Natural Science Report, Ochanomizu University, Vol. 26, No. 2, 1975 63
お茶の水女子大学 自然科学報告 第26巻 第2号

Static Model Calculation of Pion-Nucleon Scattering

Takashi Itoh

Department of Physics, Faculty of Science
Ochanomizu University, Tokyo

(Received September 12, 1975)

The p-wave pion-nucleon scattering phase-shifts are computed by the Chew-Low static model for pion incident energy of 0–300 MeV. The square of the unrenormalized coupling constant is taken to be $f^2=0.2$, and the cutoff is made at $k_{\max}=6\,\mu$. The computed 3, 3 phase-shift passes through 90° about at the right energy. The other phase-shifts computed are small in rough agreement with experiment.

1. Introduction

In 1956 Chew and Low[1,2] developed a static model of pion-nucleon interaction, and by the analysis of the pion-nucleon scattering and photopion production data they determined the coupling constant and the cutoff parameter. Their method was to derive some interrelations between the scattering and production amplitudes and the cross sections and to compare them with experiment.

The numerical calculations of the pion-nucleon scattering in the static model have also been carried out by several workers.[3-5] The calculations of the p-wave phase-shifts by Chew[3] and by Salzman and Snyder[4] are based on the perturbation expansion of the transition matrix. The applicability of the perturbation method, however, is not free from a question because the coupling constant is not small. G. Salzman and F. Salzman[5] solved the Low equation for the pion-nucleon scattering in the one-pion approximation. The Low equation involves the cross sections at all energies, and since the pion production is not negligible at higher energies, the one-pion approximation may not be quite legitimate.

In view of the above situation, it appears to be worthwhile to calculate the pion-nucleon phase-shifts in the static model by the Tamm-Dancoff method.[6,7] This is undertaken in the present paper.

2. Chew-Low model

The Chew-Low static model of pion-nucleon interaction is based on the Hamiltonian*)

*) We use units $\hbar=1$, $c=1$.

$$H = H_0 + H_I \,, \tag{1}$$

where

$$H_0 = \sum_k \omega_k a_k^\dagger a_k \,, \tag{2}$$

$$H_I = \sum_k (V_k a_k + V_k^\dagger a_k^\dagger) \,, \tag{3}$$

and

$$\omega_k = \sqrt{\mu^2 + k^2} \,, \tag{4}$$

$$V_k = N\sqrt{4\pi}\,\frac{f}{\mu}\,\tau_\lambda\, i\boldsymbol{k}\cdot\boldsymbol{\sigma}\,\frac{v(k)}{\sqrt{2\omega_k}} \,. \tag{5}$$

H_0 is the kinetic energy plus the rest energy of the pions and H_I is the interaction term. a_k and $a_k^\dagger$ are the annihilation and creation operators of the pion. $k=\boldsymbol{k}, \lambda$ designates the momentum $\boldsymbol{k}$ and the isospin state λ of the pion. $\boldsymbol{\sigma}$ and τ_λ are the spin and isospin operators of the nucleon. The nucleon is treated as a fixed source. f is the dimensionless coupling constant. μ is the pion mass. $v(k)$ is the cutoff function, and $N=(\text{quantization volume})^{-\frac{1}{2}}$

The pion-nucleon scattering is described by the state vector

$$|p\alpha+\rangle = a_p^\dagger |\alpha\rangle + \chi \,, \tag{6}$$

where

$$\chi = (E_p + i\eta - H)^{-1} V_p |\alpha\rangle \,, \tag{7}$$

and

$$E_p = \omega_p + E_s \,. \tag{8}$$

$|\alpha\rangle$ is the real nucleon state and E_s is the energy of this state. η is an infinitesimal positive quantity. $a_p^\dagger|\alpha\rangle$ in (6) is the incident wave part and χ is the scattered wave part.

We have to solve the equation

$$(E_p + i\eta - H)\chi = V_p |\alpha\rangle \tag{9}$$

for χ. For this purpose we expand χ in a complete set of state vectors. An obviously complete set is $|\beta)$, $a_q^\dagger|\beta)$, $a_q^\dagger a_r^\dagger|\beta), \cdots$, where $|\beta)$ is the bare nucleon state. This set is inconvenient for our purpose, because it has no particular relation to the real nucleon state. Another set $|\beta\rangle$, $a_q^\dagger|\beta\rangle$, $a_q^\dagger a_r^\dagger|\beta\rangle, \cdots$ appears to be attractive, but whether this set is complete or not is not known. As a compromise, we adopt here the complete set $|\beta\rangle$, $a_q^\dagger|\beta\rangle$, $a_q^\dagger a_r^\dagger|\beta)$, $a_q^\dagger a_r^\dagger a_s^\dagger|\beta), \cdots$.

3. Spherical wave representation

According to (3) and (5) the nucleon interacts only with p-wave pions, so that it is convenient to employ the spherical wave representation. Actually we need only the p-wave part of H_0. Omitting the other parts we can write the Hamiltonian as

$$H = \sum_{i,\lambda} \int dk \left[\omega_k a_{i\lambda}^\dagger(k) a_{i\lambda}(k) + V(k)\,\sigma_i \tau_\lambda \left\{a_{i\lambda}(k) + a_{i\lambda}^\dagger(k)\right\}\right] , \tag{10}$$

where
$$V(k)=\frac{f}{\mu}\frac{k^2 v(k)}{\sqrt{3\pi\omega_k}}\,. \tag{11}$$

$a_{i\lambda}(k)$ and $a_{i\lambda}^{\dagger}(k)$ are the annihilation and creation operators of the p-wave pion. k is the magnitude of the momentum. i, λ are the angular momentum and isospin indices, each of which takes on three values x, y, z. Concerning the interaction term, the above transformation—the transformation from the plane wave representation to the spherical wave representation—is achieved by

$$\sum_k \to \frac{1}{(2\pi)^3 N^2}\int k^2\, dk\, d\Omega\,, \tag{12}$$

$$\int \boldsymbol{e}_i\cdot\boldsymbol{k}\, a_{\boldsymbol{k},\lambda}\, d\Omega \to -i\sqrt{\frac{4\pi}{3}}\,(2\pi)^{\frac{3}{2}} N\, a_{i\lambda}(k)\,, \tag{13}$$

where Ω is the solid angle spanned by the direction of $\boldsymbol{k}$, and $\boldsymbol{e}_i$ is the unit vector along the axis i.

4. Nucleon ground state

Before proceeding to the problem of pion-nucleon scattering, a brief explanation is to be given concerning the nucleon ground state, *i.e.*, the real nucleon state. The nucleon ground state has been investigated by several authors.[8-10] Although the exact solution is not obtainable in a closed form, rather good approximations have been devised. In the present paper we use the approximate solution given by the present author in a previous paper.[10] In this approximation method, we first introduce the reduced Hamiltonian H_r which is defined by

$$H_r=\Omega_0\Big[\sum_{i,\lambda} a_{i\lambda}^{\dagger}\, a_{i\lambda}+\sqrt{I}\,(A+A^{\dagger})\Big]\,, \tag{14}$$

where
$$a_{i\lambda}=\int f(k)\, a_{i\lambda}(k)\, dk\,, \tag{15}$$

$$A=\sum_{i,\lambda}\sigma_i\,\tau_\lambda\, a_{i\lambda}\,, \tag{16}$$

$$\Omega_0=\int \omega_k f^2(k)\, dk\,, \tag{17}$$

$$\sqrt{I}=\Omega_0^{-1}\int V(k) f(k)\, dk\,, \tag{18}$$

and $f(k)$ is a real function normalized according to

$$\int f^2(k)\, dk=1\,. \tag{19}$$

Instead of directly handling H the minimization of the expectation value of H_r is tried, regarding $f(k)$ as a variational function. A good approximation to the nucleon ground state vector has thus been obtained in the form

$$|\alpha\rangle=(b_0-b_1 A^\dagger+b_2 A^{\dagger 2}-\cdots)|\alpha), \tag{20}$$

$$f(k)=\text{const.}\times k\,v(k), \tag{21}$$

where $b_0, b_1, b_2, \cdots$ are constants. In the present paper the rather simple form

$$|\alpha\rangle=(b_0-b_1 A^\dagger+b_2 A^{\dagger 2})|\alpha) \tag{22}$$

is adopted, because we can not expect a very accurate prediction by the present simple model in any case. As in the previous paper the unrenormalized coupling constant is fixed at $f^2=0.2$, and the cutoff is made at $k_{\max}=6\,\mu$. The optimum values of b_0, b_1 and b_2 are $b_0=0.685$, $b_1=0.218$, $b_2=0.034$.*)

5. Pion-nucleon scattering

In Sec. 2 we have briefly described the theory of pion-nucleon scattering using the plane wave representation. Now we consider the same problem in the spherical wave representation. We need to consider only the p-wave scattering. This is described by the state vector

$$\psi=a_{i\lambda}^\dagger(p)|\alpha\rangle+\chi, \tag{23}$$

where the first term $a_{i\lambda}^\dagger(p)|\alpha\rangle$ represents the incident wave and the second term χ represents the scattered wave. The scattered wave amplitude χ is to be determined by solving the equation

$$(E_p+i\eta-H)\chi=V(p)\,\sigma_i\,\tau_\lambda|\alpha\rangle. \tag{24}$$

In the present representation the statement made at the end of Sec. 2 means that χ is expanded in $|\beta\rangle$, $a_{j\mu}^\dagger(q)|\beta\rangle$, $a_{j\mu}^\dagger(q)\,a_{k\nu}^\dagger(r)|\beta)$, $a_{j\mu}^\dagger(q)\,a_{k\nu}^\dagger(r)\,a_{l\kappa}^\dagger(s)|\beta)$, $\cdots$.

We now show that in a certain approximation χ can be expressed as a linear combination of

$$\begin{aligned}
&\phi_1=|\beta\rangle,\\
&\phi_2=\int dq\,\frac{f(q)}{\omega_p+i\eta-\omega_q}\,a_{j\mu}^\dagger(q)|\beta\rangle,\\
&\phi_3=a_{j\mu}^\dagger a_{k\nu}^\dagger|\beta),\\
&\phi_4=a_{j\mu}^\dagger a_{k\nu}^\dagger a_{l\kappa}^\dagger|\beta),\\
&\cdots\cdots\cdots\cdots\cdots,
\end{aligned} \tag{25}$$

where $f(q)$ is the function defined by (21) and (19), and $a_{j\mu}^\dagger$ is the operator defined by (15). First, we consider the operation of $(E_p+i\eta-H)$ on ϕ_1 and ϕ_2. Since $|\beta\rangle$ is the eigenvector of H with the eigenvalue E_s, we have

*) The b_j's are related to the c_j's of ref. 10 by $b_j=(-1)^j c_j/\sqrt{N_j}$.

$$(E_p+i\eta-H)\,\phi_1=\omega_p\,|\,\beta\,\rangle\,, \tag{26}$$

$$(E_p+i\eta-H)\,\phi_2=a^\dagger_{j\mu}\,|\,\beta\,\rangle-K\sigma_j\,\tau_\mu\,|\,\beta\,\rangle\,, \tag{27}$$

where

$$K=\int\frac{f(q)\,V(q)}{\omega_p+i\eta-\omega_q}\,dq\,. \tag{28}$$

Next we let $(E_p+i\eta-H)$ operate on $\phi_3, \phi_4, \cdots$. We regard $\phi_3, \phi_4, \cdots$ as higher order terms and use the approximation

$$(E_p+i\eta-H)\,\phi_3=(E_p+i\eta-H_r)\,\phi_3\,,\ \text{etc.}. \tag{29}$$

This approximation will be relatively good for lower incident energies of the pion, because ϕ_3, etc. are not very important at lower energies. In this approximation we have

$$(E_p+i\eta-H)\,\phi_3=L\Omega_0\,a^\dagger_{j\mu}\,a^\dagger_{k\nu}\,|\,\beta)-M\Omega_0\,(\sigma_j\,\tau_\mu\,a^\dagger_{k\nu}$$
$$+\sigma_k\,\tau_\nu\,a^\dagger_{j\mu})\,|\,\beta)-M\Omega_0\,a^\dagger_{j\mu}\,a^\dagger_{k\nu}\,A^\dagger\,|\,\beta)\,,\ \text{etc.}, \tag{30}$$

where

$$L=(E_p-2\Omega_0)/\Omega_0\,,\qquad M=\sqrt{I}\,. \tag{31}$$

If we insert (22) into (26) and (27), the right hand sides become linear combinations of the terms of the form $a^\dagger_{j\mu}\,a^\dagger_{k\nu}\cdots|\,\beta)$. Also the right hand side of (30) consists of the terms of the same type. The same is true of the right hand side of (24) if (22) is inserted. Thus, in this approximation, Eq. (24) cah be solved by expressing χ as a linear combination of $\phi_1, \phi_2, \phi_3, \phi_4, \cdots$.

The above method will be applied in the following sections to the eigenstates of the total angular momentum and the total isospin. The eigenstate is designated by $2I$, $2J$, where I is the total isospin and J is the total angular momentum. The bare nucleon state is denoted by $|\,s\,t)$, where s and t denote the z-components of the spin and the isospin of the nucleon. The real nucleon state is denoted by $|\,s\,t\,\rangle$. The two values of $s=\pm\frac{1}{2}$ and $t=\pm\frac{1}{2}$ are denoted by $\pm$, so that the nucleon states are represented by $|\pm\pm)$ and $|\pm\pm\rangle$. The following notation is used:

$$\sigma_\pm=(\sigma_x\pm i\sigma_y)/2\,, \tag{32}$$

$$a_{\pm\lambda}=(a_{x\lambda}\mp ia_{y\lambda})/\sqrt{2}\,,\ \text{etc.}. \tag{33}$$

6. 3, 3 wave scattering

The 3, 3 state has a 16 fold degeneracy. We here consider the state with the z-components of the total angular momentum and the total isospin both equal to $+\frac{3}{2}$. The scattering is described by the state vector

$$\phi=a^\dagger_{++}(p)\,|++\rangle+V(p)\,\chi\,. \tag{34}$$

In the second term the factor $V(p)$ is ruled out for convenience. The equation to be satisfied by χ is

$$(E_p+i\eta-H)\,\chi=2\sigma_+\tau_+\,|\,+\,+\,\rangle\,. \tag{35}$$

Following the prescription of Sec. 5 we express χ as

$$\chi=c_1\phi_1+c_2\phi_2+c_3\phi_3\,, \tag{36}$$

where
$$\phi_1=\int dq\,\frac{f(q)}{\omega_p+i\eta-\omega_q}\,a^\dagger_{++}(q)\,|\,+\,+\,\rangle\,, \tag{37}$$

$$\phi_2=\Omega_0^{-1}X_2\,,\qquad\qquad \phi_3=\Omega_0^{-1}X_3\,, \tag{38}$$

and
$$X_2=a^\dagger_{++}\,A^\dagger\,|\,+\,+)\,, \tag{39}$$

$$X_3=\sum_{i,\,\lambda}a^\dagger_{+\lambda}\,a^\dagger_{i+}\,\sigma_i\,\tau_\lambda\,|\,+\,+)\,. \tag{40}$$

We also use

$$X_1=a^\dagger_{++}\,|\,+\,+)\,. \tag{41}$$

Applying the approximation of Sec. 5 we have

$$(E_p+i\eta-H)\,\phi_1=b_0X_1-b_1X_2-K(-4b_1X_1+8b_2X_2-8b_2X_3)-\cdots, \tag{42}$$

$$(E_p+i\eta-H)\,\phi_2=LX_2-13MX_1-\cdots, \tag{43}$$

$$(E_p+i\eta-H)\,\phi_3=LX_3-12MX_1-\cdots, \tag{44}$$

$$2\sigma_+\tau_+\,|\,+\,+\,\rangle=-4b_1X_1+8b_2X_2-8b_2X_3\,, \tag{45}$$

where the dots represent the terms of the form $a^\dagger_{j\mu}\,a^\dagger_{k\nu}\,a^\dagger_{l\kappa}\,|\,\beta)$. Inserting these into (35) we have

$$\begin{aligned}&[4b_1(1+Kc_1)+b_0c_1-13Mc_2-12Mc_3]\,X_1\\&\quad+[-8b_2(1+Kc_1)-b_1c_1+Lc_2]\,X_2\\&\quad+[8b_2(1+Kc_1)+Lc_3]\,X_3=0\,.\end{aligned} \tag{46}$$

We have neglected the terms represented by the dots. Solving Eq. (46) we obtain

$$c_1=-\frac{1}{Q+K}\,,\qquad\qquad c_2=\frac{8b_2Q-b_1}{L(Q+K)}\,,$$

$$c_3=-\frac{8b_2Q}{L(Q+K)}\,, \tag{47}$$

where
$$Q=\frac{Lb_0-13Mb_1}{4(Lb_1-2Mb_2)}\,. \tag{48}$$

According to (34), (36) and (37) the phase-shift δ is given by

$$e^{2i\delta}-1=-2\pi i\,V(p)\,f(p)\,(\omega_p/p)\,c_1\,. \tag{49}$$

Using (28) we have

$$e^{2i\delta}-1=2ic_1\,\mathrm{Im}\,K\,. \tag{50}$$

From (47) and (50) we finally obtain

$$\tan \delta = -\mathrm{Im}\, K/(Q+\mathrm{Re}\, K)\,. \tag{51}$$

Thus, the phase-shift turns out to be real. This result is the consequence of the approximation (29). At higher energies the approximation becomes inadequate, and we have to deal with the inelastic effects.

7. 3, 1 and 1, 3 wave scattering

Since the present model is symmetric with respect to the angular momentum and isospin, the 3, 1 and 1, 3 phase-shifts are equal to each other, so that we here consider only the 1, 3 wave scattering. In particular we consider the state in which the z-component of the total angular momentum is $+\frac{3}{2}$ while the z-component of the total isospin is arbitrary.

The scattering is described by the state vector

$$\phi = \sum_{\lambda} \sum_{t'} a^{\dagger}_{+\lambda}(p) \,|+t'\rangle (+t' \,|\, \tau_\lambda \,|+t) + V(p)\,\chi\,. \tag{52}$$

The equation for χ is

$$(E_p + i\eta - H)\,\chi = \sqrt{2}\sum_{\lambda}\sum_{t'} \sigma_+ \tau_\lambda \,|+t'\rangle (+t' \,|\, \tau_\lambda \,|+t\,). \tag{53}$$

We express χ as

$$\chi = c_1\phi_1 + c_2\phi_2 + c_3\phi_3\,, \tag{54}$$

where

$$\phi_1 = \sum_{\lambda}\sum_{t'} \int dq \frac{f(q)}{\omega_p + i\eta - \omega_q}\, a^{\dagger}_{+\lambda}(q)\,|+t'\rangle (+t' \,|\, \tau_\lambda \,|+t)\,, \tag{55}$$

$$\phi_2 = \Omega_0^{-1} X_2\,, \qquad \phi_3 = \Omega_0^{-1} X_3\,, \tag{56}$$

and

$$X_1 = \sum_{\lambda} a^{\dagger}_{+\lambda}\tau_\lambda \,|+t)\,, \tag{57}$$

$$X_2 = \sum_{\lambda} a^{\dagger}_{+\lambda} A^{\dagger} \tau_\lambda \,|+t)\,, \tag{58}$$

$$X_3 = \sum_{\lambda} a^{\dagger}_{+\lambda} \tau_\lambda A^{\dagger} \,|+t)\,. \tag{59}$$

Using the approximation of Sec. 5 we have

$$(E_p + i\eta - H)\,\phi_1 = b_0 X_1 - b_1 X_2 - K(2b_1 X_1 + 2b_2 X_2 - 2b_2 X_3) - \cdots\,, \tag{60}$$

$$(E_p + i\eta - H)\,\phi_2 = L X_2 - 7 M X_1 - \cdots\,, \tag{61}$$

$$(E_p + i\eta - H)\,\phi_3 = L X_3 - 3 M X_1 - \cdots\,, \tag{62}$$

$$\sqrt{2}\sum_{\lambda}\sum_{t'} \sigma_+ \tau_\lambda \,|+t'\rangle (+t' \,|\, \tau_\lambda \,|+t) = 2b_1 X_1 + 2b_2 X_2 - 2b_2 X_3\,. \tag{63}$$

Inserting these into (53) and neglecting the terms represented by dots in (60), (61) and (62) we have

$$[-2b_1(1+Kc_1)+b_0 c_1-7Mc_2-3Mc_3]\,X_1$$
$$+[-2b_2(1+Kc_1)-b_1 c_1+Lc_2]\,X_2+[2b_2(1+Kc_1)+Lc_3]\,X_3=0\,. \tag{64}$$

Solving this equation we obtain

$$c_1=-\frac{1}{Q+K}\,, \qquad c_2=\frac{2b_2 Q-b_1}{L(Q+K)}\,,$$
$$c_3=-\frac{2b_2 Q}{L(Q+K)}\,, \tag{65}$$

where
$$Q=\frac{7Mb_1-Lb_0}{2Lb_1+8Mb_2}\,. \tag{66}$$

The phase-shift is given by

$$\tan\delta=-\mathrm{Im}\,K/(Q+\mathrm{Re}\,K)\,. \tag{67}$$

8. 1, 1 wave scattering

The 1, 1 wave scattering is described by

$$\psi=\sum_{i,\lambda}\sum_{s',t'} a^\dagger_{i\lambda}(p)\,|\,s'\,t'\,\rangle\,(s'\,t'\,|\,\sigma_i\,\tau_\lambda\,|\,st)+V(p)\,\chi\,. \tag{68}$$

The equation for χ is

$$(E_p+i\eta-H)\,\chi=\sum_{i,\lambda}\sum_{s',t'}\sigma_i\,\tau_\lambda\,|\,s'\,t'\,\rangle\,(s'\,t'\,|\,\sigma_i\,\tau_\lambda\,|\,st)\,. \tag{69}$$

We express χ as

$$\chi=c_1\,\phi_1+c_2\,\phi_2+c_3\,\phi_3+c_4\,\phi_4\,, \tag{70}$$

where
$$\phi_1=\omega_p^{-1}\,|\,st\,\rangle\,, \tag{71}$$

$$\phi_2=\sum_{i,\lambda}\sum_{s',t'}\int dq\,\frac{f(q)}{\omega_p+i\eta-\omega_q}\,a^\dagger_{i\lambda}(q)|\,s'\,t'\,\rangle\,(s'\;\;t'\,|\,\sigma_i\,\tau_\lambda\,|\,st)\,, \tag{72}$$

$$\phi_3=\Omega_0^{-1}\,X_3\,, \qquad \phi_4=\Omega_0^{-1}\,X_4\,, \tag{73}$$

and
$$X_1=|\,st)\,, \qquad X_2=A^\dagger\,|\,st)\,, \tag{74}$$

$$X_3=\sum_{i,\lambda} a^\dagger_{i\lambda}\,a^\dagger_{i\lambda}\,|\,st)\,, \qquad X_4=A^{\dagger 2}\,|\,st)\,. \tag{75}$$

Using the approximation of Sec. 5 we have

$$(E_p+i\eta-H)\,\phi_1=b_0\,X_1-b_1\,X_2+b_2\,X_4\,, \tag{76}$$

$$(E_p+i\eta-H)\,\phi_2=b_0\,X_2-b_1\,X_4-K(9b_0\,X_1-b_1\,X_2+8b_2\,X_3+b_2\,X_4)-\cdots, \tag{77}$$

$$(E_p+i\eta-H)\,\phi_3=LX_3-2MX_2-\cdots, \tag{78}$$

$$(E_p+i\eta-H)\,\phi_4=LX_4-10MX_2-\cdots, \tag{79}$$

$$\sum_{i,\lambda}\sum_{s',t'}\sigma_i\,\tau_\lambda\,|\,s'\,t'\,\rangle\,(s'\,t'\,|\,\sigma_i\,\tau_\lambda\,|\,st)=9b_0\,X_1-b_1\,X_2+8b_2\,X_3+b_2\,X_4\,. \tag{80}$$

Inserting thses into (69) we have

$$[-9b_0(1+Kc_2)+b_0c_1]X_1+[b_1(1+Kc_2)-b_1c_1+b_0c_2-2Mc_3-10Mc_4]X_2$$
$$+[-8b_2(1+Kc_2)+Lc_3]X_3+[-b_2(1+Kc_2)+b_2c_1-b_1c_2+Lc_4]X_4=0\,, \tag{81}$$

from which we obtain

$$c_1=\frac{9Q}{Q+K}\,,\qquad c_2=-\frac{1}{Q+K}\,,$$
$$c_3=\frac{8b_2Q}{L(Q+K)}\,,\qquad c_4=-\frac{8b_2Q+b_1}{L(Q+K)}\,, \tag{82}$$

where
$$Q=\frac{Lb_0-10Mb_1}{8(8Mb_2-Lb_1)}\,. \tag{83}$$

The phase-shift is given by
$$\tan\delta=-\mathrm{Im}\,K/(Q+\mathrm{Re}\,K)\,. \tag{84}$$

9. Results and conclusion

The results of the computation are shown in Figs. 1-4 in comparison with the results of the phase-shift analysis of experimental data performed by Roper and Wright.[11] Roper and Wright also obtained the phase-shifts of the *s*-, *d*-, and *f*-waves, while the present model gives only the *p*-wave phase-shifts. The two curves in each of Figs. 1-3 are in qualitative agreement with each other; the 3, 3 phase-shift

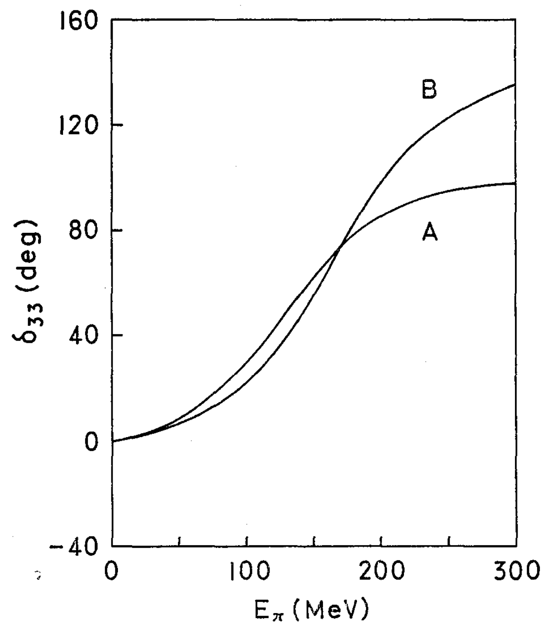

Fig. 1. The 3, 3 phase-shift δ_{33} versus the pion incident energy $E_\pi=\omega_p-\mu$. Curve A is the result of the present computation. Curve B is the solution *BO3* of the phase-shift analysis of Roper and Wright.[11]

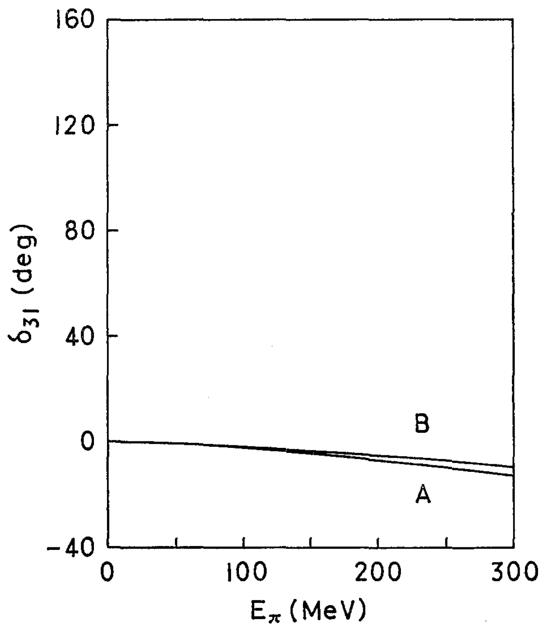

Fig. 2. The 3,1 phase-shift. The notation is the same as in Fig. 1.

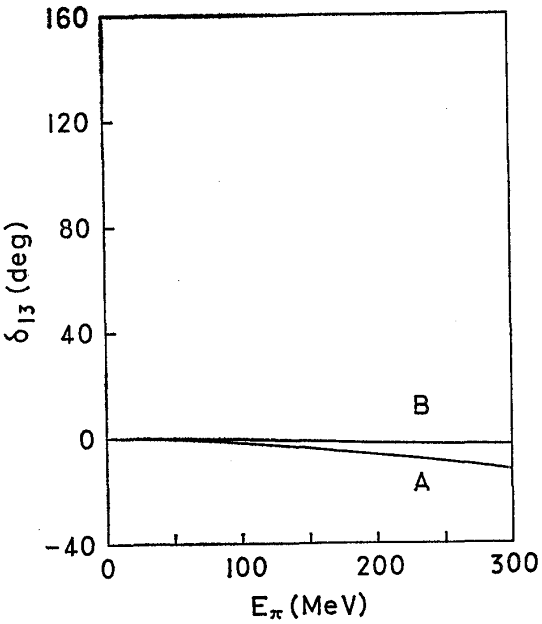

Fig. 3. The 1, 3 phase-shift. The notation is the same as in Fig. 1.

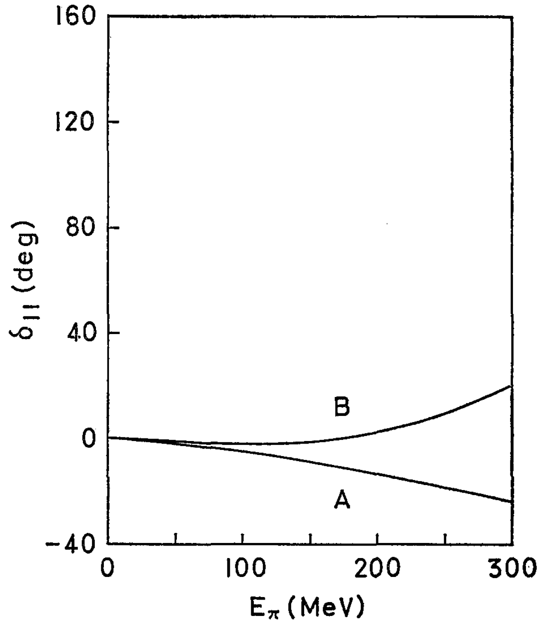

Fig. 4. The 1, 1 phase-shift. The notation is the same as in Fig. 1.

passes through 90° at about 200 MeV, while the 3, 1 and 1, 3 phas-shifts are small and negative. The agreement is not quite satisfactory in Fig. 4, *i.e.*, the experimental curve for the 1, 1 phase-shift is positive at higher energies, while the theoretical curve is negative, although both are small. In order to improve the theory it would be necessary to take account of the effects of the pion-resonances such as ρ.

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