The Structure of Turbulent Flow over Regularly Arrayed Rough Surfaces

(Wind Tunnel Experiment)

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1. Introduction

Many experimental and theoretical studies about the turbulent boundary layer including the surface layer in the atmosphere have been carried out, but it seems that in many cases of wind tunnel experiments there is a large enough fetch in the tunnel to avoid the leading edge effects.

Therefore, in the present paper, first, variations of both scale and energy spectrum of turbulence with streamwise distance in the developing region, that is, in a shorter fetch of the turbulent boundary layer over regularly arrayed rough surfaces, then the similarity of the shapes of energy spectra in the fully developed region and Kolmogorov constant are examined experimentally only for the streamwise component of turbulence under the neutral condition.

2. Experimental procedure

The basic facility consists of the small wind tunnel as shown in the previous paper of Nemoto et al. (1979). Two kinds of height of aluminum or celluloid strips were used for the roughness element, each height (h) of these strips was 5 and 10 mm, respectively.

Separation distance between nearest-neighbour roughness elements, or strips (d), downwind location of measurement from the first strip (x), and height of measurement from the floor level (z) were varied for each experiment respectively. All velocity data were obtained by constant-temperature hot-wire anemometry. That is, the hot-wire probe (diameter: 5 μ, length: about 2 mm) was used and the output of the anemometer, that is, the fluctuating wind velocity component in the streamwise direction, was recorded on a magnetic tape with the data recorder. The playback output of the data recorder was processed by an analog data analyzer, which consisted of power supply unit, mean meter unit, sigma meter unit and five band pass sigma meter units (the char-

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acteristics of the band pass sigma meter units for estimating energy spectrum will be published by M. Kato). The mean meter unit was used for obtaining the mean velocity for the averaging time of 50 sec, which was also that of all other units. Five basic frequency bands and their center frequencies for the sigma meter units used here are as shown in Table 1. Hence, when the tape speed of the data recorder is reduced to 1/2, each value of σ $(=\sqrt{\overline{u^2}})$ for another five center frequencies of 0.8, 3.2, 12.8, 51.2 and 204.8 Hz, respectively, are obtained, and here u is turbulent component of the velocity in the streamwise direction.

Table 1. Frequency bands and their center frequencies

Frequency band (Hz)	Center frequency (Hz)
0.2— 0.8	0.4
0.8— 3.2	1.6
3.2— 12.8	6.4
12.8— 51.2	25.6
51.2—204.8	102.4

From the combined operation of the data recorder and the analog data analyzer, values of σ for each center frequency of 10 or more than that may be obtained. Hence the spectrum distribution (hereafter, for simplicity, "spectrum" is often used instead of "energy spectrum") may be approximated by the following equation, that is,

$$E(n) \doteqdot \frac{\overline{u_i^2}}{\left(\sum_{i=1}^{10} \overline{u_i^2}\right) \Delta \log n_c} = \frac{G(n)}{\Delta \log n_c}$$

where $E(n)$ is nondimensional energy spectrum function for frequency n, n_c center frequency, difference between nearest-neighbour center frequencies expressed by the logarithmic scale $\Delta \log n_c = 0.301$ in the present experiment because of $(n_c)_{i+1}/(n_c)_i = 2$ and $\overline{u_i^2}$ indicates a partial turbulence kinetic energy expressing contribution due to the frequencies contained in the i-th frequency band. Throughout the present paper, $G(n) = \overline{u_i^2} / \sum_{i=1}^{10} \overline{u_i^2}$ is used to express the spectrum instead of $E(n)$.

On the other hand, the mean velocity can be easily obtained with the mean meter unit, then its vertical profile can also be obtained and it shows a logarithmic profile, hence from these velocity profiles roughness parameter (z_0) and friction velocity (u_*) may be estimated for each case and the profile data are also utilized for estimating Kolmogorov constant from the spectrum distribution in the inertial subrange obtained here.

3. Experimental results

3.1. Variation of roughness parameter with streamwise distance in the developing region of turbulence

As shown in Fig. 1, roughness parameter (z_0) varies with streamwise distance (x) in the region of shorter fetch (that is, in the developing region of turbulence), but it tends to take a constant value in the region of a large enough fetch. These results have already been described in the previous paper (Nemoto et al., 1979). The value for z_0 reaches maximum at about x=100 cm. On the other hand, as the friction velocity (u_*) can be obtained from the logarithmic profile of mean wind velocity as already mentioned, the vertical flux of

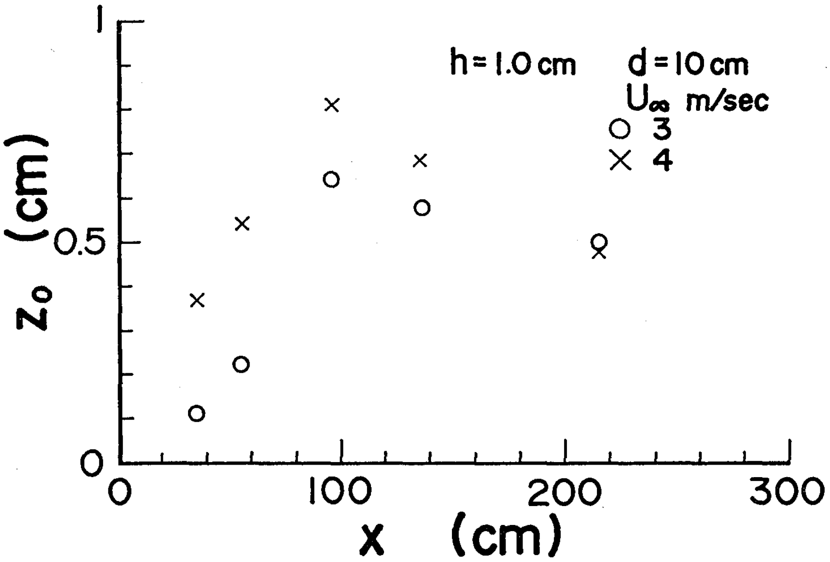

Fig. 1. Variation of roughness parameter (z_0) with streamwise distance (x).

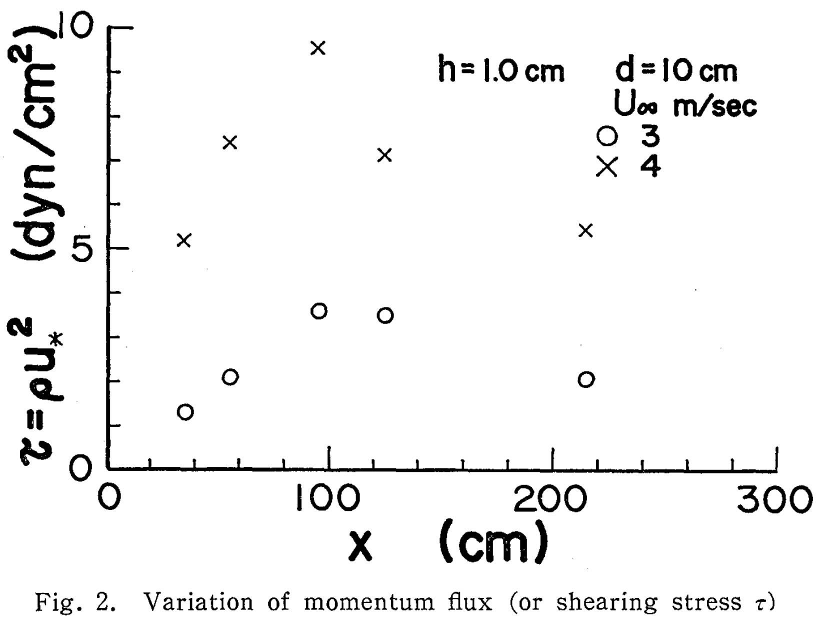

Fig. 2. Variation of momentum flux (or shearing stress τ) with streamwise distance (x).

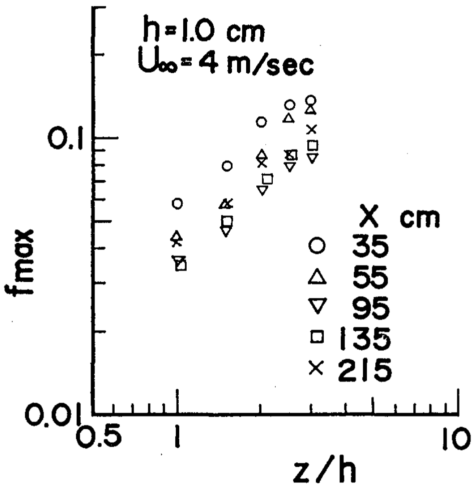

Fig. 3. An example of variation of f_{max} (the value of f at the peak value of $G(n)$) with height (z) and streamwise distance (x) for a single strip.

momentum (τ) at each streamwise distance may also be estimated as shown in Fig. 2. It is seen from this figure that τ also has the maximum value at about $x=100$ cm. The conditions of experiments are shown in each figure.

Fig. 3 shows an example of variation of f_{max} with both x and z in the case where a single strip is set on the floor. In the figure f_{max} is a value of nondimensional frequency $f=nz/U_z$ at the peak value of the spectrum, where U_z is mean wind velocity at a height of z. It is seen from this figure that f_{max} takes a minimum value at each height at $x=95$ cm (near 100 cm). That is, the results show that the horizontal eddy size corresponding to the peak frequency of spectrum distribution becomes maximum, because the eddy size (λ_z) corresponding to f_{max} is expressed as $\lambda_z=z/f_{max}$ for a height of z. It is obvious from the above results that the effect of the first strip remains over a wide downstream region also in the case of many strips regularly arrayed.

3.2. Comparison of the distribution of turbulence kinetic energy in the streamwise direction in the case of a single strip with that in the case of many strips regularly arryed

Fig. 4 shows the distributions of turbulence kinetic energy in the streamwise direction for the two cases of a single strip and many strips regularly arrayed for the two cases of tunnel velocities, $U_\infty=2$ and 3 m/sec, for $h=1.0$ cm, $z=2.0$ cm. In these figures the ordinate shows nondimensional turbulence kinetic energy and the abscissa nondimensional streamwise distance. The figure in the left hand side shows that the turbulence kinetic energy decreases very rapidly to the downstream distance of $x=55$ cm for a single strip. On the other hand, the figure in the right hand side shows the distribution of turbulence kinetic energy over strips regularly arrayed. In the latter case the decay of the turbulence kinetic energy cannot be recognized, and it rather increases with the streamwise distance x and order of the magnitude of the turbulence kinetic energy is different considerably for the two cases, that is, the turbulence kinetic energy for a single strip is about two figures smaller than that for many regularly arrayed strips at $x=100$ cm. The same results, of course, are obtained for the height of 3 cm (the figure is omitted here). Hence, it is obvious that many strips which are regularly arrayed are more effective for the experiment to simulate the flow over the relatively flat ground surface with a certain degree of roughness, to avoid the decay of the turbulence kinetic energy ($\overline{u_z^2}$,

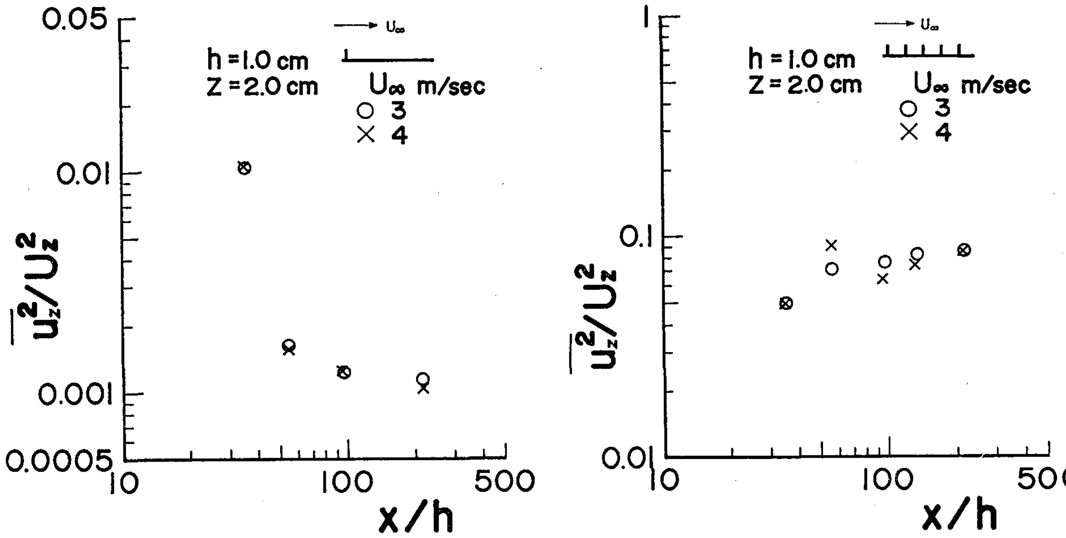

Fig. 4. Comparison of variation of turbulence kinetic energy with streamwise distance for the case of a single strip with that for the case of regularly arrayed strips.

which is the total turbulence kinetic energy at a height of z for the averaging time of 50 sec and measured with the sigma meter unit of the frequency band from DC to 1 K Hz).

3.3. Variation of shape of energy spectrum with x

Fig. 5 shows spectrum distributions against nondimensional frequency $f=nz/U_z$ at each height of 1.0, 1.5, 2.0, 2.5, 3.0, 3.5 cm under the conditions of $U_\infty=2$ m/sec, $h=0.5$ cm, $d=5$ cm, $x=107.5$ cm, where each value of $G(n)$ is plotted against the center frequency of each frequency band. It is seen from this figure that each spectrum distribution at each height shifts toward the higher frequency side with height. Hence it seems difficult to find the spectrum function in the empirical form as a function of f, which is able to express all the data measured.

However, if the frequency f/f_{max}, normalized by f_{max}, is used instead of f, it will be seen that all the data at each height may be expected to be expressed by an empirical formula as shown in Fig. 6 and the solid line in the figure is the one calculated by the empirical formula (1) to be described later and shows the spectrum at $x=215$ cm. Although the measurements were made at $x=35$, 55, 95, 135, 215 cm, only the two spectra at $x=35$ cm and 215 cm (by the curve) are shown to avoid complication of the figure. The spectra at $x=55$, 95, 135 cm are able to be written between the two spectra shown here and three spectra mentioned above approach to the one at $x=215$ cm in turn with increase of x and the spectrum at $x=135$ cm almost coincides with the one at $x=215$ cm, where the curve at $x=215$ cm shows the one calculated by the following formula :

$$G(n)=0.76\frac{f/f_{max}}{(1+f/f_{max})^2} \tag{1}$$

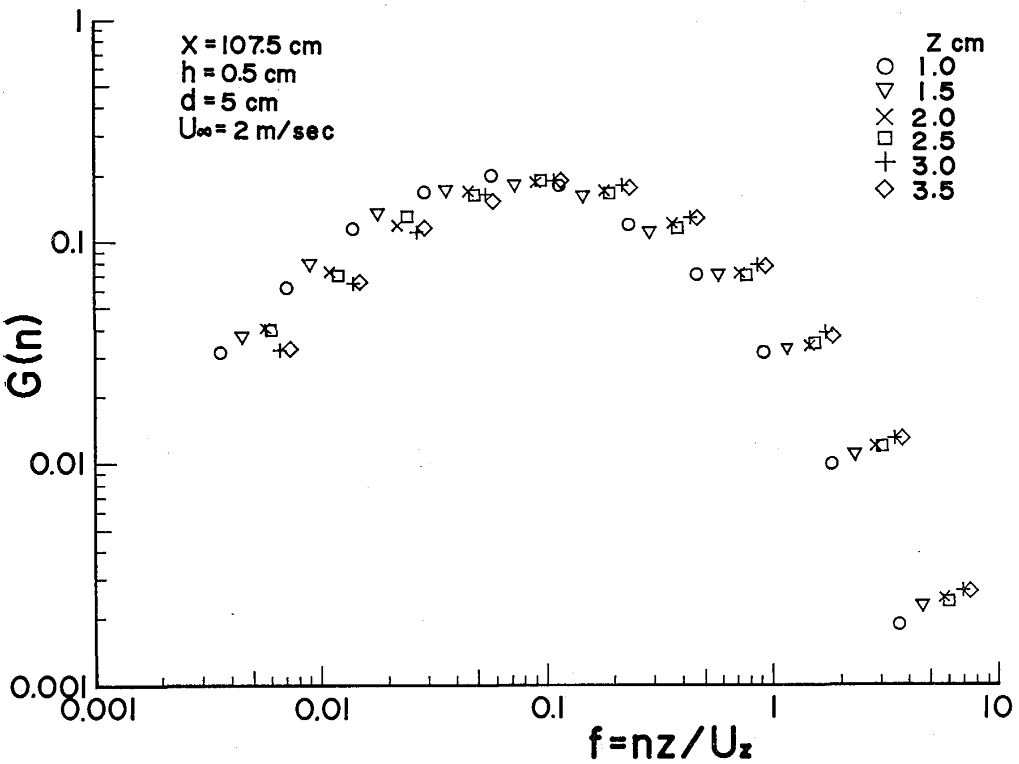

Fig. 5. Nondimensional spectrum $G(n)$ as function of nondimensional frequency (f).

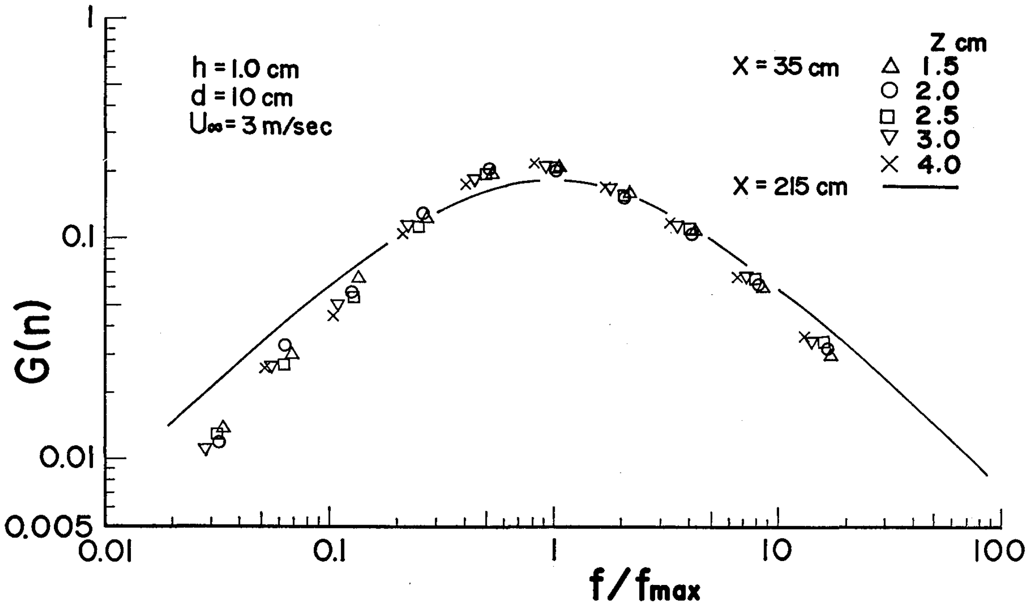

Fig. 6. Comparison of spectrum shape at $x=35$ cm with that at $x=215$ cm.

which fits to the measured values as will be shown later (Fig. 10).

Then it may be seen from Fig. 6 that, (i) the value of spectrum $G(n)$ at the peak frequency f_{max} decreases with x, (ii) the values of spectrum in the lower frequency region increase with x, (iii) the values of spectrum in the higher frequency region increase with x.

3.4. Similarity of turbulence structure

The similarity of the turbulence structure was examined by changing h, d, x and U_∞. The spectra for each height shown in the right upper side of the figure under the conditions of h=0.5 cm, d=5 cm, x=107.5 cm and U_∞=2 m/sec are shown in the same Fig. 7 and the curve in the figure indicates the empirical formula (1). On the other hand, in Fig. 8, all the spectra for each height shown in

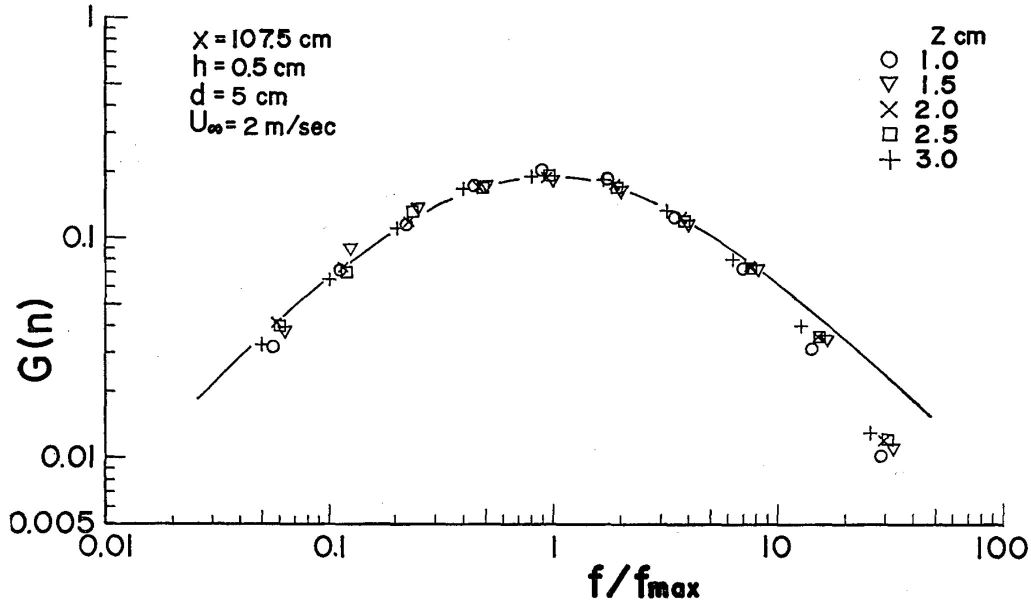

Fig. 7. Nondimensional spectra as function of normalized frequency f/f_{max} under the condition shown in the left upper side of the figure. The curve shows the generalized form of spectrum expressed by Eq. 1.

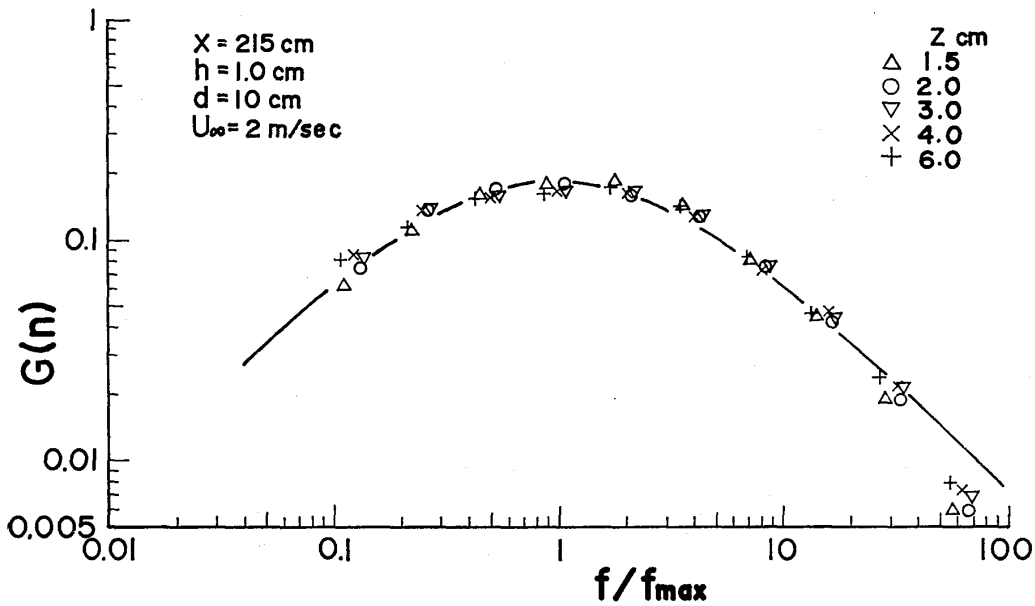

Fig. 8. Nondimensional spectra as function of f/f_{max} under the condition shown in the left upper side of the figure. In this case, x, h and d are respectively twice those in the case shown in Fig. 7. The curve indicates Eq. 1.

the right upper side of the figure under the conditions of twice as many of each geometrical scale of h, d and x, that is, $h=1.0$ cm, $d=10$ cm, $x=215$ cm for the same tunnel velocity ($U_\infty=2$ m/sec) are shown and the curve in the figure also indicates the empirical formula (1). Figs. 9 and 10 are the same comparison for $U_\infty=3$ m/sec, the two curves in these figures are also that calculated by the empirical formula (1). It is obviously seen from these results that the turbulence

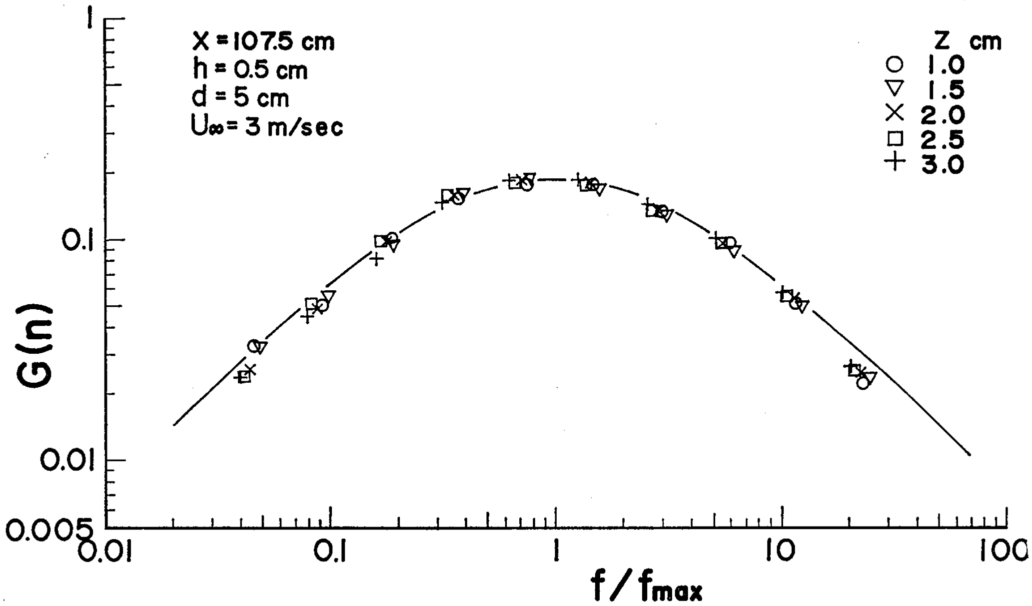

Fig. 9. Nondimensional spectra as function of f/f_{max} under the condition shown in the left upper side of the figure. The conditions are the same as those of Fig. 7 except $U_\infty=3$ m/sec. The curve indicates Eq. 1.

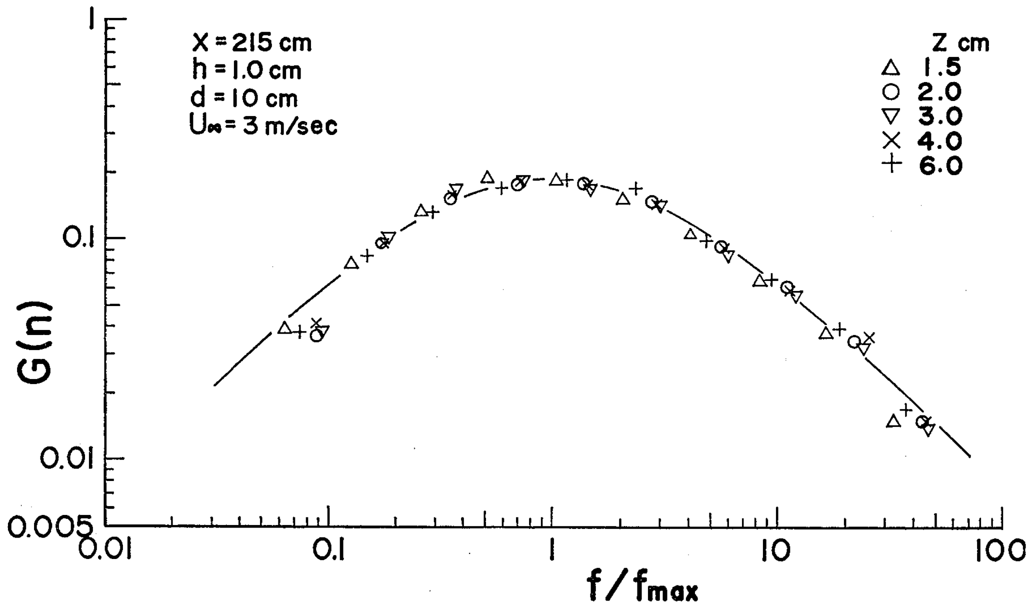

Fig. 10. Nondimensional spectra as function of f/f_{max} under the condition shown in the left upper side of the figure. The conditions are the same as those of Fig. 8 except $U_\infty=3$ m/sec. The curve indicates Eq. 1.

structure for each case is similar to each other with respect to the streamwise component. Unfortunately, vertical and lateral components of velocity were not measured, so that discussion about these components could not be made. But it may be expected that the similarity would be held for them too.

On the other hand, Lumley and Panofsky (1964) suggested the generalized form as follows:

$$E(n)=\frac{f/f_{max}}{(1+f/f_{max})^2} \tag{2}$$

This form is applicable to the vertical component of turbulence in the atmosphere, but the formula obtained in the present study is similar to that quite well. This may be due to the fact that the turbulent flow is generated in the wind tunnel by regularly arrayed roughness elements.

3.5. Kolmogorov constant

The energy spectrum function in the inertial subrange is generally expressed as follows:

$$F(k)=A\varepsilon^{2/3}k^{-5/3} \tag{3}$$

where $F(k)$: one-dimensional energy spectrum function (cm^3/sec^2)
ε: dissipation rate of turbulence kinetic energy per unit mass and unit time (cm^2/sec^3)
k: wave number, $2\pi n/U_z$ ($rad \cdot cm^{-1}$)
A: Kolmogorov constant for one-dimensional energy spectrum

Then we first transform the expression (3) in order to estimate Kolmogorov constant A by applying the experimental data obtained here. That is, Equation (3) is written as

$$kF(k)=A\varepsilon^{2/3}k^{-2/3}$$

and the left hand side of the above equation is equal to u_n^2, being the contribution to the turbulence kinetic energy due to turbulence of wave number k or frequency n, then

$$u_n^2=A\varepsilon^{2/3}\left(\frac{2\pi}{U_z}\right)^{-2/3}n^{-2/3}$$

can be derived, and as only the neutral condition is dealt with, $\varepsilon=u_*/\kappa z$ (κ: von Kármán constant 0.4) can be applied to the above equation, then

$$u_n^2=A\kappa^{-2/3}(2\pi)^{-2/3}u_*^2\left(\frac{U_z}{nz}\right)^{2/3}$$

Next, by dividing both sides by $\overline{u_z^2}$ (total kinetic energy of x component of turbulence at a height of z, as already mentioned),

$$\frac{u_n^2}{\overline{u_z^2}}=A\kappa^{-2/3}(2\pi)^{-2/3}\frac{u_*^2}{\overline{u_z^2}}f^{-2/3}$$

$$=E(n)\div\frac{G(n)}{\Delta\log n_c}=\frac{G(n)}{0.301}$$

is derived. On the other hand, $U_z=(u_*/\kappa)\ln(z/z_0)$ helds in this case, hence the above equation is further transformed into

$$\frac{u_n^2}{\overline{u_z^2}}=A\kappa^{4/3}(2\pi)^{-2/3}\frac{U_z^2}{\overline{u_z^2}}\left(\ln\frac{z}{z_0}\right)^{-2}f^{-2/3}$$

then finally,

$$G(n)=0.163A\frac{u_*^2}{\overline{u_z^2}}f^{-2/3}$$

or (4)

$$G(n)=0.0261A\frac{U_z^2}{\overline{u_z^2}}\left(\ln\frac{z}{z_0}\right)^{-2}f^{-2/3}$$

can be derived. As all the quantities except constant A can be obtained from the present experiments, A may be estimated from Equation (4). The results are shown in Fig. 11. It may be seen from this figure that A somewhat varies with x, z and U_∞. Although its reason cannot be found definitely now, it may be infered to be due to the fact that quantities (e. g., z_0) to be estimated from the velocity profile and $\overline{u_z^2}$ estimated by the present method have some errors. The average value of A is 0.55 and coincides with that proposed by Busch (1973) as a reasonable average for the atmosphere. However it will still be necessary to reexamine A in more detail by other methods.

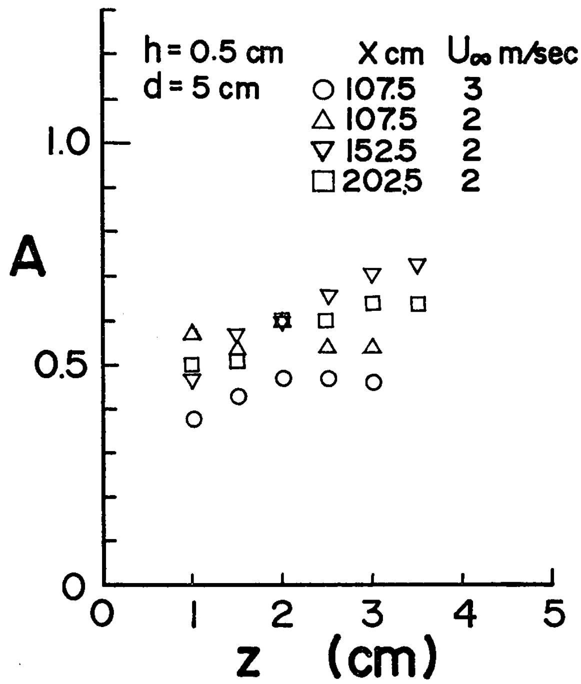

Fig. 11. Values of Kolmogorov constant A estimated at each height under the conditions shown in the upper side of the figure.

4. Summary

The results obtained in the present experiments are summarized as follows:

1) Nondimensional energy spectra vary with x gradually in the developing stage of turbulence, that is,

(i) the value of spectrum $G(n)$ at the peak frequency f_{max} decreases with x,
(ii) the values of spectrum in both lower and higher frequency regions increase with x,

but do not in the fully developed one and tend to take an invariable shape.

2) The effect of the first strip remains to the wide streamwise region even in the case of regularly arrayed strips.

3) The shape of the generalized energy spectrum of u component of turbulence in the fully developed region in the wind tunnel is similar to that of w component in the atmosphere.

4) Kolmogorov constant cannot be definitely determined from the present experiment, although an average value of 0.55 is estimated. Further study about this problem will be necessary.

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