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A Method of Calculations of the Higher Order Partial Cumulants in the Problem of Low Field Resonance

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A theory of low field resonance is formulated with the use of an expansion formula of the damping theory: That is, the time convolution equation is applied to this problem. In this treatment, the so-called "partial cumulants" of somewhat complicated structure should be calculated. To get the explicit forms of the "partial cumulants", computer calculations are preferable. The use of a computer makes it possible to treat a lot of terms and put the same class of terms into a single are. When the computer calculation is done, then the calculation time is entirely shortened and the higher order terms are treated quite easily.

In this paper we present the corresponding FORTRAN programs for these calculations and give an argorism and a method to use them.

§ 1. Introduction

The expansion formulas of the damping theory[1] are known as a general method of the nonequilibrium statistical mechanics. Shibata and Arimitsu[1] obtained the two kinds of formulas: One is the time convolution equation (TCE) and the other is the time convolutionless equation (TCLE). One should choose one of these equations depending on the nature of the problem.

In some problems, strength of the coherent motion and the fluctuation acting on it becomes the same order of magnitude, so that a usual simple purturbation theory can give an incorrect result. For instance, a problem of exciton migration[2),3)] and that of low field resonance[4)] belong to this category. Even for these problems, we obtained the satisfactory results by means of the TCE.

Particularly, in the theory of low field resonance, we could take all orders of perturbations into account and get an exact solution for a certain model. In the course of the calculations, we treated special form of the "partial cumulants". A general forms of the "partial cumulants" were

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already written down with the use of computer[5]. However, in our present problem, we must treat quite complicated terms including the "partial cumulants". Therefore, a further computational investigation is required.

In this paper we present a method to calculate the special forms of "partial cumulants" as an application of the method presented in ref. 5, and give the results up to the 12-th order. Once the expressions up to the 12-th order is known, then the arbitrary order terms can be written down consistently. Consequently, the "Green function" or the power spectrum is constructed in the form of an infinite continued fraction.

In the next section, we present the basic formulation of the theory of the low field resonance and clarify the nature of terms to be calculated. Then we explain how the problem is treated with use of the computer explicitly (Sec. 3).

In the last section we give the conclusion.

§ 2. Summary of the basic formulation

2-1. Model Hamiltonian

Our system is described by the Hamiltonian

$$\mathscr{H}(t)=\mathscr{H}_0+\mathscr{H}_1(t) \tag{2.1}$$

where

$$\mathscr{H}_0=\gamma' h_0 S_z \tag{2.2}$$

and

$$\mathscr{H}_1(t)=\gamma' \sum_{\mu} H_\mu(t)\, S_\mu \tag{2.3}$$

In these expressions, h_0 represent the static field along z-direction. The fluctuating field $H_\mu(t)$ acting on the relevant spin S_μ is assumed to be a Gaussian Markoffian process with zero mean:

$$<H_\mu(t)>_B = 0, \tag{2.4}$$

$$<H_\mu(t)H_{\mu'}(t')>_B = \delta_{\mu\mu'}(\hbar/\gamma')^2 \Delta_\mu^2\, \mathrm{e}^{-\tau_\mu|t-t'|}, \tag{2.5}$$

$$<H_\mu(t)H_\mu(t_1)\ldots H_\mu(t_{n-1})>_{B.,\,C.} = 0, \quad (n\geq 3). \tag{2.6}$$

where μ, μ' can take x and y and $\Delta_x=\Delta_y\equiv\Delta_\perp$.

We rewrite (2.3) as

$$\mathscr{H}_1(t)=\gamma' \sum_{\nu} H_{\bar{\nu}}(t)\, S_{\nu} \tag{2.7}$$

for the convenience in the following formulation. In (2.7) ν takes +, −, and moreover $\nu = +$ (−) is understood to represent $\nu = +1$ (−1). We have also introduced the notation $\bar{\nu}=-\nu$. In (2.7) $H_+(t)$ and $H_-(t)$ are defined by

$$H_{\pm}(t)=\frac{1}{2}\{H_x(t)\pm \mathrm{i}\, H_y(t)\}\,. \tag{2.8}$$

Similarly we have put

$$S_{\pm} = S_x \pm \mathrm{i}\, S_y$$

and

$$S_0 = S_z$$

2-2. Formulation

Here we give an outline of the formulation. As for details, we refer the reader to the reference 4.

Time-evolution of the system is determined by the Liouville-von Neumann equation of density matrix. First we take an average of the density matrix with the use of the time-convolution equation (TCE) with the expansion formulae. From the moment equations of spin operators S_+ and S_z, we can discuss the power spectrum and so on.

The general form of the moment equation is given by

$$\langle \dot{S}_{\nu'}\rangle_t = \mathrm{i}\nu'\omega_0(-)^{\nu'+1}\langle S_{\nu'}\rangle_t + \sum_n \sum_{\nu} (\gamma'/\mathrm{i}\hbar)^n \prod_{j=0}^{n-2} \int_0^{t_j} \mathrm{d}t_{j+1}\, \kappa_j\, \mathrm{e}^{\mathrm{i}\lambda_j \omega_0 (t_j - t_{j+1})} \times \Big\langle \prod_{j=0}^{n-1} H_{\bar{\nu}_j}(t_j)\Big\rangle_{B.,\,P.\,C.}\, \kappa_{n-1}\langle S_{\lambda_{n-1}}\rangle_{t_{n-1}} \tag{2.9}$$

where ν represents the set $\{\nu', \nu_0, \nu_1, \ldots\}$ with $t_0 \equiv t$ and $\omega_0 = \gamma' h_0/\hbar$. Moreover we have

$$\lambda_j = \nu' + \nu_0 + \nu_1 + \ldots + \nu_j \tag{2.10}$$

where $\lambda_{-1} = \nu'$, and

$$\kappa_j = (-)^{\lambda_j}(\lambda_{j-1} - \nu_j)\,. \tag{2.11}$$

We have also a "conservation low":

$$\nu_0 + \nu_1 + \cdots + \nu_{n-1} = 0 \ . \tag{2.12}$$

Owing to the rules (2.10) (2.11) (2.12), we can get an explicit form of the expression (2.9). For example, even when a single element of the set $\{\kappa_j, j=0,1,\ldots,n\text{-}1\}$ is equal to zero, the corresponding term to the set $\{\nu,\ \nu_0,\ \nu_1,\ \ldots\}$ vanishes. Therefore we may consider restricted combinations of ν', ν_0, ν_1, ... From these consideration we calculate the partial cumulant:

$$<\prod_{j=0}^{n-1} H_{\bar{\nu}_j}(t_j)>_{B.,P.C.} \tag{2.13}$$

in (2.9). As the set $\{\bar{\nu}_0, \bar{\nu}_1, \ldots, \bar{\nu}_{n-1}\}$ in (2.9) has a various combinations of + and −, (2.13) is divided into a lot of moments composed of H_x and H_y. So it is a very tedious work to calculate the "partial cumulants" (2.13).

To remove these difficulties, we should use the computer: we present a method of calculations for the explicit representation corresponding to the given ν's by the FORTRAN programs.

§ 3. Calculation of "partial cumulants"

3-1. Treatment in the computer

In this section we give a method to calculate the special type of "partial cumulants".

The desired "partial cumulants" are shown in (2.13) in the form

$$<\prod_{j=0}^{n-1} H_{\bar{\nu}_j}(t_j)>_{B.,P.C.} \ ,$$

which satisfy the conservation rule (2.12) and condition $\kappa_j \neq 0$. We will discuss FORTRAN programs to obtain the explicit expressions of the "partial cumulants" (2.13) corresponding to the values of $\{\bar{\nu}_j, j=0,1,\ldots,n\text{-}1\}$.

These programs are composed of three parts: First we notice that the "partial cumulants" (2.13) consist of the multiplication of $H_{\pm}(t_j)$'s which are defined as $H_x(t_j) \pm \mathrm{i} H_y(t_j)$. On the other hand, because the moments of $H_\mu(t_j)$'s are defined for $\mu=x, y$, we have to decompose the "partial cumulants" (2.13) into the terms consisting of $H_x(t_j)$'s and $H_y(t_j)$'s; that is, we must treat 2^n terms. Each term is made up of H_x or H_y corresponding the time from t_0 ($\equiv t$) to t_{n-1}. So we consider the n "boxes" corresponding to these time points which should be occupied by H_x or H_y. The first step

of these calculations is to give all possible permutations of $H_x(t_j)$'s and $H_y(t_j)$'s.

A method to obtain these permutations is the same as the one adopted in ref. 5. When a "box" is occupied by H_x (H_y) , we assign "0" ("1") to the "box". Therefore we have two possible configurations each of which is associated with a corresponding binary digit. As the term consisting of the odd numbers of H_x or H_y has no contribution, so we remove these terms. We call this program "LFR1".

Next step is to obtain the diagrams appearing in the calculation of the "partial cumulants" of the stochastic variables $H_\mu(\mu=x,y)$'s which are assumed to be Gaussian Markoffian. Such diagrams were considered in some details in ref. 3, but here we give a simple explanation of the procedure. When we calculate the term

$$<\prod_{j=0}^{n-1} H_\mu(t_j)>_{B.,\,P.\,C.} \tag{3.1}$$

which consists of the same stochastic variable $H_\mu(t_j)$ ($t_0 \geq t_1 \geq \ldots \geq t_{n-1}$), we treat it as follows:

The general form of (3.1) is composed of the linear combination of irreducible diagrams. These diagrams are constructed from the permutation of the same numbers of "+" and "−" which indicate the sign of t_j's (see, (3.2) for the general expression). The first member always takes the value of "+" and the last one "−". For example, the second order diagram gives

(+−) ,

and the fourth order diagrams are

(++−−)

and

(+−+−) ,

and so on. But from the nature of the "partial cumulants", "reducible" diagrams are removed from the outset. The simbol "+" must be paired with "−" located in the right hand side of it. If the pair is developed in the diagram, we call it "reducible". In the examples mentioned above, (+−+−) is a "reducible" diagram.

In the computer calculations, we assign +1 to the simbol "+" and −1 to "−". When we repeat summations from the left of the diagram, even if one of

the intermediate sum becomes zero, we identify the diagram to be "reducible". The final sum is of course equal to zero because the same numbers of +1 and −1 are contained. The permutations of +1 and −1 are obtained with the use of the binary digit. We call this program "LFR2".

The last procedure is to give the weights of the diagrams with the use of the above results. First we determine the explicit form of (2.13); that is, the combination of H_+ and H_- should be found. In our treatment we make an input of the value of ν_j's. For example, in the fourth order, we make an input the simbols "++−−" and "+−+−", etc. Then we check the order and conservation rule (2.12). If we make a mistake in the first input, we must refrain. Next each of the diagrams obtained in the program "LFR2" should be read. Corresponding to the diagram, each of the purmutations of x and y obtained in "LFR1" is separated into two groups which are "x"-group and "y"-group. We make a pair of "+" and "−" in the diagram only inside of the group, because H_x and H_y are independent stochastic variables. In these treatments, when the numbers of "+" and "−" corresponding to each group are not equal, the term has no contribution to the diagram. Moreover, a term of $H_y(t_j)$ has a factor of ± i corresponding to $H_{\pm}(t_j)$. Finally we get the number of combinations of "+" and "−", and the factor coming from $H_y(t_j)$'s. We can thus obtain all the weight of the considering diagram by multiplying these values by $(1/2)^n$ (n the order of diagram). For other diagrams, we repeat the same procedure. We call this program "LFR11".

Hence, each of the diagrams is written down as follows:

$$(\text{weight}) \times \left(\frac{\hbar}{\gamma'} \Delta_\perp\right)^n \mathrm{e}^{-\gamma_\perp(+t_0+\cdots-t_{n-1})} \,. \tag{3.2}$$

Simbols "+" and "−" in the diagram represent the sign of the corresponding times in the exponent. For example, in the second order, we have

$$(+-) = \left(\frac{\hbar}{\gamma'} \Delta_\perp\right)^2 \mathrm{e}^{-\gamma_\perp(+t_0-t_1)} \,,$$

and in the fourth order:

$$(++--) = \left(\frac{\hbar}{\gamma'} \Delta_\perp\right)^4 \mathrm{e}^{-\gamma_\perp(+t_0+t_1-t_2-t_3)} \,.$$

3-2. Programs list

LFR1

```
 1          DIMENSION IA(12)
 2 C
 3          WRITE(2,200)
 4 200      FORMAT('ORDER(2-12)=')
 5          READ(1,201) ID
 6 201      FORMAT(I2)
 7          JF=2**ID
 8          IC=0
 9 C
10          REWIND 5
11          WRITE(5) JF,ID
12 C
13          DO 30 J=1,JF
14 C
15          DO 10 I=1,ID
16          IA(I)=0
17 10       CONTINUE
18 C
19          J0=J-1
20          DO 15 I=1,ID
21          L=ID+1-I
22          IA(L)=J0-J0/2*2
23          J0=J0/2
24          IF(J0.LT.1) GO TO 2
25 15       CONTINUE
26 C
27 2        IT=0
28          DO 20 I=1,ID
29          IT=IT+IA(I)
30 20       CONTINUE
31          JT=IT/2*2
32          IF(JT.NE.IT) GO TO 30
33          IC=IC+1
34          WRITE(5) (IA(I),I=1,ID)
35          WRITE(2,100) IC, (IA(I),I=1,ID)
36 100      FORMAT(I5,2X,12I1)
37 C
38 30       CONTINUE
39          ENDFILE 5
40 C
41          REWIND 5
42          WRITE(5) IC,ID
43 C
44 99       STOP
45          END
```

LFR2

```
 1          DIMENSION IA(12),A(12)
 2 C
 3          WRITE(2,200)
 4 200      FORMAT('ORDER(2-12)=@')
 5          READ(1,201) ID
 6 201      FORMAT(I2)
 7          JF=2**ID
 8          J0=JF/2+1
 9          IC=0
10 C
11          REWIND 9
12          WRITE(9) JF,ID
13 C
14          DO 30 J=J0,JF
15 C
16          DO 10 I=1,ID
17          IA(I)=0
18          A(I)=' '
19 10       CONTINUE
20 C
21          J0=J-1
22          DO 15 I=1,ID
23          L=ID+1-I
24          IA(L)=J0-J0/2*2
25          J0=J0/2
26          IF(J0.LT.1) GO TO 2
27 15       CONTINUE
28 C
29 2        DO 20 I=1,ID
30          IF(IA(I).EQ.0) IA(I)=-1
31 20       CONTINUE
32          IF(IA(1).NE.1) GO TO 30
33          IF(IA(ID).NE.-1) GO TO 30
34 C
35          IT=0
36          DO 25 I=1,ID
37          IT=IT+IA(I)
38          IF(I.EQ.ID) GO TO 25
39          IF(IT.EQ.0) GO TO 30
40 25       CONTINUE
41          IF(IT.NE.0) GO TO 30
42          IC=IC+1
43 C
44          DO 27 I=1,ID
45          IF(IA(I).EQ.1) A(I)='+'
46          IF(IA(I).EQ.-1) A(I)='-'
47 27       CONTINUE
48          WRITE(9) (IA(I),I=1,ID)
49          WRITE(2,100) IC, (A(I),I=1,ID)
```

```
 50 100    FORMAT(I5,2X,12A1)
 51 C
 52 30     CONTINUE
 53        ENDFILE 9
 54 C
 55        REWIND 9
 56        WRITE(9) IC,ID
 57 C
 58 99     STOP
 59        END
```

LFR11

```
  1 C**** CALCULATION OF CORELATION OF MIXED TERM(X,Y) ****
  2 C
  3         DIMENSION IA(12),IB(12),IN(12),LC(2),MC(2)
  4        *   ,A(12),B(12)
  5         COMPLEX CI,CFA
  6 C
  7         CI=CMPLX(0.,1.)
  8 C
  9         WRITE(2,330)
 10 330     FORMAT('ORDER=@')
 11         READ(1,331) ID
 12 331     FORMAT(I2)
 13         WRITE(2,350)
 14 350     FORMAT('INPUT THE LOGICAL UNIT OF OUTPUT@')
 15         READ(1,351) IOP
 16 351     FORMAT(I1)
 17         WRITE(IOP,360) ID
 18 360     FORMAT(1H ,'** CORELATION **  ORDER=',I2)
 19 C
 20 1       WRITE(2,100)
 21 100     FORMAT('INPUT THE KIND OF CORELATION(+OR-)@')
 22         READ(1,101) (A(I),I=1,12)
 23 101     FORMAT(12A1)
 24         DO 60 I=1,12
 25         IF(A(I).NE.' ') GO TO 3
 26 60      CONTINUE
 27         GO TO 99
 28 C
 29 3       WRITE(IOP,370) (A(I),I=1,12)
 30 370     FORMAT(//1H ,5X,'<',12A1,'>pc'/)
 31 C
 32         DO 10 I=1,12
 33         IF(A(I).EQ.' ') GO TO 2
 34 10      CONTINUE
```

```
      2       ICO=I-1
              IF(ICO.NE.ID) GO TO 1
      C
              IC=0
              DO 20 I=1,ID
              IF(A(I).EQ.'+') IN(I)=+1
              IF(A(I).EQ.'-') IN(I)=-1
              IC=IC+IN(I)
     20       CONTINUE
              IF(IC.NE.0) GO TO 1
      C
              REWIND 5
              READ(5) JF,ID0
              IF(ID0.NE.ID) GO TO 99
              WRITE(2,400) JF
    400       FORMAT(1H ,'  CORELATION TERMS=',I5)
      C
              REWIND 9
              READ(9) LF,ID0
              IF(ID0.NE.ID) GO TO 99
              WRITE(2,401) LF
    401       FORMAT(1H ,'  DIAGRAMS=         ',I5/)
      C
      C
              DO 50 L=1,LF
              READ(9) (IB(I),I=1,ID)
      C
              REWIND 5
              READ(5) NDA,NDB
              IFF=0
              DO 40 J=1,JF
              READ(5) (IA(I),I=1,ID)
      C
              IC=0
              DO 30 I=1,ID
              IC=IC+IA(I)*IB(I)
     30       CONTINUE
              JC=IC/2*2
              IF(JC.NE.IC) GO TO 40
      C
              DO 33 I=1,2
              LC(I)=0
              MC(I)=1
     33       CONTINUE
              DO 35 I=1,ID
              N=ID+1-I
              NN=IA(N)+1
              IF(IB(N).EQ.1) MC(NN)=MC(NN)*LC(NN)
              LC(NN)=LC(NN)-IB(N)
     35       CONTINUE
      C
```

```
      CFA=CMPLX(1.,0.)
      DO 37 I=1,ID
      IF(IA(I).NE.1) GO TO 37
      CFA=CFA*CI*IN(I)
37    CONTINUE
      CFA=CFA*MC(1)*MC(2)
C
      IFA=REAL(CFA)
      IFF=IFF+IFA
40    CONTINUE
C
      DO 45 I=1,ID
      IF(IB(I).EQ.1) B(I)='+'
      IF(IB(I).EQ.-1) B(I)='-'
45    CONTINUE
C
      IFF=IFF/2**(ID/2)
      WRITE(IOP,500) IFF,(B(I),I=1,12)
500   FORMAT(1H ,I7,'(',12A1,')',)
C
50    CONTINUE
      GO TO 1
C
99    STOP
      END
```

3-3. Examples

We present here the results with the use of the programs given in the previous subsection. We consider two cases: One is the case corresponding to $\nu'=+$ in (2.9), and the other is the case of $\nu'=0$.

(i) The case of $\nu'=+$

```
** CORELATION **  ORDER= 2

     <+-           >p c

      1(+-)
```

```
** CORELATION **  ORDER= 4
      <+-+-          >pc
       1(++--)
      <++--          >pc
       2(++--)

** CORELATION **  ORDER= 6
      <+-+-+-        >pc
       1(++-+--)
       2(+++---)
      <+-++--        >pc
       2(++-+--)
       2(+++---)
      <++--+-        >pc
       2(++-+--)
       2(+++---)
      <++-+--        >pc
       4(++-+--)
       2(+++---)

** CORELATION **  ORDER= 8
      <+-+-+-+-      >pc                <++--+-+-      >pc
       1(++-+-+--)                        2(++-+-+--)
       2(++-++---)                        4(++-++---)
       2(+++--+--)                        2(+++--+--)
       4(+++-+---)                        4(+++-+---)
```

```
 4(++++----)                          4(++++----)
<+-+-++--      >pc                   <++--++--      >pc
 2(++-+-+--)                          4(++-+-+--)
 2(++-++---)                          4(++-++---)
 4(+++--+--)                          4(+++--+--)
 4(+++-+---)                          4(+++-+---)
 4(++++----)                          4(++++----)
<+-++--+-      >pc                   <++-+--+-      >pc
 2(++-+-+--)                          4(++-+-+--)
 2(++-++---)                          4(++-++---)
 2(+++--+--)                          2(+++--+--)
 2(+++-+---)                          2(+++-+---)
 6(++++----)                          6(++++----)
<+-++-+--      >pc                   <++-+-+--      >pc
 4(++-+-+--)                          8(++-+-+--)
 2(++-++---)                          4(++-++---)
 4(+++--+--)                          4(+++--+--)
 2(+++-+---)                          2(+++-+---)
 6(++++----)                          6(++++----)
```

From these results we can write down the explicit form of (2.9) for $\nu'=+$:

$$\langle \dot{S}_+ \rangle_t = \mathrm{i}\,\omega_0 \langle S_+ \rangle_t$$

$$+\mathrm{i}^2 \Delta_\perp^2 \int_0^t \mathrm{d}t_1\, \mathrm{e}^{-\gamma_\perp (t-t_1)} \langle S_+ \rangle_{t_1}$$

$$+\mathrm{i}^4 \Delta_\perp^4 \int_0^t \mathrm{d}t_1 \int_0^{t_1} \mathrm{d}t_2 \int_0^{t_2} \mathrm{d}t_3 \left[\mathrm{e}^{-\mathrm{i}\omega_0 (t_1 - t_2)} \mathrm{e}^{-\gamma_\perp (t + t_1 - t_2 - t_3)} \langle S_+ \rangle_{t_3} \right.$$

$$+e^{i\omega_0(t_1-t_2)}e^{-\gamma_\perp(t+t_1-t_2-t_3)}\langle S_+\rangle_{t_3}\Big]$$

$$+\ldots. \tag{3.3}$$

Equation (3.3) can be solved immediately by the method of Laplace-transform. As the higher order term of (3.3) is systematic, so we can write down the exact result in the form of intinite continued fraction (see ref. 4, for more details).

(ii) The case of $\nu'=0$

```
** CORELATION **  ORDER= 2
      <+-           >p c
       1(+-)
      <-+           >p c
       1(+-)


** CORELATION **  ORDER= 4
      <+-+-         >p c
       1(++--)
      <+--+         >p c
       1(++--)
      <-++-         >p c
       1(++--)
      <-+-+         >p c
       1(++--)
```

```
** CORELATION **  ORDER= 6

      <+-+-+-       >p c

       1(++-+--)

       2(+++---)

      <+-+--+       >p c

       1(++-+--)

       2(+++---)

      <+--++-       >p c

       1(++-+--)

       2(+++---)

      <+--+-+       >p c

       1(++-+--)

       2(+++---)

      <-++-+-       >p c

       1(++-+--)

       2(+++---)

      <-++--+       >p c

       1(++-+--)

       2(+++---)

      <-+-++-       >p c

       1(++-+--)

       2(+++---)

      <-+-+-+       >p c

       1(++-+--)

       2(+++---)
```

```
** CORELATION **  ORDER= 8

     <+-+-+-+-     >pc          <+--++-+-     >pc
      1(++-+-+--)                1(++-+-+--)
      2(++-++---)                2(++-++---)
      2(+++--+--)                2(+++--+--)
      4(+++-+---)                4(+++-+---)
      4(++++----)                4(++++----)
     <+-+-+--+     >pc          <+--++--+     >pc
      1(++-+-+--)                1(++-+-+--)
      2(++-++---)                2(++-++---)
      2(+++--+--)                2(+++--+--)
      4(+++-+---)                4(+++-+---)
      4(++++----)                4(++++----)
     <+-+--++-     >pc          <+--+-++-     >pc
      1(++-+-+--)                1(++-+-+--)
      2(++-++---)                2(++-++---)
      2(+++--+--)                2(+++--+--)
      4(+++-+---)                4(+++-+---)
      4(++++----)                4(++++----)
     <+-+--+-+     >pc          <+--+-+-+     >pc
      1(++-+-+--)                1(++-+-+--)
      2(++-++---)                2(++-++---)
      2(+++--+--)                2(+++--+--)
      4(+++-+---)                4(+++-+---)
      4(++++----)                4(++++----)
```

<-++-+-+- >pc

1(++-+-+--)

2(++-++---)

2(+++--+--)

4(+++-+---)

4(++++----)

<-++-+--+ >pc

1(++-+-+--)

2(++-++---)

2(+++--+--)

4(+++-+---)

4(++++----)

<-++--++- >pc

1(++-+-+--)

2(++-++---)

2(+++--+--)

4(+++-+---)

4(++++----)

<-++--+-+ >pc

1(++-+-+--)

2(++-++---)

2(+++--+--)

4(+++-+---)

4(++++----)

<-+-++-+- >pc

1(++-+-+--)

2(++-++---)

2(+++--+--)

4(+++-+---)

4(++++----)

<-+-++--+ >pc

1(++-+-+--)

2(++-++---)

2(+++--+--)

4(+++-+---)

4(++++----)

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1(++-+-+--)

2(++-++---)

2(+++--+--)

4(+++-+---)

4(++++----)

<-+-+-+-+ >pc

1(++-+-+--)

2(++-++---)

2(+++--+--)

4(+++-+---)

4(++++----)

From these results we can write down the explicit form of (2.9) for $\nu'=0$:

$$\begin{aligned}\langle \dot{S}_z\rangle_t = {} & \mathrm{i}^2 \Delta_\perp^2 \int_0^t \mathrm{d}t_1 \big[\mathrm{e}^{-\mathrm{i}\omega_0(t-t_1)} \mathrm{e}^{-r_\perp(t-t_1)} \langle S_z\rangle_{t_1} \\ & + \mathrm{e}^{\mathrm{i}\omega_0(t-t_1)} \mathrm{e}^{-r_\perp(t-t_1)} \langle S_z\rangle_{t_1} \big] \\ & + \mathrm{i}^4 \Delta_\perp^4 \int_0^t \mathrm{d}t_1 \int_0^{t_1} \mathrm{d}t_2 \int_0^{t_2} \mathrm{d}t_3 \big[\mathrm{e}^{-\mathrm{i}\omega_0(t-t_1+t_2-t_3)} \mathrm{e}^{-r_\perp(t+t_1-t_2-t_3)} \langle S_z\rangle_{t_3} \\ & + \mathrm{e}^{\mathrm{i}\omega_0(t-t_1-t_2+t_3)} \mathrm{e}^{-r_\perp(t+t_1-t_2-t_3)} \langle S_z\rangle_{t_3} \\ & + \mathrm{e}^{-\mathrm{i}\omega_0(-t+t_1+t_2-t_3)} \mathrm{e}^{-r_\perp(t+t_1-t_2-t_3)} \langle S_z\rangle_{t_3} \\ & + \mathrm{e}^{-\mathrm{i}\omega_0(-t+t_1-t_2+t_3)} \mathrm{e}^{-r_\perp(t+t_1-t_2-t_3)} \langle S_z\rangle_{t_3} \\ & + \cdots \end{aligned} \tag{3.4}$$

Similarly, eq. (3.4) can be solved immediately by the method of Laplace-transform. Final result is expressed in the form of infinite continued fraction (see ref. 4).

§ 4. Conclusion

In this work we present a method to obtain the explicit representation of the special form of the "partial cumulants" (2.13) in the theory of low field resonance.

In this theory we consider moment equations (2.9) which have somewhat complicated forms. But they can be solved by the method of Laplace-transform and can be expressed in the form of the infinite continued fraction. To obtain the general form of infinite continued fraction, it is necessary to get the explicit representation of the "partial cumulants" up to the 12-th order. However, we have to treat a lot of terms in the higher order. For example, in the eighth order, we must consider 128 terms for a set of $\{\bar{\nu}_j\}$, and calculate the weights of 5 diagrams. If we try to solve without the aid of the computer, it is a hopeless task to complete the calculations. Thus, it will be impossible to calculate the higher order terms than the eighth order.

In the previous work we have developed the method to write down the explicit form of "partial cumulants" and "ordered cumulants"[5]. Applying the method to the present problem, we could discuss the exact treatment of the theory of low field resonance[4]. Thus the usefulness of the systematic use of the computer has been shown explicitly for this kind of problems.

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