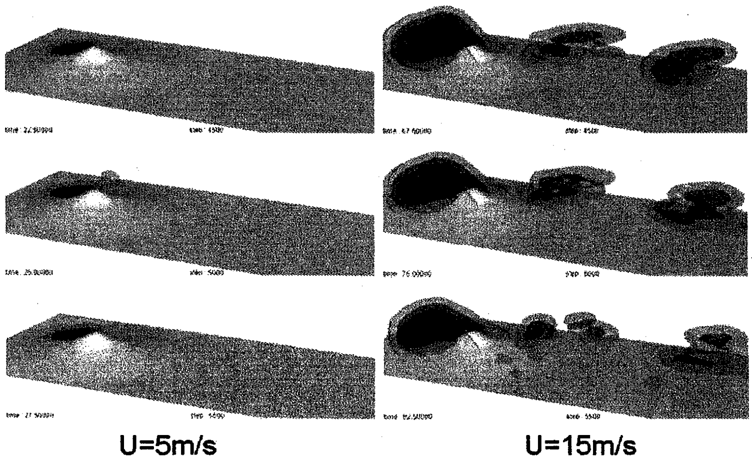


Fig. 14 The effect of wind speed on the formation of clouds.

Finally, Fig.14 shows the effect of the wind speed on the generation of the clouds. As the speed increases, the clouds distribute widely and their sizes also become large and their shapes become more flat.

4 SUMMARY

In this study, we proposed a simple model to simulate the generation of clouds and formation of the typical clouds above or behind a mountain numerically. The flow is assumed to be stratified. The incompressible Navier-Stokes equation and the density equation are solved under the Boussinsq approximation. The standard MAC method is used as a numerical method, and Mt. Fuji is chosen for the computation. In the simulation, small particles are put in the flow field. Each particle has its own temperature and humidity and is moved according to the velocity at its position. If the temperature of the particle becomes lower than temperature of the dew point, the particle is assumed to become a water drop and is displayed as a cloud. Although various assumptions are employed and the number of the grid points is not enough, we can simulate the typical clouds frequently observed in the real situation. The results are also acceptable from a physical point of view. The effects of the humidity, the adiabatic lapse rate, the wind velocity and the Froude number on the formation of the clouds are also investigated.

In this computation, the exchange of lateral heat between vapor and water drop is ignored. Taking the effect of lateral heat into account is left for future works.

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