The correction of "Measurable Norms and Related Conditions in Some Examples"

Keiko Harai
Graduate School of Humanities and Sciences, Ochanomizu University,
Tokyo 112-8610, Japan

We made a mistake in the Natural Science Report of the Ochanomizu University Vol.54 No.1(2003), 1-14. So we correct error that.

We proof that (*iii*) implies (*iv*) in Theorem 3.1. By (*iii*), there exists a monotone increasing sequence $\{P_n\}$ which strongly converges to I in $\mathscr{F}$ satisfying that for an arbitrary $\varepsilon > 0$, there exsits n_0 such that $k \geq n_0$ implies $\mu(\{\|P_kx - P_{n_0}x\| > \varepsilon\}) < \varepsilon$. (For simplification, we denote $\{x \in H; \|P_kx - P_{n_0}x\| > \varepsilon\}$ by $\{\|P_kx - P_{n_0}x\| > \varepsilon\}$.) By the triangle inequality, we have $\|P_kx\| \leq \|P_{n_0}x\| + \|P_kx - P_{n_0}x\|$. So we have that for an arbitrary $\varepsilon > 0$, there exsits n_0 such that $k \geq n_0$ implies

$$\mu(\{\|P_kx\| \leq \|P_{n_0}x\| + \varepsilon\}) \geq 1 - \varepsilon.$$

For each j $(j = 1, 2, \ldots, n_0 - 1)$, there exists $M_j > 0$ such that

$$(\mu \circ P_j{}^{-1})(|t_j| > M_j) < \frac{\varepsilon}{2^j}$$

where $t_j \in \mathbb{R}^{l_j}$, $l_j = \dim P_jH$, and there exists $M_{n_0} > 0$ such that

$$(\mu \circ P_{n_0}{}^{-1})(|t_{n_0}| > M_{n_0}) < \frac{\varepsilon}{2^{n_0+1}}$$

since $(\mu \circ P_j{}^{-1})$ is a measure on the finite dimensional space.
Let $M \geq \max\{M_1, M_2, \ldots, M_{n_0}\} + \varepsilon$ and $N > M$, then

$$\begin{aligned}
\mu(\{\sup_{1\leq k\leq n} \|P_kx\| > N\}) &\leq \mu(\{\sup_{1\leq k\leq n_0} \|P_kx\| > N\}) + \mu(\{\sup_{n_0<k\leq n} \|P_kx\| > N\}) \\
&\leq \mu(\{\sup_{1\leq k\leq n_0} \|P_kx\| > N\}) + \mu(\{\|P_{n_0}x\| + \varepsilon > N\}) + \varepsilon \\
&\leq \sum_{k=1}^{n_0} \mu(\{\|P_kx\| > N\}) + \mu(\{\|P_{n_0}x\| + \varepsilon > N\}) + \varepsilon \\
&\leq \sum_{k=1}^{n_0} (\mu \circ P_k{}^{-1})(|t_k| > M_k) + \mu(\{\|P_{n_0}x\| > M_{n_0}\}) + \varepsilon \\
&\leq \sum_{k=1}^{n_0} \frac{\varepsilon}{2^k} + \frac{\varepsilon}{2^{n_0+1}} + \varepsilon \\
&\leq 2\varepsilon.
\end{aligned}$$