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Tables of Cartesian Coordinates of Semiregular Polyhedra and Two Rhombic Polyhedra

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Abstract

Analytical and numerical cartesian coordinates, together with several quantities describing the geometric characteristics and perspective drawings, of semiregular polyhdra inscribed in 2×2×2 cube are given. The data of the rhombic dodecahedron and tricosahedron are also supplemented.

Recently the need for drawing polyhedra is increasingly growing. For this purpose a number of convenient tools are being available and the cartesian coordinates of polyhedra can also be obtained [1-3]. However, in most cases those data are usually numerical but not analytical, and the high symmetry of polyhedra are usually not carefully taken into account. It has been pointed out that almost all the classical documents dealing with polyhedra cannot escape from numerical and even analytical errors [4].

The authors have been trying to prepare an educational software for self-teaching the geometrical and topological features of polyhedra. The above-mentioned issue is nothing else but the motivation of this tabulation.

In many standard textbooks of polyhedra, characteristic quantities are calculated for polyhedra with unit edge-length (namely, with a unit edge length, E=1) [5-7]. We have chosen those semiregular polyhedra and two special rhombic polyhedra inscribed in the 2×2×2 cube in order to supply convenient data for designing more complicated polydedra originating from regular and semiregular polyhedra. Of course, conversion can be executed quite easily by using the numerical value of E/R or R/E, where R denotes the radius of the sphere inscribing the polyhedron concerned.

The following Tables list the name (also in Japanese), conventional designation (sequence of the numbers of edges of polygons meeting at each vertex), numbers of vertices (V), edges (E), faces (F), component polygons (f_n), analytical expressions of the cartesian coordinates of vertices, radius of the sphere inscribing the polyhedron (R), distances from the center of the polyhedron to the center of polygon faces (R_n), surface area (S), areas of polygons (S_n), volume (V), volumes of polygonal pyramids formed by the center of polyhedron and the component faces (V_n), length of each edge (E), E/R, and R/E, in this order followed by the table of numerical values of the cartesian coordinates of {V} and other characteristics, where S_0 and V_0, respectively, represent the area and volume of the polyhedron-inscribing sphere. Note that both the values of S/S_0 and V/V_0 indicate the sphericity of the polyhedron approaching to unity from below, while $(S/S_0)/(V_0/V)^{2/3}$ approaches to unity from above.

Snub cube and snub dodecahedron have different properties from the majority of the semiregular polydhedra in several respects. Namely, the cartesian coordinates of their vertices can be obtained as the solutions of cubic equations [1], which are too much complicated to be expressed in compact forms. Thus only numerical values are given. Secondly, as they do

not have any mirror symmetry, the vertex expressions are given in a different way from other polyhedra. Namely, in this paper the combined use of ± signs is meant to take all the possible combination except for the above two cases. The set of Chinese characters (複号同順) given in the pages of the two snub polyhedra means that those signs are to be taken simultaneously from above or from below. The cartesian coordinates of the regular polyhedra can be found in [4].

Finally three different perspective drawings are given for each polyhedron so that the readers might get fresh impression to the good-old family of semiregular polyhedra.

References

[1] S. Wolfram, *Mathematica*, Addison-Wesley, Redwood, California (1988).

[2] See for example the website of G. Hart: http://www.georgehart.com/

[3] J. Sekiguchi, *Mathematical Study of Polyhedra and Graphics* (in Japanese), Makino, Tokyo (1996).

[4] H. Hosoya and Y. Maruyama, Efficient generation of the Cartesian coordinates of truncated icosahedron and related polyhedra, *J. Mol. Graph. Model.*, in press.

[5] H.M. Cundy, A.R. Rollett, *Mathematical Models*, Oxford Univ. Press, London (1952).

[6] R. Williams, *The Geometrical Foundation of Natural Structure*, Dover, New York (1979).

[7] S. Hitotsumatsu, *Sei-Tamentai o Toku* (in Japanese) (*Analysis of Regular Polyhedra*), Tokai Univ. Press, Tokyo (1983).

Truncated tetrahedron（角切り四面体）3, 6^2

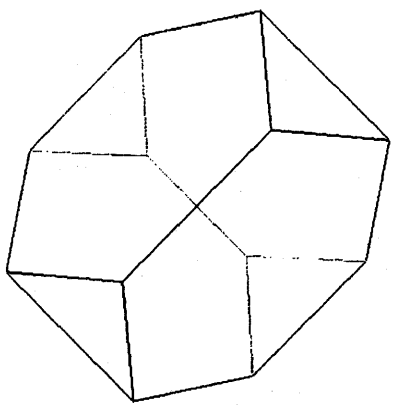
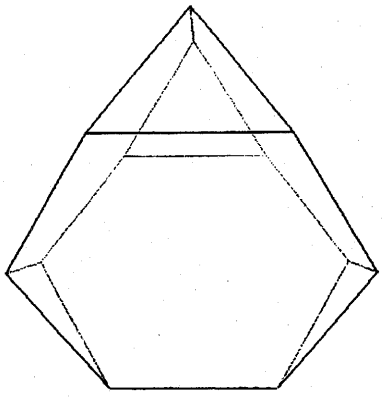
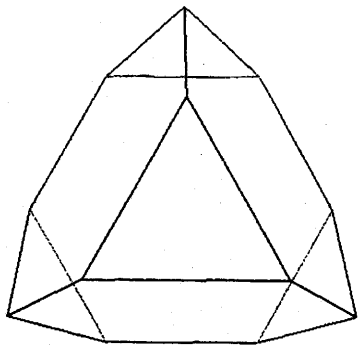
V = 12 E = 18 F = 8 $f_3 = 4$ $f_6 = 4$

(1/3,1/3,1) (−1/3,−1/3,1) (1,1/3,1/3) (1,−1/3,−1/3) (1/3,1,1/3) (−1/3,1,−1/3)
(−1,−1/3,1/3) (−1,1/3,−1/3) (−1/3,−1,1/3) (1/3,−1,−1/3) (1/3,−1/3,−1) (−1/3,1/3,−1)

$R = \sqrt{11}/3$ $R_3 = 5\sqrt{3}/9$ $R_6 = \sqrt{3}/3$
$S = 56\sqrt{3}/9$ $S_3 = 2\sqrt{3}/9$ $S_6 = 4\sqrt{3}/3$
$V = 184/81$ $V_3 = 10/81$ $V_6 = 4/9$
$E = 2\sqrt{2}/3$ $E/R = \sqrt{8/11}$ $R/E = \sqrt{11/8}$

x	y	z
0.333333	0.333333	1
−0.333333	−0.333333	1
1	0.333333	0.333333
1	−0.333333	−0.333333
0.333333	1	0.333333
−0.333333	1	−0.333333
−1	−0.333333	0.333333
−1	0.333333	−0.333333
−0.333333	−1	0.333333
0.333333	−1	−0.333333
0.333333	−0.333333	−1
−0.333333	0.333333	−1

R_3	0.962250	S_3	0.384900	V_3	0.123457	E	0.942809
R_6	0.577350	S_6	2.309401	V_6	0.444444	E/R	0.852803
R	1.105542	S	10.777205	V	2.271605	R/E	1.172604
		S/S_0	0.701691	V/V_0	0.401346	$(S/S_0)(V_0/V)^{2/3}$	1.289635
		S/E^2	12.124356	V/E^3	2.710576		

Cuboctahedron（立方八面体） $(3,4)^2$

V = 12 E = 24 F = 14 f_3 = 8 f_4 = 6

$(0,\pm1,\pm1)$ $(\pm1,0,\pm1)$ $(\pm1,\pm1,0)$

$R=\sqrt{2}$ $R_3=2\sqrt{3}/3$ $R_4=1$

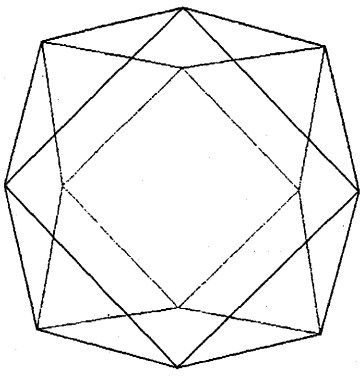
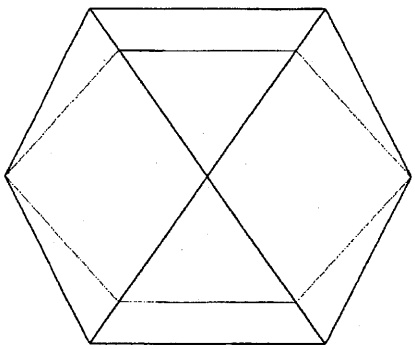
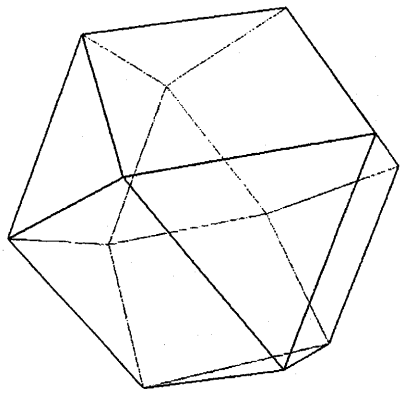
$S=4\left(3+\sqrt{3}\right)$ $S_3=\sqrt{3}/2$ $S_4=2$

$V=20/3$ $V_3=1/3$ $V_4=2/3$

$E=\sqrt{2}$ $E/R=1$ $R/E=1$

x	y	z
0	±1	±1
±1	0	±1
±1	±1	0

R_3	1.154701	S_3	0.866025	V_3	0.333333	E	1.414214
R_4	1	S_4	2	V_4	0.666667	E/R	1
R	1.414214	S	18.928203	V	6.666667	R/E	1
		S/S_0	0.753129	V/V_0	0.562698	$(S/S_0)(V_0/V)^{2/3}$	1.104976
		S/E^2	9.464102	V/E^3	2.357023		

Truncated cube（角切り立方体） $3, 8^2$

V = 24 E = 36 F = 14 $f_3 = 8$ $f_8 = 6$

$(\pm a, \pm 1, \pm 1)$ $(\pm 1, \pm a, \pm 1)$ $(\pm 1, \pm 1, \pm a)$ $\quad a = -1 + \sqrt{2}$

$R = \sqrt{5 - 2\sqrt{2}}$ $\quad R_3 = \sqrt{3}(1 + \sqrt{2})/3$ $\quad R_8 = 1$

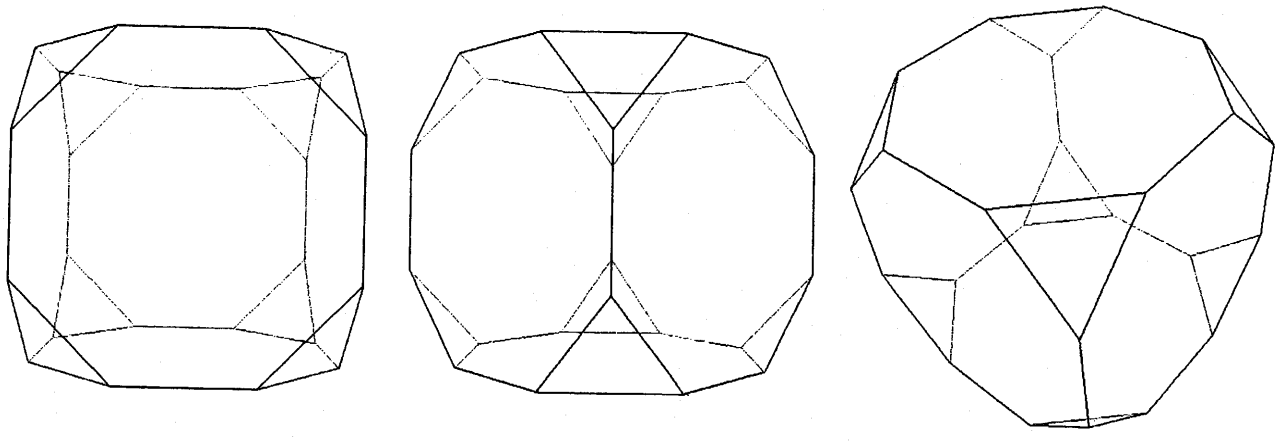
$S = 8(6\sqrt{2} + 3\sqrt{3} - 2\sqrt{6} - 6)$ $\quad S_3 = \sqrt{3}(3 - 2\sqrt{2})$ $\quad S_8 = 8(\sqrt{2} - 1)$

$V = 56(\sqrt{2} - 1)/3$ $\quad V_3 = (\sqrt{2} - 1)/3$ $\quad V_8 = 8(\sqrt{2} - 1)/3$

$E = 2(\sqrt{2} - 1)$ $\quad E/R = \sqrt{4(7 - 4\sqrt{2})/17}$ $\quad R/E = \sqrt{(7 + 4\sqrt{2})/4}$

x	y	z
±0.414214	±1	±1
±1	±0.414214	±1
±1	±1	±0.414214

R_3	1.393847	S_3	0.297173	V_3	0.138071	E	0.828427
R_8	1	S_8	3.313708	V_8	1.104569	E/R	0.562169
R	1.473626	S	22.259634	V	7.731986	R/E	1.778824
		S/S_0	0.815706	V/V_0	0.576821	$(S/S_0)(V_0/V)^{2/3}$	1.177172
		S/E^2	32.434664	V/E^3	13.599663		



Truncated octahedron（角切り八面体） $4, 6^2$

V = 24 E = 36 F = 14 $f_4 = 6$ $f_6 = 8$

$(\pm 1/2, 0, \pm 1)$ $(0, \pm 1/2, \pm 1)$ $(\pm 1, 0, \pm 1/2)$ $(0, \pm 1, \pm 1/2)$ $(\pm 1, \pm 1/2, 0)$ $(\pm 1/2, \pm 1, 0)$

$R = \sqrt{5}/2$ $R_4 = 1$ $R_6 = \sqrt{3}/2$

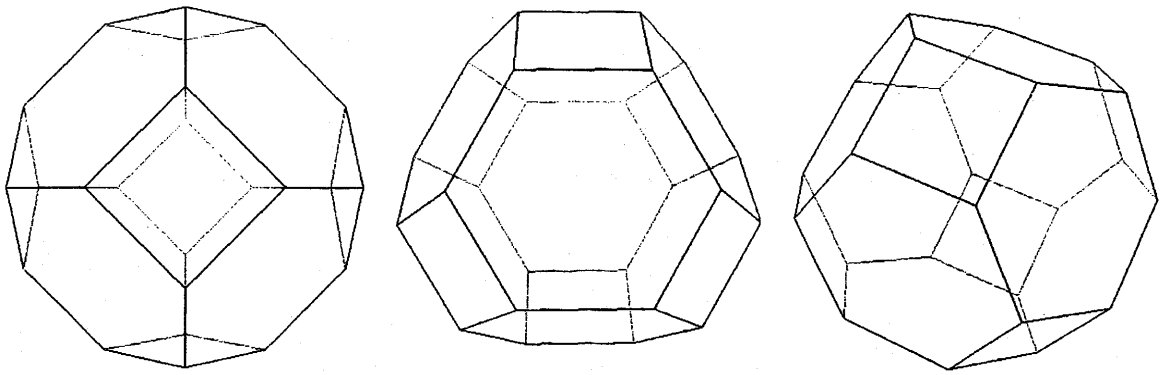
$S = 3(1 + 2\sqrt{3})$ $S_4 = 1/2$ $S_6 = 3\sqrt{3}/4$

$V = 4$ $V_4 = 1/6$ $V_6 = 3/8$

$E = \sqrt{2}/2$ $E/R = \sqrt{2/5}$ $R/E = \sqrt{5/2}$

x	y	z
±0.5	0	±1
0	±0.5	±1
±1	0	±0.5
0	±1	±0.5
±1	±0.5	0
±0.5	±1	0

R_4	1	S_4	0.5	V_4	0.166667	E	0.707107
R_6	0.866025	S_6	1.299038	V_6	0.375	E/R	0.632456
R	1.118034	S	13.392305	V	4	R/E	1.581139
		S/S_0	0.852581	V/V_0	0.683292	$(S/S_0)(V_0/V)^{2/3}$	1.099000
		S/E^2	26.784610	V/E^3	11.313708		



Rhombicuboctahedron（斜立方八面体） $3, 4^3$

V = 24 E = 48 F = 26 $f_3 = 8$ $f_4 = 18$

$(\pm a, \pm a, \pm 1)$ $(\pm 1, \pm a, \pm a)$ $(\pm a, \pm 1, \pm a)$ $\quad a = -1 + \sqrt{2}$

$R = \sqrt{7 - 4\sqrt{2}}$ $\quad R_3 = \sqrt{3}(2\sqrt{2} - 1)/3$ $\quad R_4 = 1$

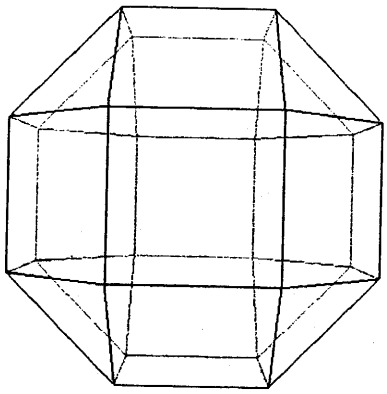
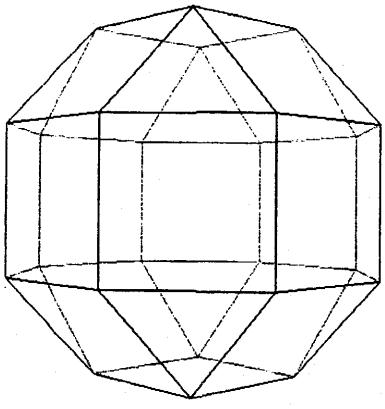
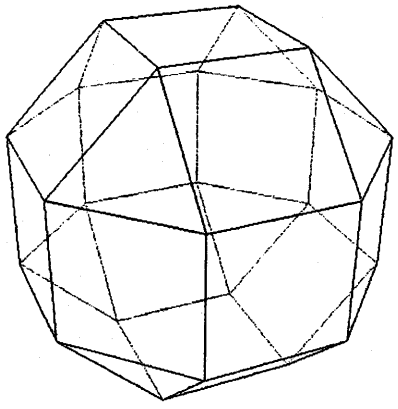
$S = 8(3 - 2\sqrt{2})(9 + \sqrt{3})$ $\quad S_3 = \sqrt{3}(3 - 2\sqrt{2})$ $\quad S_4 = 4(3 - 2\sqrt{2})$

$V = 16(8 - 5\sqrt{2})/3$ $\quad V_3 = (8\sqrt{2} - 11)/3$ $\quad V_4 = 4(3 - 2\sqrt{2})/3$

$E = 2(\sqrt{2} - 1)$ $\quad E/R = \sqrt{4(5 - 2\sqrt{2})/17}$ $\quad R/E = \sqrt{(5 + 2\sqrt{2})/4}$

x	y	z
±0.414214	±0.414214	±1
±1	±0.414214	±0.414214
±0.414214	±1	±0.414214

R_3	1.055643	S_3	0.297173	V_3	0.104569	E	0.828427
R_4	1	S_4	0.686292	V_4	0.228764	E/R	0.714813
R	1.158942	S	14.730631	V	4.954305	R/E	1.398966
		S/S_0	0.872747	V/V_0	0.759818	$(S/S_0)(V_0/V)^{2/3}$	1.048131
		S/E^2	21.464102	V/E^3	8.714045		

Rhombicuboctahedron2（斜立方八面体２） $3,4^3$

V = 24 E = 48 F = 26 $f_3 = 8$ $f_4 = 18$

$(\pm a,\pm a,\pm 1)$ $(\pm 1,\pm a,\pm a)$ $(\pm a,-1,\pm a)$ $(\pm a\sqrt{2},1,0)$ $(0,1,\pm a\sqrt{2})$ $\quad a=-1+\sqrt{2}$

$R=\sqrt{7-4\sqrt{2}}$ $\quad R_3=\sqrt{3}(2\sqrt{2}-1)/3$ $\quad R_4=1$

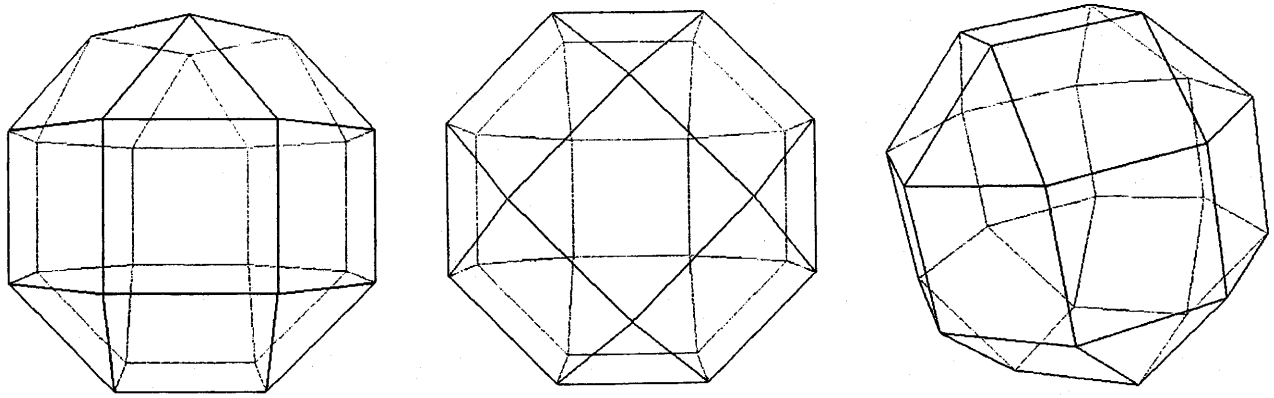
$S=8(3-2\sqrt{2})(9+\sqrt{3})$ $\quad S_3=\sqrt{3}(3-2\sqrt{2})$ $\quad S_4=4(3-2\sqrt{2})$

$V=16(8-5\sqrt{2})/3$ $\quad V_3=(8\sqrt{2-11})/3$ $\quad V_4=4(3-2\sqrt{2})/3$

$E=2(\sqrt{2-1})$ $\quad E/R=\sqrt{4(5-2\sqrt{2})/17}$ $\quad R/E=\sqrt{(5+2\sqrt{2})/4}$

x	y	z
±0.414214	±0.414214	±1
±1	±0.414214	±0.414214
±0.414214	−1	±0.414214
±0.585786	1	0
0	1	±0.585786

R_3	1.055643	S_3	0.297173	V_3	0.104569	E	0.828427
R_4	1	S_4	0.686292	V_4	0.228764	E/R	0.714813
R	1.158942	S	14.730631	V	4.954305	R/E	1.398966
		S/S_0	0.872747	V/V_0	0.759818	$(S/S_0)(V_0/V)^{2/3}$	1.048131
		S/E^2	21.464102	V/E^3	8.714045		



Truncated cuboctahedron（角切り立方八面体）4,6,8

V = 48 E = 72 F = 26 $f_4 = 12$ $f_6 = 8$ $f_8 = 12$

$(\pm a,\pm b,\pm 1)$ $(\pm b,\pm a,\pm 1)$ $(\pm 1,\pm b,\pm a)$ $(\pm b,\pm 1,\pm a)$ $(\pm 1,\pm a,\pm b)$ $(\pm a,\pm 1,\pm b)$

$$a = \left(3+\sqrt{2}\right)/7 \qquad b = \left(-1+2\sqrt{2}\right)/7$$

$$R = \sqrt{69+2\sqrt{2}}/7 \qquad R_4 = \left(1+5\sqrt{2}\right)/7 \qquad R_6 = \sqrt{3}\left(3+\sqrt{2}\right)/7$$

$$R_8 = 1$$

$$S = 48\left(10\sqrt{2}+9\sqrt{3}-4\sqrt{6}\right)/49 \qquad S_4 = 4\left(9-4\sqrt{2}\right)/49 \qquad S_6 = 6\sqrt{3}\left(9-4\sqrt{2}\right)/49$$

$$S_8 = 8\left(1+5\sqrt{2}\right)/49$$

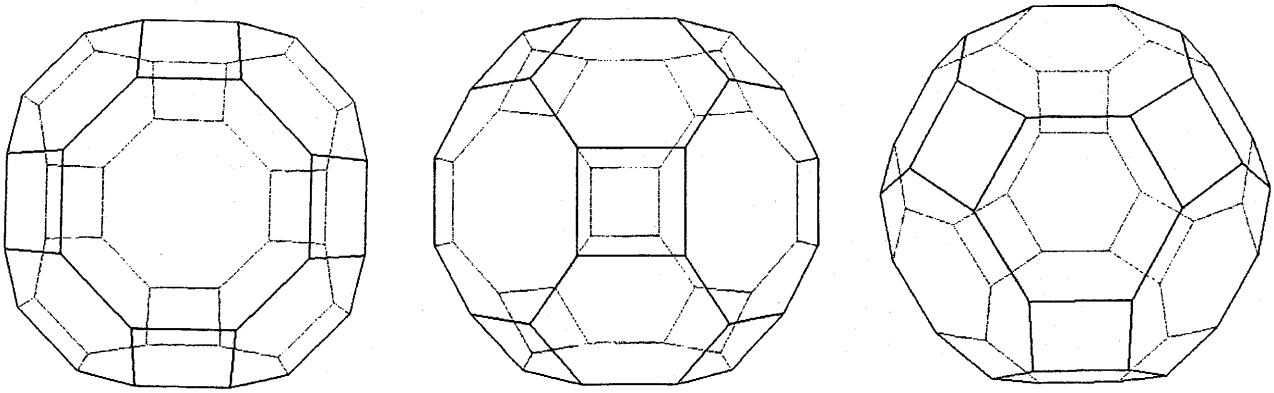
$$V = 16\left(33+67\sqrt{2}\right)/343 \qquad V_4 = 4\left(41\sqrt{2}-36\right)/1029 \qquad V_6 = 6\left(19-3\sqrt{2}\right)/343$$

$$V_8 = 8\left(1+5\sqrt{2}\right)/147$$

$$E = 2\left(2\sqrt{2}-1\right)/7 \qquad E/R = 2\sqrt{\left(13-6\sqrt{2}\right)/97} \qquad R/E = \sqrt{13+6\sqrt{2}}/2$$

x	y	z
±0.630602	±0.261204	±1
±0.261204	±0.630602	±1
±1	±0.261204	±0.630602
±0.261204	±1	±0.630602
±1	±0.630602	±0.261204
±0.630602	±1	±0.261204

R_4	1.153010	S_4	0.272910	V_4	0.0854529	E	0.522408
R_6	1.092235	S_6	0.709041	V_6	0.258146	E/R	0.431479
R_8	1	S_8	1.317725	V_8	0.439242	R/E	2.317611
R	1.210738	S	16.853595	V	5.959291		
		S/S_0	0.914919	V/V_0	0.801596	$(S/S_0)(V_0/V)^{2/3}$	1.060259
		S/E^2	61.755172	V/E^3	41.798990		



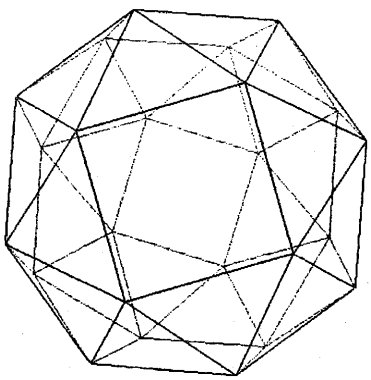
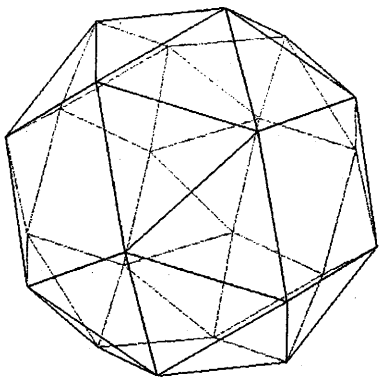
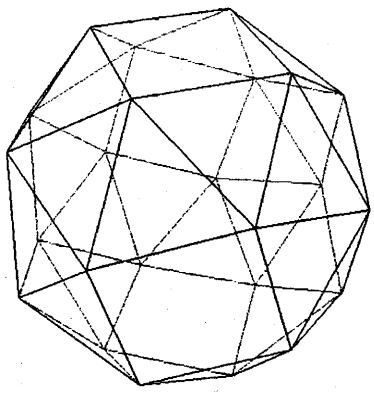
Snub cube（ねじれ立方体）$3^4,4$

V = 24　E = 60　F = 38　f_3 =32　f_4 = 6

x	y	z
1	±0.295598	∓0.543689
1	±0.543689	±0.295598
−1	±0.295598	±0.543689
−1	±0.543689	∓0.295598
0.295598	±1	±0.543689
-0.295598	±1	∓0.543689
0.543689	±1	∓0.295598
−0.543689	±1	±0.295598
0.295598	±0.543689	∓1
-0.295598	±0.543689	±1
0.543689	±0.295598	±1
−0.543689	±0.295598	∓1

(複号同順)

R_3	1.061913	S_3	0.331667	V_3	0.117400	E	0.875187
R_4	1	S_4	0.765952	V_4	0.255317	E/R	0.744206
R	1.176000	S	15.209051	V	5.288717	R/E	1.343713
		S/S_0	0.875140	V/V_0	0.776318	$(S/S_0)(V_0/V)^{2/3}$	1.036059
		S/E^2	19.856406	V/E^3	7.889476		

Icosidodecahedron (十二・二十面体) $(3,5)^2$

V = 30 E = 60 F = 32 f_3 = 20 f_5 = 12

$(\pm 1/2,\pm\tau/2,\pm\tau^{-1}/2)$ $(\pm\tau^{-1}/2,\pm 1/2,\pm\tau/2)$ $(\pm\tau/2,\pm\tau^{-1}/2,\pm 1/2)$

$(\pm 1,0,0)$ $(0,\pm 1,0)$ $(0,0,\pm 1)$ $\tau = (\sqrt{5}+1)/2$

$R = 1$ $R_3 = \tau/\sqrt{3}$ $R_5 = \sqrt{(5+\sqrt{5}/10}$

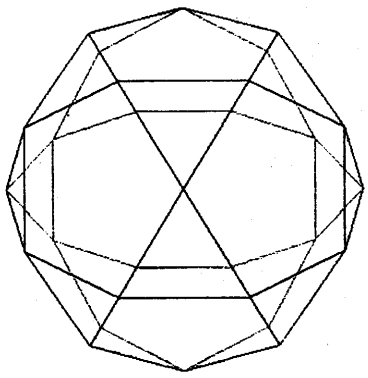
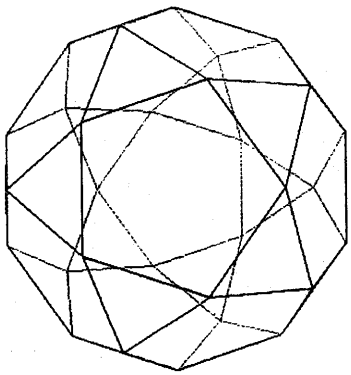
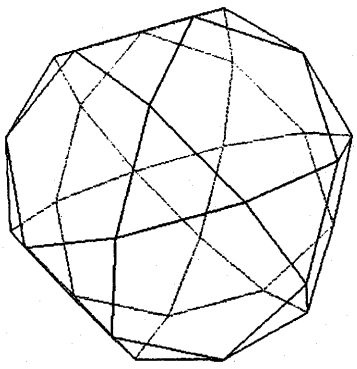
$S = \left(5\sqrt{3}(3-\sqrt{5})+3\sqrt{10(5-\sqrt{5})}\right)/2$ $S_3 = \sqrt{3}(3-\sqrt{5})/8$ $S_5 = \sqrt{10(5-\sqrt{5})}/8$

$V = (11\sqrt{5}-5)/6$ $V_3 = (\sqrt{5}-1)/24$ $V_5 = \sqrt{5}/12$

$E = \tau^{-1}$ $E/R = \tau^{-1}$ $R/E = \tau$

x	y	z
±0.5	±0.809017	±0.309017
±0.309017	±0.5	±0.809017
±0.809017	±0.309017	±0.5
±1	0	0
0	±1	0
0	0	±1

R_3	0.934172	S_3	0.165396	V_3	0.0515028	E	0.618034
R_5	0.850651	S_5	0.657164	V_5	0.186339	E/R	0.618034
R	1	S	11.193889	V	3.266125	R/E	1.618034
		S/S_0	0.890781	V/V_0	0.779730	$(S/S_0)(V_0/V)^{2/3}$	1.051498
		S/E^2	29.305983	V/E^3	13.835526		

Truncated dodecahedron （角切り十二面体） $3,10^2$

V = 60 E = 90 F = 32 $f_3 = 20$ $f_{10} = 12$

$(\pm a,\pm(3a-1)/2,\pm 2a)$ $(\pm 2a,\pm a,\pm(3a-1)/2)$ $(\pm(3a-1)/2,\pm 2a,\pm a)$

$(\pm(1-a),\pm a,\pm(1+a)/2)$ $(\pm(1+a)/2,\pm(1-a),\pm a)$ $(\pm a,\pm(1+a)/2,\pm(1-a))$

$(\pm a\tau^{-2},0,\pm 1)$ $(\pm 1,\pm a\tau^{-2},0)$ $(0,\pm 1,\pm a\tau^{-2})$ $\quad a = \sqrt{5}/5$

$R = \sqrt{(17-3\sqrt{5})/10}$ $\quad R_3 = \sqrt{3}(15+\sqrt{5})/30$ $\quad R_{10} = \sqrt{(5+\sqrt{5})/10}$

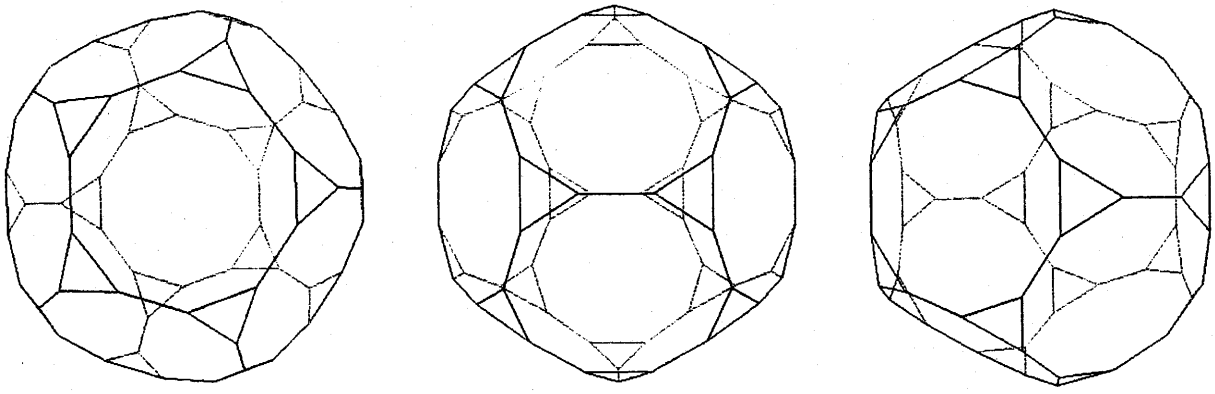
$S = 2\sqrt{3}(7-3\sqrt{5})+12\sqrt{50-22\sqrt{5}}$ $\quad S_3 = \sqrt{3}(7-3\sqrt{5})/10$ $\quad S_{10} = \sqrt{50-22\sqrt{5}}$

$V = 2(45-19\sqrt{5})/15+4\sqrt{2(7-3\sqrt{5})}$ $\quad V_3 = (45-19\sqrt{5})/150$ $\quad V_{10} = \sqrt{2(7-3\sqrt{5})}/3$

$E = (3\sqrt{5}-5)/5$ $\quad E/R = \sqrt{(74-30\sqrt{5})/61}$ $\quad R/E = \sqrt{74+30\sqrt{5}}/4$

x	y	z
±0.447214	±0.170820	±0.894427
±0.894427	±0.447214	±0.170820
±0.170820	±0.894427	±0.447214
±0.552786	±0.447214	±0.723607
±0.723607	±0.552786	±0.447214
±0.447214	±0.723607	±0.552786
±0.170820	0	±1
±1	±0.170820	0
0	±1	±0.170820

R_3	0.995125	S_3	0.050541	V_3	0.016765	E	0.341641
R_{10}	0.850651	S_{10}	0.898056	V_{10}	0.254644	E/R	0.336763
R	1.014468	S	11.787483	V	3.391023	R/E	2.969449
		S/S_0	0.911423	V/V_0	0.775363	$(S/S_0)(V_0/V)^{2/3}$	1.079899
		S/E^2	100.990763	V/E^3	85.03966		



Truncated icosahedron （角切り二十面体） $5,6^2$

V=60 E=90 F=32 $f_5=12$ $f_6=20$

$(\pm 1/3,\pm a,\pm b)$ $(\pm b,\pm 1/3,\pm a)$ $(\pm a,\pm b,\pm 1/3)$

$(\pm 2/3,\pm c,\pm b/2)$ $(\pm b/2,\pm 2/3,\pm c)$ $(\pm c,\pm b/2,\pm 2/3)$

$(\pm 1,\pm b/2,0)$ $(0,\pm 1,\pm b/2)$ $(\pm b/2,0\pm 1)$ $a=(3+\sqrt{5})/5$ $b=(\sqrt{5}-1)/3$

$R=\sqrt{(21-\sqrt{5})/18}$ $R_5=\sqrt{(85-\sqrt{5})/90}$ $R_6=\sqrt{3}(\sqrt{5}+1)/6$

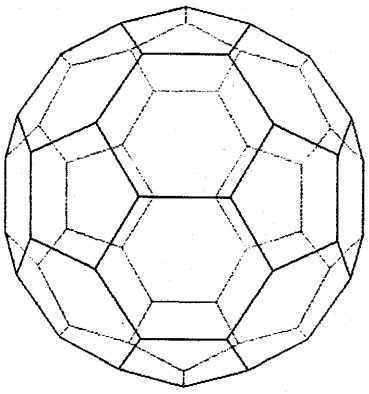
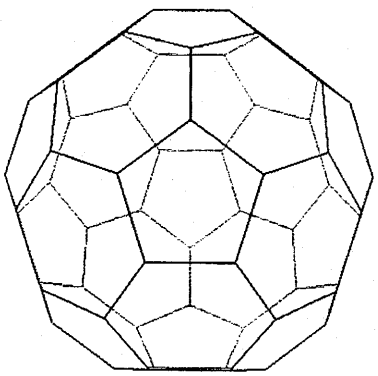
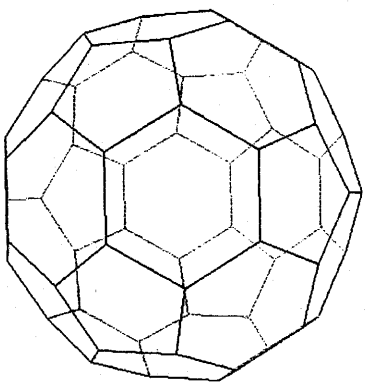
$S=20\sqrt{3}(3-\sqrt{5})/3+2\sqrt{10(5-\sqrt{5})}/3$ $S_5=\sqrt{10(5-\sqrt{5})}/18$ $S_6=(3-\sqrt{5})/\sqrt{3}$

$V=(78\sqrt{5}-70)/27$ $V_5=(9\sqrt{5}-5)/162$ $V_6=(\sqrt{5}-1)/9$

$E=(\sqrt{5}-1)/3$ $E/R=\sqrt{2(29-9\sqrt{5})/109}$ $R/E=\sqrt{(29+9\sqrt{5})/8}$

x	y	z
±0.333333	±0.872678	±0.412023
±0.412023	±0.333333	±0.872678
±0.872678	±0.412023	±0.333333
±0.666667	±0.745356	±0.206011
±0.206011	±0.666667	±0.745356
±0.745356	±0.206011	±0.666667
±1	±0.206011	0
0	±1	±0.206011
±0.206011	0	±1

R_5	0.958957	S_5	0.292073	V_5	0.093362	E	0.412023
R_6	0.934172	S_5	0.441056	V_6	0.137341	E/R	0.403548
R	1.021000	S	12.326001	V	3.867159	R/E	2.478019
		S/S_0	0.940938	V/V_0	0.867414	$(S/S_0)(V_0/V)^{2/3}$	1.034531
		S/E^2	72.607253	V/E^3	55.287731		

Rhombicosidodecahedron (斜十二・二十面体) 3,4,5,4

V = 60 E = 120 F = 62 $f_3 = 20$ $f_4 = 30$ $f_5 = 12$

$$(\pm 2\tau^{-2},\ \pm\tau^{-2},\pm\tau^{-1})\quad (\pm\tau^{-2},\pm\tau^{-1},\pm 2\tau^{-2})\quad (\pm\tau^{-1},\pm 2\tau^{-2},\ \pm\tau^{-2})$$
$$(\pm 1,\ \pm(\sqrt{5}-2),\ \pm(\sqrt{5}-2))\quad (\pm(\sqrt{5}-2),\ \pm(\sqrt{5}-2),\pm 1)\quad (\pm(\sqrt{5}-2),\ \pm 1,\ \pm(\sqrt{5}-2))$$
$$(0,\ \pm(3\sqrt{5}-5)/2,\ \pm\tau^{-1})\quad (\pm(3\sqrt{5}-5)/2,\ \tau^{-1},\ 0)\quad (\pm\tau^{-1},\ 0,\ \pm(3\sqrt{5}-5)/2)$$

$$R = \sqrt{19-8\sqrt{5}} \qquad R_3 = \sqrt{3}(4-\sqrt{5})/3 \qquad R_4 = 1 \qquad R_5 = 3\sqrt{(5-2\sqrt{5})/5}$$

$$S = 20\sqrt{3}(9-4\sqrt{5}) + 120(9-4\sqrt{5}) + 12\sqrt{5(85-8\sqrt{5})}$$

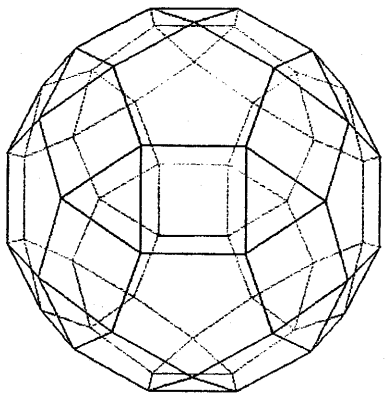
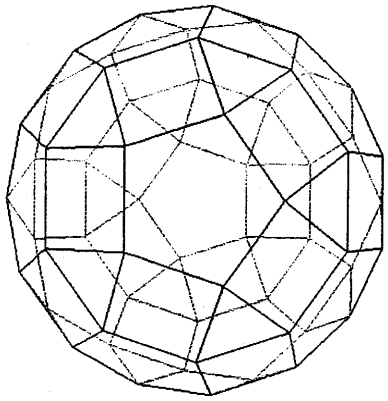
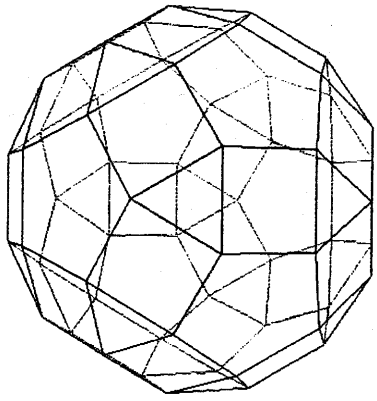
$$S_3 = \sqrt{3}(9-4\sqrt{5}) \qquad S_4 = 4(9-4\sqrt{5}) \qquad S_5 = \sqrt{5(85-38\sqrt{5})}$$

$$V = 8(185-82\sqrt{5})/3 \qquad V_3 = (56-25\sqrt{5})/3 \qquad V_4 = 4(9-4\sqrt{5})/3 \qquad V_5 = 9\sqrt{5}-20$$

$$E = 2(\sqrt{5}-2) \qquad E/R = 2\sqrt{(11-4\sqrt{5})/41} \qquad R/E = \sqrt{11+4\sqrt{5}}/2$$

x	y	z
±0.763932	±0.381966	±0.618034
±0.381966	±0.618034	±0.763932
±0.618034	±0.763932	±0.381966
±1	±0.236068	±0.236068
±0.236068	±0.236068	±1
±0.236068	±1	±0.236068
0	±0.854102	±0.618034
±0.854102	±0.618034	0
±0.618034	0	±0.854102

R_3	1.018407	S_3	0.096524	V_3	0.032767	E	0.472136
R_4	1	S_4	0.222912	V_4	0.074304	E/R	0.447838
R_5	0.974759	S_5	0.383516	V_5	0.124612	R/E	2.232951
R	1.054256	S	13.220037	V	4.379802		
		S/S_0	0.946521	V/V_0	0.892334	$(S/S_0)(V_0/V)^{2/3}$	1.021203
		S/E^2	59.305984	V/E^3	41.615325		

Truncated icosidodecahedron （角切り十二・二十面体） 4,6,10

$V=120$ $E=180$ $F=62$ $f_4=30$ $f_6=20$ $f_{10}=12$

$(\pm(3\sqrt{5}+1)/11,\pm(21-3\sqrt{5})/22,\pm(3\sqrt{5}+1)/22)$ $(\pm(3\sqrt{5}+1)/22,\pm(3\sqrt{5}+1)/11,\pm(21-3\sqrt{5})/22)$
$(\pm(21-3\sqrt{5})/22,\pm(3\sqrt{5}+1)/22,\pm(3\sqrt{5}+1)/11)$
$(\pm(7\sqrt{5}-5)/22,\pm(\sqrt{5}+15)/22,\pm(7-\sqrt{5})/11)$ $(\pm(7-\sqrt{5})/11,\pm(7\sqrt{5}-5)/22,\pm(\sqrt{5}+15)/22)$
$(\pm(\sqrt{5}+15)/22,\pm(7-\sqrt{5})/11,\pm(7\sqrt{5}-5)/22)$
$(\pm(4\sqrt{5}-6)/11,\pm(9+5\sqrt{5})/22,\pm(3\sqrt{5}+1)/22)$ $(\pm(3\sqrt{5}+1)/22,\pm(4\sqrt{5}-6)/11,\pm(9+5\sqrt{5})/22)$
$(\pm(9+5\sqrt{5})/22,\pm(3\sqrt{5}+1)/22,\pm(4\sqrt{5}-6)/11)$
$(\pm(4\sqrt{5}-6)/11,\pm(5\sqrt{5}-2)/11,\pm(\sqrt{5}+4)/11)$ $(\pm(\sqrt{5}+4)/11,\pm(4\sqrt{5}-6)/11,\pm(5\sqrt{5}-2)/11)$
$(\pm(5\sqrt{5}-2)/11,\pm(\sqrt{5}+4)/11,\pm(4\sqrt{5}-6)/11)$
$(\pm(2\sqrt{5}-3)/11,\pm1,\pm(2\sqrt{5}-3)/11)$ $(\pm(2\sqrt{5}-3)/11,\pm(2\sqrt{5}-3)/11,\pm1)$
$(\pm1,\pm(2\sqrt{5}-3)/11,\pm(2\sqrt{5}-3)/11)$

$$R=\sqrt{179-24\sqrt{5}}/11$$

$$R_4=1 \quad R_6=\sqrt{3}(4+\sqrt{5})/11 \qquad R_{10}=\sqrt{5(25-2\sqrt{5})}/11$$

$$S=120(29-12\sqrt{5})(1+\sqrt{3}+\sqrt{5+2\sqrt{5}})/121$$

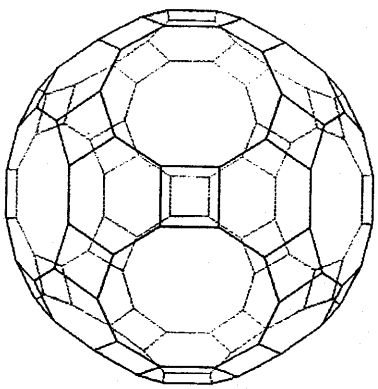
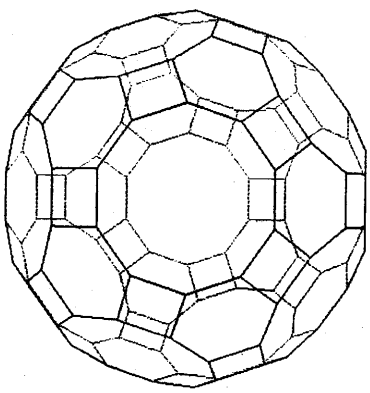
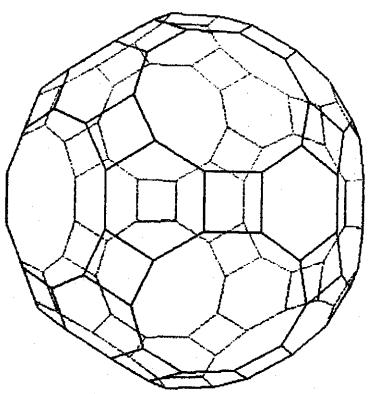
$$S_4=4(29-12\sqrt{5})/121 \quad S_6=6\sqrt{3}(29-12\sqrt{5})/121 \quad S_{10}=10(29-12\sqrt{5})\sqrt{5+2\sqrt{5}}/121$$

$$V=40(767-284\sqrt{5})/1331$$

$$V_4=4(29-12\sqrt{5})/363 \quad V_6=6(56-19\sqrt{5})/1331 \quad V_{10}=50(56-19\sqrt{5})/3993$$

$$E=2(2\sqrt{5}-3)/11$$

$$E/R=2\sqrt{(29-12\sqrt{5})/(179-24\sqrt{5})} \qquad R/E=\sqrt{(179-24\sqrt{5})/(29-12\sqrt{5})}/2$$

x	y	z
±0.700746	±0.649627	±0.350373
±0.350373	±0.700746	±0.649627
±0.649627	±0.350373	±0.700746
±0.484203	±0.783458	±0.433085
±0.433085	±0.484203	±0.783458
±0.783458	±0.433085	±0.484203
±0.267661	±0.917288	±0.350373
±0.350373	±0.267661	±0.917288
±0.917288	±0.350373	±0.267661
±0.267661	±0.834576	±0.566915
±0.566915	±0.267661	±0.834576
±0.834576	±0.566915	±0.267661
±0.133831	±1	±0.133831
±0.133831	±0.133831	±1
±1	±0.133831	±0.133831

R_4	1	S_4	0.071642	V_4	0.023881	E	0.267661
R_6	0.981926	S_6	0.186133	V_6	0.060923	E/R	0.262992
R_{10}	0.921010	S_{10}	0.551232	V_{10}	0.169230	R/E	3.802394
R	1.017753	S	12.486707	V	3.965641		
		S/S_0	0.959298	V/V_0	0.898044	$(S/S_0)(V_0/V)^{2/3}$	1.030596
		S/E^2	174.292000	V/E^3	206.803400		

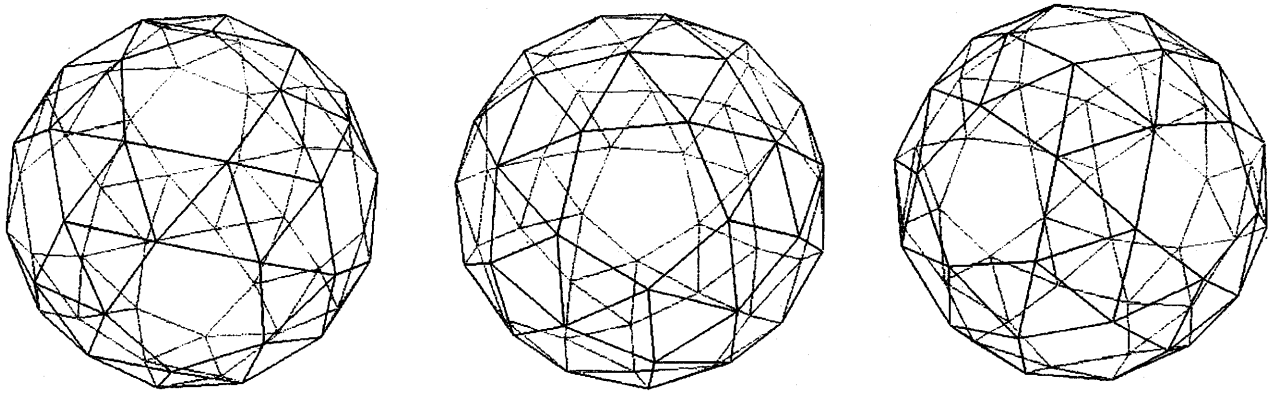
Snub dodecahedron （ねじれ十二面体） 3,5

V = 60 E = 150 F = 92 f_3 = 80 f_5 = 12

x	*y*	*z*
0.52605079	±0.78534843	±0.40416227
-0.52605079	±0.78534843	∓0.40416227
0.40416227	±0.52605079	±0.78534843
-0.40416227	±0.52605079	∓0.78534843
0.78534843	±0.40416227	±0.52605079
-0.78534843	±0.40416227	∓0.52605079
0.17873726	±1.00000000	±0.15780283
-0.17873726	±1.00000000	∓0.15780283
0.15780283	±0.17873726	±1.00000000
-0.15780283	±0.17873726	∓1.00000000
1.00000000	±0.15780283	±0.17873726
-1.00000000	±0.15780283	∓0.17873726
0.83268556	±0.59583773	±0.09198319
-0.83268556	±0.59583773	∓0.09198319
0.09198319	±0.83268556	±0.59583773
-0.09198319	±0.83268556	∓0.59583773
0.59583773	±0.09198319	±0.83268556
-0.59583773	±0.09198319	∓0.83268556
0.27072046	±0.94315126	∓0.30663476
-0.27072046	±0.94315126	±0.30663476
0.30663476	±0.27072046	∓0.94315126
-0.30663476	±0.27072046	±0.94315126
0.94315126	±0.30663476	∓0.27072046
-0.94315126	±0.30663476	±0.27072046
0.34731353	±0.67488273	∓0.69336524
-0.34731353	±0.67488273	±0.69336524
0.69336524	±0.34731353	∓0.67488273
-0.69336524	±0.34731353	±0.67488273
0.67488273	±0.69336524	∓0.34731353
-0.67488273	±0.69336524	±0.34731353

（複号同順）

R_3	0.990480	S_3	0.098465	V_3	0.032509	E	0.476859
R_5	0.944619	S_5	0.391228	V_5	0.123187	E/R	0.463857
R	0.476859	S	12.571927	V	4.078978	R/E	2.155837
		S/S_0	0.946628	V/V_0	0.896280	$(S/S_0)(V_0/V)^{2/3}$	1.018318
		S/E^2	55.286742	V/E^3	37.616684		



Rhombic dodecahedron（菱形十二面体） V(3,4)^2

V = 14 v_3 = 8 v_4 = 6 E = 24 F = 12

$(0,0,\pm1)$ $(0,\pm1,0)$ $(\pm1,0,0)$ $(\pm1/2,\pm1/2,\pm1/2)$

$R = 1$ $R_4 = \sqrt{2}/2$

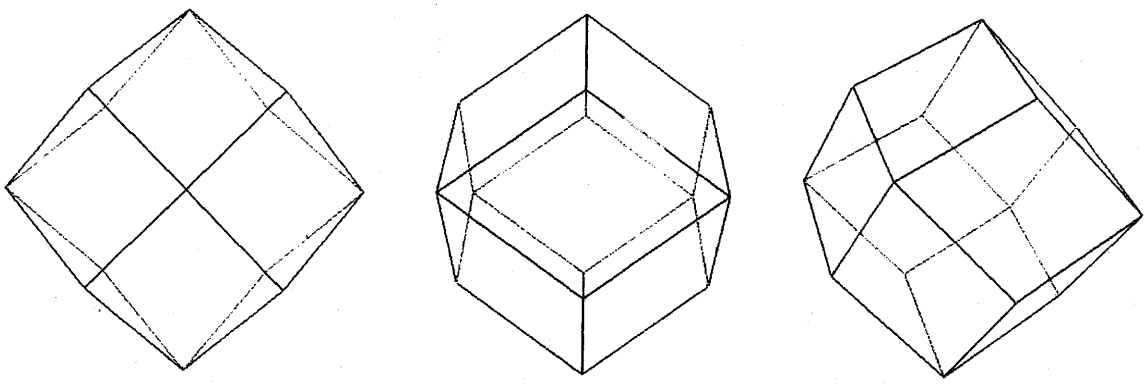
$S = 6\sqrt{2}$ $S_4 = \sqrt{2}/2$

$V = 2$ $V_4 = 1/6$

$E = \sqrt{3}/2$ $E/R = \sqrt{3}/2$ $R/E = 2\sqrt{3}/3$

x	y	z
0	0	±1
0	±1	0
±1	0	0
±0.5	±0.5	±0.5

R_4	0.707107	S_4	0.707107	V_4	0.166667	E	0.866025
R	1	S	8.485281	V	2	E/R	0.866025
						R/E	1.154701
		S/S_0	0.675237	V/V_0	0.477465	$(S/S_0)(V_0/V)^{2/3}$	1.105339
		S/E^2	11.313708	V/E^3	3.079201		



Rhombic triacontahedron （菱形三十面体） V(3,5)²

V = 32 v_3 = 20 v_5 = 12 E = 60 F = 30

$(\pm 1,\pm\tau^{-1},0)$ $(0,\pm 1,\pm\tau^{-1})$ $(\pm\tau^{-1},0,\pm 1)$ $(\pm\tau^{-2},\pm 1,0)$ $(\pm 1,0,\pm\tau^{-2})$ $(0,\pm\tau^{-2},\pm 1)$ $(\pm\tau^{-1},\pm\tau^{-1},\pm\tau^{-1})$

$R = \sqrt{(5-\sqrt{5})/2}$ $R_4 = 1$

$S = 60(\sqrt{5}-2)$ $S_4 = 2(\sqrt{5}-2)$

$V = 20(\sqrt{5}-2)$ $V_4 = 2(\sqrt{5}-2)/3$

$E = \sqrt{5-2\sqrt{5}}$ $E/R = \sqrt{(3-\sqrt{5})/2}$ $R/E = \sqrt{(3+\sqrt{5})/2}$

x	y	z
±1	±0.618034	0
0	±1	±0.618034
±0.618034	0	±1
±0.381966	±1	0
±1	0	±0.381966
0	±0.381966	±1
±0.618034	±0.618034	±0.618034

R_4	1	S_4	0.472136	V_4	0.157379	E	0.726543
R	1.175571	S	14.164079	V	4.721360	E/R	0.618034
						R/E	1.618034
		S/S_0	0.815607	V/V_0	0.693797	$(S/S_0)(V_0/V)^{2/3}$	1.040701
		S/E^2	26.832816	V/E^3	12.310735