A remark on the p-radonifying maps

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(Received, March 28, 1978)

In this short note, we shall investigate the connection between p-radonifying maps (cf. [2]) and $(*)_p$-conditions. The $(*)_p$-condition was introduced by Y. Takahashi (cf. [5]) and acts an important role in the measure theory in infinite dimensional spaces.

Throughout this note, let E, F be Banach spaces, E' the topological dual space of E and $L(E, F)$ the Banach space of all continuous linear maps from E into F. Next we shall introduce some symbols as special subsets of $L(E, F)$. We denote by $\Pi_p(E, F)$ the set of all p-absolutely summing maps, by $R_p(E, F)$ the set of all p-radonifying maps and by $UD_p(E, F)$ the set of all universally p-decomposable maps.

Remark 1. We consider only the case of $1 \leq p < \infty$.

Remark 2. We shall add a few more words in explanation of the word "universally". Let Ω be a topological Hausdorff space, μ a Radon probability on Ω and $L^P(\Omega, \mu)$ be the space of all μ-measurable functions on Ω, $f(\omega)$, such that

$$\left(\int_\Omega |f(\omega)|^P d\mu\right)^{1/P} < \infty.$$

Then we say that u is a universally p-decomposable map if for every $\gamma \in L(F, L^P(\Omega, \mu))$, $\gamma \circ u$ is always a p-decomposable map. (cf. [1])

Lemma 1. (cf. [2] and [3]) *If u is p-radonifying, then u is p-absolutely summing.*

Lemma 2. (cf. [2] and [3]) *If $1 < p < \infty$, then $R_p(E, F) = \Pi_p(E, F)$.*

Lemma 3. *If E has a $(*)_p$-condition, then $R_p(E, F) \subset UD_p(E, F)$ for every Banach space F.*

Proof. By Lemma 1, $R_p(E, F) \subset \Pi_p(E, F)$. For every $u \in R_p(E, F)$ we have $u^* \in \Pi_p(F', E')$ where u^* is the adjoint operator of u, because E has a $(*)_p$-condition. By [1], we can conclude that $u \in UD_p(E, F)$.

Theorem. *Let E be a reflexive Banach space and $1 < p < \infty$. Then* *the following are equivalent:*

(1) *E has a $(*)_p$-condition;*

(1)′ $u \in \Pi_p(E, l^p)$ implies $u^* \in \Pi_p(l^q, E')$ *and vice versa, where* $1/p+1/q=1$;

(2) $R_p(E, F) \subset UD_p(E, F)$ *for every Banach space* F;

(2)′ $R_p(E, l^p) \subset UD_p(E, l^p)$;

(2)″ $R_p(E, l^p) = UD_p(E, l^p)$.

PROOF. We shall give the proof in the following order:

1. (1) $\Longleftrightarrow$ (1)′ $\Rightarrow$ (2)″ $\Rightarrow$ (2)′ $\Rightarrow$ (1);
2. (1) $\Rightarrow$ (2) $\Rightarrow$ (2)′ $\Rightarrow$ (1).

The equivalence of (1) and (1)′ is obviously verified using the character of l^q which has a $(*)_p$-condition. Next let us show that (1)′ implies (2)″. The first half of (1)′ is equivalent to the definition of a $(*)_p$-condition, therefore we have $R_p(E, l^p) \subset UD_p(E, l^p)$. Now let u be an arbitrary element of $UD_p(E, l^p)$. To conclude the proof it is sufficient to see that u^* is included in $R_p(l^q, E')$. Given a cylindrical measure λ of type p on l^q, there corresponds a continuous linear random function $f: l^p \longrightarrow L^p(\Omega, \mu)$ associated with λ. By the assumption, the composition $f \circ u^{**} = f \circ u$ is a p-decomposable map. The random function $f \circ u$ induces the cylindrical measure $u^*(\lambda)$, and moreover the p-decomposability of $f \circ u$ implies that $u^*(\lambda)$ is a Radon probability of order p on E' (cf. [2] and [4]). This means $u^* \in R_p(l^q, E')$. It is trivial that (2)″ implies (2)′, and also it is easily seen that (2)′ implies (1) by the above conclusion, that is, $u \in UD_p(E, l^p)$ implies $u^* \in R_p(l^q, E')$.

Thus we can complete the proof. Indeed, in order to conclude the second part, it is sufficient to see that (1) implies (2). However it is the very result of Lemma 3.

References

1) S. Kwapień, *On a theorem of L. Schwartz and its applications to absolutely summing operators*, Studia Math. **38** (1970), 193-201.
2) L. Schwartz, *Séminaire L. Schwartz* 1969-1970, École Polytechnique.
3) L. Schwartz, *Seminar Schwartz*, Notes on Pure Mathematics 7 (1973), 1-64.
4) L. Schwartz, *Radon measures on arbitrary topological spaces and cylindrical measures*, Oxford University Press, 1973.
5) Y. Takahashi, *Some remarks on p-absolutely summing operators*, Hokkaido Math. J. **5** (1976), 308-315.