An Empirical Formula for the Thickness of the Turbulent Boundary Layer over Rough Surface (Wind Tunnel Experiment)

Shigeru Nemoto, Kaoru Takenouchi* and Akiko Toyoda*

Environmental Science, Faculty of Science, Ochanomizu University, Tokyo

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§ 1. Introduction.

It seems that the thickness of a turbulent boundary layer formed over the rough surface which is artificially made with windbreaks is a function of roughness parameter, distance between windbreaks and distance from leading edge of the windbreak. On the other hand, the roughness parameter is also a function of windbreak height, distance between windbreaks, distance from leading edge of the windbreak and uniform velocity of a wind tunnel, in the downstream region near the leading edge of the windbreak.

Many experimental and theoretical studies about the turbulent boundary layer including the internal boundary layer problem have been carried out, but in many cases there is a large enough fetch in the tunnel to avoid leading edge effects. Recently, Sugawara et al. (1978) have examined variation of the thickness of the turbulent boundary layer with downstream distance but they do not deal with the downstream region near the leading edge as we do. Iqbal et al. (1977) have dealt with mainly the roughness effects of multiple windbreaks. It seems that no practical expression of the boundary-layer thickness in the downstream region near the leading edge has been found.

Then we try, in the present paper, to express the boundary-layer thickness in the downstream region near the leading edge ($x/z_0 < 10^3$) by an empirical formula under the condition of neutral state.

§ 2. Experimental procedure.

The basic facility consists of a small wind tunnel of the suction type (Fig. 1). Its test section is 30 cm high, 30 cm wide and 3 m long.

Three kinds of height of aluminum strips (celluloid was used only for h=0.5 cm) were used for the windbreaks, each height (h) of these strips was 5, 10 and 13 mm, respectively.

* Department of Physics.

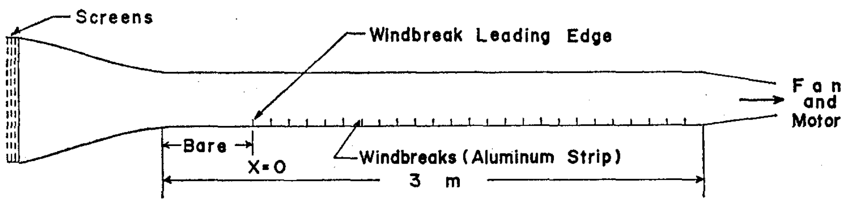

Fig. 1. Outline of the wind tunnel: test-section, length 3 m, width 30 cm, height 30 cm.

Three kinds of distance between strips, that is, d=5, 10 and 15 cm were adopted. Hence spacing ratio d/h ranges from 3.8 to 30.

Four kinds of wind velocity (U_∞=1, 2, 3, 4 m/sec, approximately) were selected. Experiments have been carried out by combination of the above mentioned conditions.

All velocity data were obtained by constant-temperature hot-wire anemometry. That is, the hot-wire (diameter: 5 μ, length: about 2 mm) was traversed vertically (speed of traverse is 18 mm/min) and the output of the anemometer was recorded on a chart. This is a trace of fluctuating wind velocity, so the curve which passes through the center of fluctuating trace was drawn by the eye and mean wind velocities at each height were obtained from this averaged curve. The vertical distribution of mean wind velocities obtained from the above-mentioned method was compared with that obtained from averaged values (averaging time: 2 min) at each height of 1, 2, 3, 4, 5, 6, 8 and 9 cm in Fig. 2. In this case, the experimental condition is as follows: h=5 mm, d=10 cm, distance downwind from the leading edge x=235 cm (d/h=20). It is seen from the figure that the two cases coincide with each other very well. Hence, we adopted this traversing method in the present experiment.

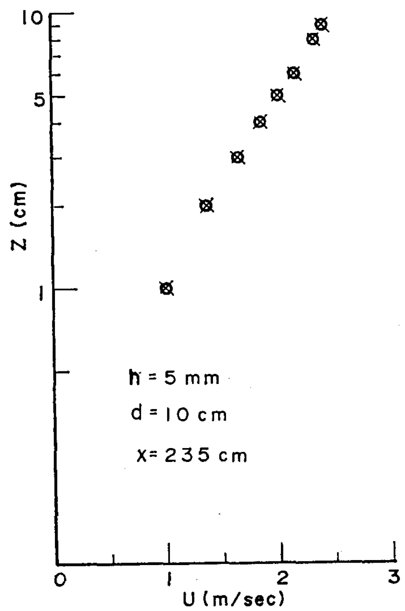

Fig. 2. Comparison of the two methods of measurement of mean wind velocity.
○: Traversing method
×: Averaged value for 2 min

In order to avoid the effect of vortices from individual barriers (strips), velocity traverses were taken only in the range of height (z) about $1.5\,h<z\leqq\delta$. Velocity data for $3\,h<z$ were employed to obtain values of roughness parameter z_0 and friction velocity v_*. But no logarithmic wind profile was obtained at a distance shorter than x=35 cm.

The boundary-layer thickness δ was estimated from the velocity traces. That is, the height at which the turbulence could not be recognized was adopted as δ. As the boundary is changing intermittently, some errors are unavoidable in this traversing method. The vertical distribution of mean wind velocity was measured at the middle position of every distance between windbreaks.

§ 3. Experimental results.

Variations of the boundary-layer thickness $\delta(x)$ with x obtained from the experiments are shown in Figs. 3, 4 and 5, where x indicates a distance downwind from the leading edge of windbreak as shown in Fig. 1. The ordinate shows non-dimensional boundary-layer thickness δ/z_0 and the abscissa non-dimensional downstream distance x/z_0, where z_0 means roughness parameter. The curves shown in these figures are those calculated from the

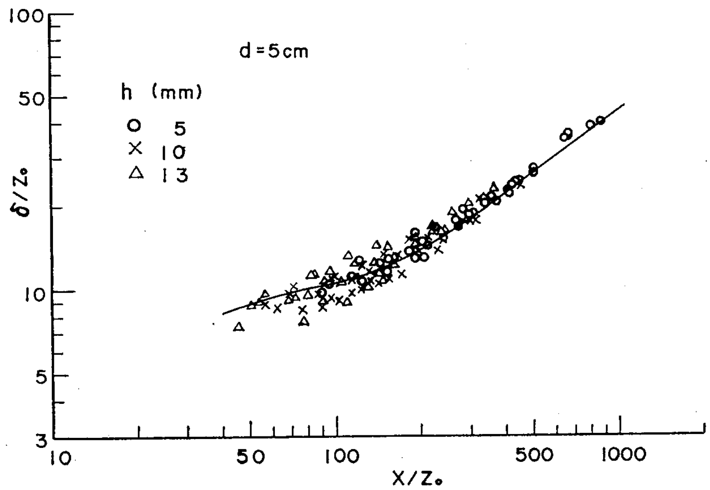

Fig. 3. Variation of the thickness of the boundary-layer with distance downwind from the leading edge for the case of $d=5$ cm ($h=5$, 10, 13 mm).

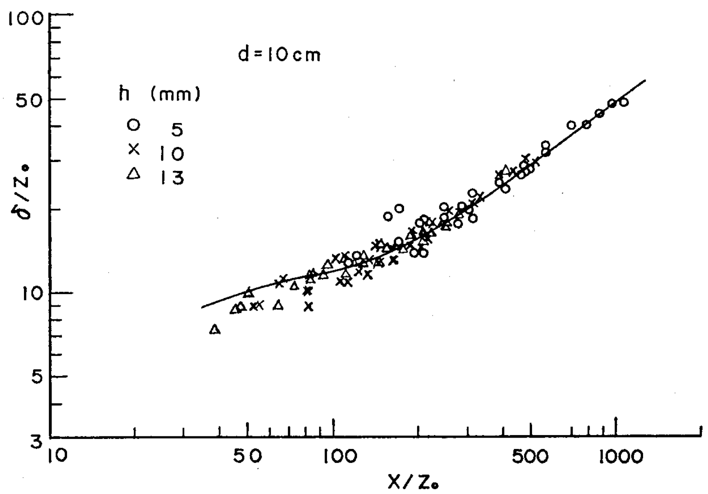

Fig. 4. Variation of the thickness of the boundary-layer with distance downwind from the leading edge for the case of $d=10$ cm ($h=5$, 10, 13 mm).

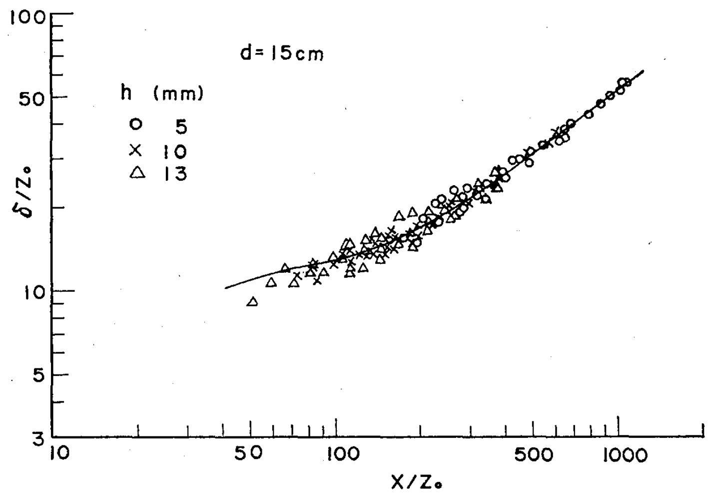

Fig. 5. Variation of the thickness of the boundary-layer with distance downwind from the leading edge for the case of $d=15$ cm ($h=5$, 10, 13 mm).

empirical formula to be described later in the paper.

It is evident from these figures that the curves express the experimental results very well and deviate from the straight line in the region of smaller x ($x/z_0<2\times10^2$, approximately). It is also seen from these figures that the curve shifts on the whole upward (toward increasing direction of δ/z_0) with increase of d.

§ 4. Empirical formula.

It is known that the velocity profile in the turbulent boundary layer over the smooth flat plate is expressed by the power law as given in the following form,

$$\frac{u}{U_\infty}=\left(\frac{z}{\delta}\right)^{1/7} \quad (u \text{ is wind velocity at a height of } z)$$

and if the turbulent boundary layer generates from the leading edge of the flat plate $\delta(x)$ is given as follows:

$$\delta(x)=0.37\left(\frac{\nu}{U_\infty}\right)^{1/5}x^{4/5} \tag{1}$$

Then, by using vertical eddy viscosity coefficient

$$K_z=\kappa v_* z_0 \quad (\kappa=0.4, \text{ Kármán constant}) \tag{2}$$

in place of molecular viscosity coefficient ν in the above-mentioned formula and further by dividing the both sides by z_0, Equation (1) is transformed into

$$\frac{\delta(x)}{z_0}=\alpha\kappa^{1/5}\left(\frac{v_*}{U_\infty}\right)^{1/5}\left(\frac{x}{z_0}\right)^{4/5} \tag{3}$$

This expression has already been obtained by Elliott (1958), but it is here derived in a different way, where α indicates an empirical constant. This expression is applicable to the region of larger x.

In the present paper, we adopted a method of correcting Equation (3) so as to fit it well to the experimental results mentioned above. Following three corrections were made, that is,

1) correction for application to the region of smaller x,
2) correction for variation of v_*/U_∞ with x,
3) correction for variation of δ with d.

More detailed explanation on these corrections will be described in the following,

1) Figs. 3, 4 and 5 show that δ/z_0 deviates from the straight line which is applicable only to larger x, with decrease of x/z_0. The term

$$\{1+\beta e^{-\gamma(x/z_0)}\} \tag{4}$$

is supplemented for expressing this trend, where β and γ are empirical constants.

2) Fig. 6 shows variation of v_*/U_∞ with x. It is seen from this figure that the relation

$$\frac{v_*}{U_\infty}=0.57\left(\frac{x}{z_0}\right)^{-1/4} \tag{5}$$

that is,

$$\left(\frac{v_*}{U_\infty}\right)^{1/5}=0.89\left(\frac{x}{z_0}\right)^{-0.05}$$

holds between v_*/U_∞ and x/z_0.

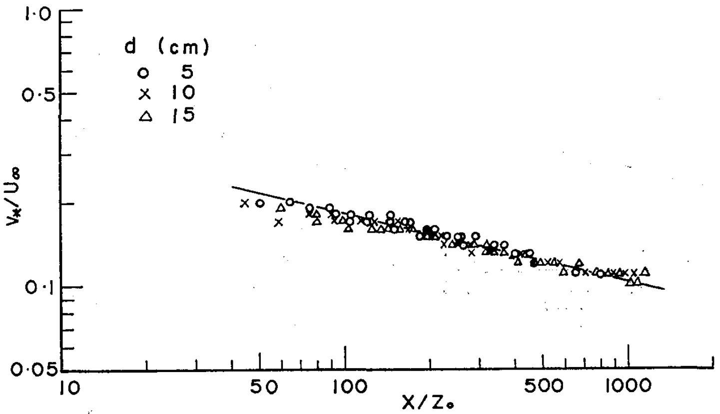

Fig. 6. Variation of non-dimensional friction velocity v_*/U_∞ with non-dimensional downstream distance x/z_0 (d=5, 10, 15 cm).

3) As seen from comparison of Figs. 3, 4 and 5, δ/z_0 shifts upward parallel on the whole with increase of the distance between barriers (d), then the term

$$0.9\left(\frac{d}{\Delta d}\right)^{0.18} \tag{6}$$

is supplemented for expressing this trend, where Δd is difference in the distance between barriers (d) and taken as 5 cm in the present experiments.

That is, three kinds of d (=5, 10 and 15 cm) are adopted. Hence, values of $d/\Delta d$ are 1, 2 and 3, respectively.

The following formula will be obtained by giving the above-mentioned three corrections, that is, 1), 2) and 3) to Equation (3), and by deciding each empirical constant,

$$\frac{\delta(x)}{z_0}=0.243\left(\frac{d}{\Delta d}\right)^{0.18}\left(\frac{x}{z_0}\right)^{3/4}\{1+2.43e^{-0.019(x/z_0)}\} \tag{7}$$

This is the empirical formula to be obtained.

But it is further necessary to obtain the relationship between z_0 and x, in order to estimate $\delta(x)$ at a downstream distance x from this formula. Because z_0 varies with x and other factors in the region of smaller x.

§5. Estimation of roughness parameter z_0

From the experimental results, values of z_0 in each case change with x in the region of smaller x, that is, z_0 increases with x to $x=x_0$ (x_0=97.5 cm in the present cases), becomes maximum there and decreases from then and takes a constant value ($z_{0\infty}$) for x larger than x' (x'=216.5 cm in the present cases).

Therefore, it was tried at first to find the relationship among $z_{0\infty}$, h and d for the region larger than x' from the experimental results. As a result, the equation

$$\left\langle\frac{h}{z_{0\infty}}\right\rangle_h=4.05\left(\frac{d}{x_0}\right)^{1/4} \tag{8}$$

was obtained as shown in Fig. 7, where $\langle\,\rangle_h$ indicates mean value with respect to h. This relationship holds independent of wind velocity. Because the flow is in equilibrium with the roughness in this region.

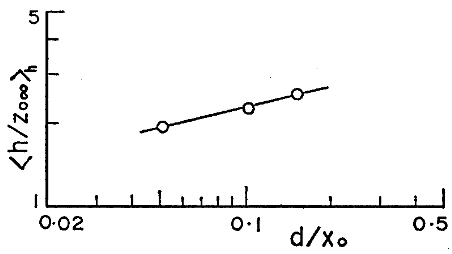

Fig. 7. Relationship between $\langle h/z_{0\infty}\rangle_h$ and d/x_0.

It is expected that z_0 is a function of h, d, U_∞ and x in the region of relatively shorter distance from the leading edge. Hence it was tried to find this functional form experimentally from the experimental results. At first, ξ and η are defined as follows:

$$\xi=\frac{h}{z_{0\infty}}\left(\frac{x_0}{d}\right)^{1/4}-\frac{h}{z_0}\left(\frac{x_0}{d}\right)^{1/4}$$

$$=h\left(\frac{x_0}{d}\right)^{1/4}\left(\frac{1}{z_{0\infty}}-\frac{1}{z_0}\right) \tag{9}$$

$$\eta=\frac{x-x_0}{x_0} \tag{10}$$

and then ζ is defined as $\zeta=\langle\langle\xi\rangle_h\rangle_d$, where $\langle\langle\,\rangle_h\rangle_d$ indicates to take average value with respect to both h and d. Fig. 8 shows the relation between ζ and η. In the figure,

○ indicates the case of $\bar{U}_\infty=1.16$ m/sec
× indicates the case of $\bar{U}_\infty=2.00$ m/sec
△ indicates the case of $\bar{U}_\infty=3.12$ m/sec
+ indicates the case of $\bar{U}_\infty=4.10$ m/sec

and $\bar{U}_\infty$ indicates the average value of 60 of U_∞ in each case of tunnel velocities of about 1, 2, 3 and 4 m/sec.

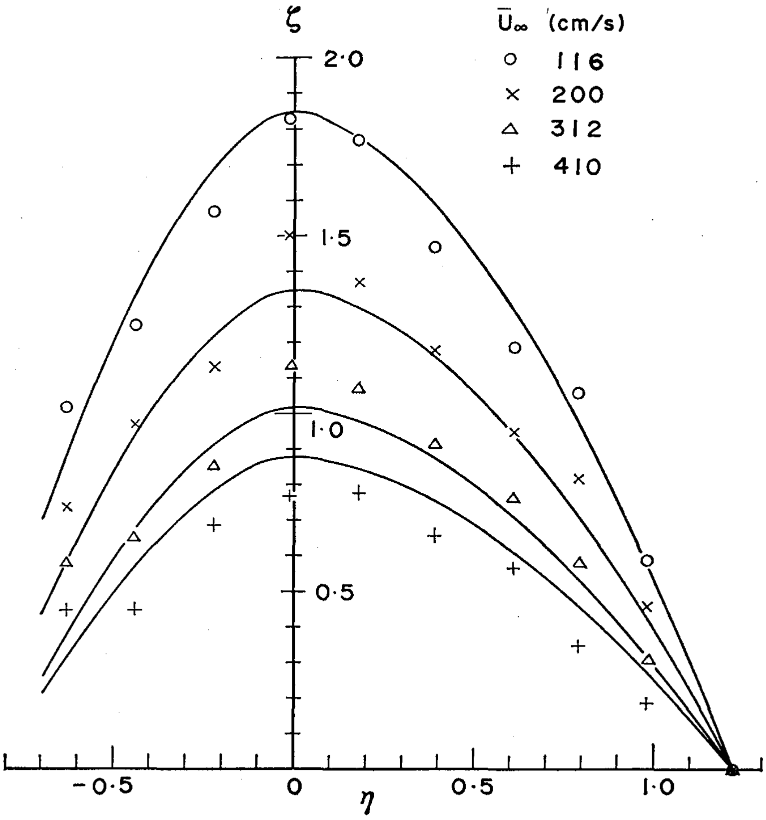

Fig. 8. Relationship between ζ and η for each case of mean wind velocity ($\bar{U}_\infty$) of 116, 200, 312 and 410 cm/sec, respectively.

Then, the empirical formula which expresses these experimental results may be obtained as follows:

1) $\zeta=(2.02-1.44\eta^{1.7})\left(\frac{U_\infty}{U_s}\right)^{-0.6}$ for $0\leqq\eta\leqq1.22$

where U_s indicates a reference wind velocity and is taken as $U_s=1$ m/sec in the present experiment, that is, empirical constants 2.02 and 1.44 are determined to satisfy $U_s=1$ m/sec. Hence U_s may be omitted from the expression. That is, ζ may be expressed as follows:

$$\zeta=(2.02-1.44\eta^{1.7})U_\infty^{-0.6} \tag{11}$$

2) $\zeta=0$ for $1.22\leqq\eta$ (12)

3) $\zeta=2.02U_\infty^{-0.6}-1.06U_\infty^{-0.42}|\eta|^{1.7}$ for $-0.63\leqq\eta\leqq0$ (13)

where U_s is also omitted in cases of 2) and 3). The results calculated by the equations (11) and (13) are shown by the curves for each case of velocities ($\bar{U}_\infty$) in the figure. Experimental values coincide with calculated ones on the whole except for the case of $\bar{U}_\infty=4.10$ m/sec, in which experimental values are somewhat different from calculated ones.

Next, equations (11), (12) and (13) are rewritten as follows:

1) $$\frac{1}{z_0}=\frac{1}{h}\left(\frac{d}{x_0}\right)^{1/4}\left[4.05-2.02U_\infty^{-0.6}+1.44U_\infty^{-0.6}\left(\frac{x-x_0}{x_0}\right)^{1.7}\right] \quad \text{for } x_0\leqq x\leqq x' \tag{14}$$

2) $$\frac{h}{z_{0\infty}}=4.05\left(\frac{d}{x_0}\right)^{1/4} \quad \text{for } x'\leqq x \tag{8}$$

3) $$\frac{1}{z_0}=\frac{1}{h}\left(\frac{d}{x_0}\right)^{1/4}\left[4.05-2.02U_\infty^{-0.6}+1.06U_\infty^{-0.42}\left|\frac{x-x_0}{x_0}\right|^{1.7}\right]$$

$$\text{for } 36.1\text{ cm}\leqq x\leqq x_0 \tag{15}$$

where the reference velocity U_s is omitted in equations (14) and (15). That is, the value of z_0 must be estimated for each region of x, as mentioned above. In the present experiment, $x_0=97.5$ cm, $x'=216.5$ cm.

Further, for larger x (216.5 cm$<x$) Equation (7) is reduced to

$$\frac{\delta(x)}{z_{0\infty}}=0.243\left(\frac{d}{\varDelta d}\right)^{0.18}\left(\frac{x}{z_{0\infty}}\right)^{3/4}$$

that is, $\delta(x)$ is proportional to $x^{3/4}$.

On the other hand, as previously mentioned, the thickness of the turbulent boundary layer over the flat plate is expressed by Equation (1), therefore, $\delta(x)$ is proportional to $x^{4/5}$.

It was impossible to confirm whether $\delta(x)$ was proportional to $x^{3/4}$ or to $x^{4/5}$ in the wind tunnel used in the present experiment as the length of the tunnel was only 3 m long. Therefore, the results of comparison by using the observational data in the field obtained by Bradley (1968) are shown in Fig. 9 (cf. Fig. 11 in Bradley's paper. Comparison of the modified region with boundary layer growth over a flat plate. But the comparison was made only for the case of tarmac-spikes transition, that is, smooth-rough transition). It is difficult to find difference in the two formulas from this figure. In the figure the mark □ indicates value observed on 26, Aug.,

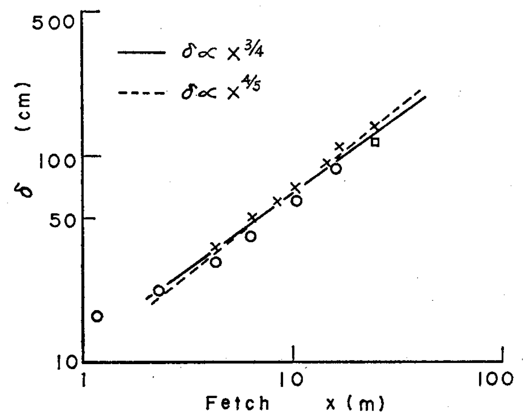

Fig. 9. Variation of the thickness of the internal boundary layer with fetch (x) for the case of tarmac-spikes transition. (After Bradley's observation: □ 26, Aug., × 27, Aug., ○ 30, Aug., 1964)

the mark $\times$ those on 27, Aug., the mark $\bigcirc$ those on 30, Aug., respectively.

§ 6. Discussion.

The difference $(\delta/z_0)_{\mathrm{cal}}-(\delta/z_0)_{\mathrm{obs}}$ for each x/z_0 was obtained and

$$\sigma=\left\langle\left[\left(\frac{\delta}{z_0}\right)_{\mathrm{cal}}-\left(\frac{\delta}{z_0}\right)_{\mathrm{obs}}\right]^2\right\rangle^{1/2}$$

was estimated for each d, where $(\delta/z_0)_{\mathrm{cal}}$ is calculated non-dimensional boundary-layer thickness and $(\delta/z_0)_{\mathrm{obs}}$ is observed one. From the results of estimation of the above-mentioned σ,

$$\sigma=1.2 \quad \text{for } d=5\,\mathrm{cm}$$

$$\sigma=1.4 \quad \text{for } d=10\,\mathrm{cm}$$

$$\sigma=1.2 \quad \text{for } d=15\,\mathrm{cm}$$

were obtained, where $\langle\,\rangle$ means that average is taken over all the experimental data, hence σ indicates a kind of standard deviation.

The roughness parameter z_0 gives maximum value at $x=97.5\,\mathrm{cm}$, but its reason cannot be found. It is planned to make its cause clear in future by examining the turbulent structure of the flow. The limits of application of the empirical formula proposed in the present paper is not confirmed clearly, but it may be considered to be valid at least under the conditions dealt with here. Further experimental tests will be necessary to establish its universality.

§ 7. Summary.

The results obtained are summarized as follows:
when d, h, Δd and U_∞ are given,

1) z_0 is estimated from Equation (15) for x in the region of

$$36.1\,\mathrm{cm}\leqq x\leqq x_0$$

2) z_0 is estimated from Equation (14) for x in the region of

$$x_0\leqq x\leqq x'$$

3) z_0 is equal to $z_{0\infty}$ from Equation (8) for x in the region of

$$x'\leqq x$$

therefore, from Equation (7), that is,

$$\frac{\delta(x)}{z_0}=0.243\left(\frac{d}{\Delta d}\right)^{0.18}\left(\frac{x}{z_0}\right)^{3/4}\{1+2.43e^{-0.019(x/z_0)}\}$$

the value of δ at a downstream distance x may be estimated.

In the present experiment, Δd was given as follows:

$$\Delta d=5\,\mathrm{cm}, \text{ that is, } d/\Delta d=1,\ 2,\ 3$$

and x_0, x' were obtained as follows:

$$x_0=97.5\ \mathrm{cm}$$

$$x'=216.5\ \mathrm{cm}$$

Further,

$$\frac{\delta(x)}{z_{0\infty}}=0.243\left(\frac{d}{\Delta d}\right)^{0.18}\left(\frac{x}{z_{0\infty}}\right)^{3/4}$$

for x in the region of $216.5\ \mathrm{cm}\leqq x$ and $\delta(x)$ is proportional to $x^{3/4}$.

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References

Bradley, E. F., 1968: A Micrometeorological Study of Velocity Profiles and Surface Drag in the Region Modified by a Change in Surface Roughness. Quart. J. Roy. Met. Soc., 94, 361-379.

Elliott, W. P., 1958: The Growth of the Atmospheric Internal Boundary Layer. Trans. Amer. Geophys. Union, 39 (6), 1048-1054,

Iqbal, M., Khatry, A. K. and Seguin, B., 1977: A Study of the Roughness Effects of Multiple Windbreaks. Boundary-Layer Met., 11 (2), 187-203.

Sugawara, K., Kitabayashi, K. and Yokoyama, O., 1978: Wind Tunnel Simulation of the Atmospheric Neutral Boundary Layer (Part I). Pollution Control, 13 (5), 321-329.