On the Concentration of a Plume after when the Wind Direction Changed

Jiro Sakagami

Environment Science, Faculty of Science,
Ochanomizu University

(Received September 12, 1975)

Introduction

It is not seldom, especially in coastal regions, that the wind which has been blowing in one direction during a certain period, suddenly dies away, and after then it blows again in another direction. A plume which has been emitted from a source in the former stage, drifts with the wind in the second stage. The concentrations of the plume at positions which were to the windward in the former stage are important in the problem of the atmospheric pollution. We consider this problem in the cases when the changes of wind direction are 1) 180°, 2) 175° and 3) 90°.

Formulation

We assume that the wind has been blowing in the direction 0ξ with the velocity u_1 m/s during T s, then it changes the direction to $0x$ and the velocity to u_2 m/s. The relations between coordinates

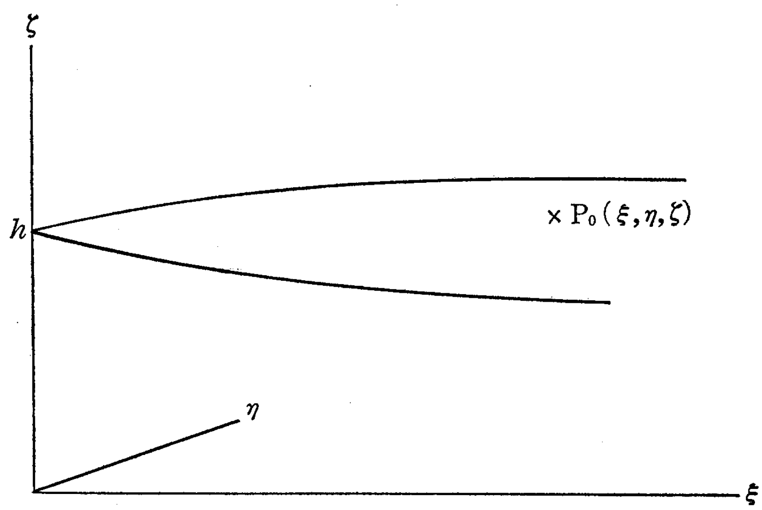


Fig. 1. Coordinates in the 1st stage.

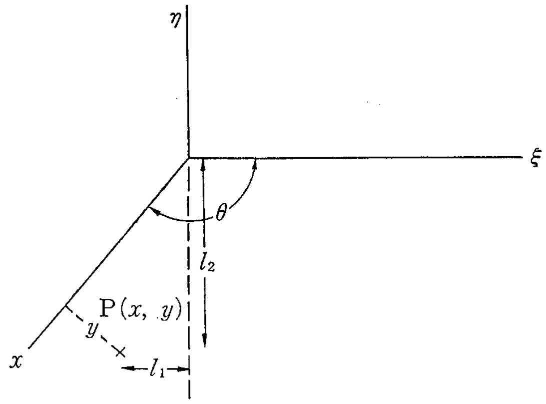


Fig. 2. Relations between (ξ, η) and (x, y).

are shown in Figs. 1 and 2. A plume is emitted from a source whose height is h m above the origin.

The concentration of a puff C_1, which has been emitted at the instant τ $(0 \leqq \tau \leqq T)$ at the point $P_0(\xi, \eta, \zeta)$ be

$$C_1(\xi, \eta, \zeta, \tau) = q d\tau\, F_1(\xi, \eta, \zeta, a, b, \tau) \qquad 1),$$

where q is the source intensity of the continuous source, so $qd\tau$ is that of the instantaneous source. a and b are defined by eq. 5) or 7).

The concentration C_2 at the instant t in the second stage, due to the puff centered at P_0, is given by

$$C_2(x, y, z, \tau, t) = \int_0^\infty d\zeta \int_{-\infty}^\infty d\eta \int_{-\infty}^\infty C_1(\xi, \eta, \zeta, \tau)\, F_2(x, y, z, \xi, \eta, \zeta, t)\, d\xi \qquad 2).$$

So the resulted concentration due to the puffs emitted during the whole period T is given by

$$C_3(x, y, z, t) = \int_0^T d\tau \int_0^\infty d\zeta \int_{-\infty}^\infty d\eta \int_{-\infty}^\infty C_1 F_2\, d\xi \qquad 3).$$

The functions F_1 and F_2 are given by next two equations:

$$F_1(\xi, \eta, \zeta, \tau) = \frac{e^{-\frac{(\xi - u_1\tau)^2 + \eta^2}{a_1} - \frac{h+\zeta}{b_1}}}{a_1 b_1 \pi}\, I_0\left(\frac{2\sqrt{h\zeta}}{b_1}\right) \qquad 4),$$

$$\left.\begin{aligned} a_1 &= q_A(\varphi_A u_1 \tau + e^{-\varphi_A u_1 \tau} - 1) \\ b_1 &= q_B(\varphi_B u_1 \tau + e^{-\varphi_B u_1 \tau} - 1) \end{aligned}\right\} \qquad 5).$$

$$F_2(x, y, z, \xi, \eta, \zeta, t) = \frac{1}{a_2 b_2 \pi}\, e^{-\frac{(x - \xi\cos\theta + \eta\sin\theta - u_2 t)^2}{a_2}}$$

$$\times\, e^{-\frac{(\xi\sin\theta + \eta\cos\theta + y)^2}{a_2}}\, e^{-\frac{\zeta + z}{b_2}}\, I_0\left(\frac{2\sqrt{\zeta z}}{b_2}\right) \qquad 6),$$

$$\left.\begin{aligned} a_2 &= q_A(\varphi_A u_2 t + e^{-\varphi_A u_2 t} - 1) \\ b_2 &= q_B(\varphi_B u_2 t + e^{-\varphi_B u_2 t} - 1) \end{aligned}\right\} \qquad 7),$$

where q_A, q_B, φ_A, φ_B, are the diffusion parameters which are shown in

Table 1 and I_0 is the 1st kind modified Bessel function of the order 0[1].*

Table 1. φ_A, $\sqrt{q_A}$, φ_B, q_B, h and ζ. h is the source height.

ζ	h	φ_A	$\sqrt{q_A}$	φ_B	q_B
0.4	0.5	4.78(-2)	4.26(1)	4.20(-2)	3.50(-1)
	10	4.78(-2)	4.26(1)	4.60(-2)	2.93(-1)
	20	4.78(-2)	4.26(1)	4.71(-2)	2.86(-1)
	30	4.78(-2)	4.26(1)	4.77(-2)	2.83(-1)
	50	4.78(-2)	4.26(1)	4.80(-2)	2.78(-1)
	70	4.78(-2)	4.26(1)	4.81(-2)	2.75(-1)
	100	4.78(-2)	4.26(1)	4.82(-2)	2.70(-1)
	150	4.78(-2)	4.26(1)	4.83(-2)	2.69(-1)
	200	4.78(-2)	4.26(1)	4.84(-2)	2.67(-1)
	300	4.78(-2)	4.26(1)	4.84(-2)	2.64(-1)
0	0.5	1.48(-2)	1.56(1)	1.10(-2)	5.30
	10	1.09(-2)	2.18(1)	2.46(-2)	1.02
	20	1.01(-2)	2.37(1)	3.00(-2)	7.00(-1)
	30	9.7 (-3)	2.48(1)	3.29(-2)	5.65(-1)
	50	9.2 (-3)	2.62(1)	3.79(-2)	4.41(-1)
	70	8.9 (-3)	2.71(1)	4.02(-2)	3.80(-1)
	100	8.6 (-3)	2.84(1)	4.27(-2)	3.39(-1)
	150	8.3 (-3)	2.94(1)	4.40(-2)	3.08(-1)
	200	8.0 (-3)	3.04(1)	4.63(-2)	2.93(-1)
	300	7.7 (-3)	3.23(1)	4.75(-2)	2.78(-1)
-0.1	0.5	4.50(-3)	7.59(1)	4.25(-3)	3.48(1)
	10	2.12(-3)	1.59(2)	1.48(-2)	2.87
	20	1.80(-3)	1.88(2)	1.98(-2)	1.61
	30	1.61(-3)	2.09(2)	2.34(-2)	1.14
	50	1.40(-3)	2.38(2)	2.87(-2)	7.55(-1)
	70	1.29(-3)	2.57(2)	3.30(-2)	5.78(-1)
	100	1.17(-3)	2.85(2)	3.70(-2)	4.59(-1)
	150	1.06(-3)	3.07(2)	4.20(-2)	3.57(-1)
	200	9.8 (-4)	3.40(2)	4.44(-2)	3.18(-1)
	300	8.8 (-4)	3.66(2)	4.78(-2)	2.79(-1)
-0.2	0.5	1.12(-3)	2.77(2)	1.30(-3)	3.73(2)
	10	2.52(-4)	1.24(3)	7.20(-3)	1.18(1)
	20	1.78(-4)	1.73(3)	1.10(-2)	5.19
	30	1.44(-4)	2.14(3)	1.40(-2)	3.21
	50	1.11(-4)	2.77(3)	1.93(-2)	1.69
	70	9.50(-5)	3.30(3)	2.38(-2)	1.11
	100	7.90(-5)	3.93(3)	2.95(-2)	7.22(-1)
	150	6.50(-5)	4.88(3)	3.74(-2)	4.50(-1)
	200	5.60(-5)	5.54(3)	4.28(-2)	3.41(-1)
	300	4.54(-5)	6.83(3)	4.78(-2)	2.94(-1)

According to the above equations, we get

$$C_3(x, y, z, t)=\int_0^T q\,d\tau\int_0^\infty d\zeta\int_{-\infty}^\infty d\eta\int_{-\infty}^\infty \frac{d\xi}{a_2 a_2 b_1 b_2 \pi}$$

$$\times \exp\Big[-\Big\{\frac{(\xi-u_1\tau)^2}{a_1}+\frac{\eta_2}{a_1}+\frac{(x-\xi\cos\theta+\eta\sin\theta-u_2 t)^2}{a_2}$$

$$\frac{(\xi\sin\theta+\eta\cos\theta+y)^2}{a_2}+\frac{h+\zeta}{b_1}+\frac{\zeta+z}{b_2}\Big\}\Big] I_0\Big(\frac{2\sqrt{h\zeta}}{b_1}\Big) I_0\Big(\frac{2\sqrt{\zeta z}}{b_2}\Big) \qquad 8),$$

$$=\int_0^T \frac{q}{(a_1+a_2)\,(b_1+b_2)\pi}\, e^{-\frac{1}{a_1+a_2}(l_2-u_2 t\sin\theta)^2}$$

$$\times\; e^{-\frac{1}{a_1+a_2}(u_1\tau+u_2 t\cos\theta-l_1)^2}\, e^{-\frac{h+z}{b_1+b_2}}\, I_0\Big(\frac{2\sqrt{hz}}{b_1+b_2}\Big)\,d\tau \qquad 9).$$

The ground level concentration is given by

$$C_3(x, y, 0, t)=\int_0^T \frac{q}{(a_1+a_2)\,(b_1+b_2)\pi}\, e^{-\frac{1}{a_1+a_2}[(l_2-u_2 t\sin\theta)^2+(u_1\tau+u_2 t\cos\theta-l_1)^2]}$$

$$\times\; e^{-\frac{h}{b_1+b_2}}\,d\tau \qquad 10).$$

*) In this paper the numerals 4.78(−2), for example, means 4.78×10^{-2}.

Approximated calculation

As the parameters a_1 and b_1 are the functions of τ, it is very tedious to carry out the integration exactly. So we integrate the eqs. 10) and 11) approximately by putting a_{10} and b_{10} instead of a_1 and b_1, where

$$\tau_0=\frac{1}{u_1}(l_1-u_2t\cos\theta) \tag{11}$$

When $\tau_0 \geqq 0$,

$$\left.\begin{aligned} a_{10}&=q_A(\varphi_A u_1\tau_0+e^{-\varphi_A u_1\tau_0}-1)\\ b_{10}&=q_B(\varphi_B u_1\tau_0+e^{-\varphi_B u_1\tau_0}-1)\end{aligned}\right\} \tag{12}$$

and when $\tau_0<0$

$$a_{10}=b_{10}=0 \tag{13}$$

Then we get approximately for eq. 9),

$$C_4=C_4(x,y,z,t)=\frac{q}{u_1\sqrt{a_{10}+a_2}\,(b_{10}+b_2)\sqrt{\pi}}\,e^{-\frac{1}{a_{10}+a_2}(l_2-u_2t\sin\theta)^2}$$
$$\times\, e^{-\frac{h+z}{b_{10}+b_2}}\,I_0\left(\frac{2\sqrt{hz}}{b_{10}+b_2}\right)\frac{1}{2}\left[\operatorname{erf}\left(\frac{u_1T+u_2t\cos\theta-l_1}{\sqrt{a_{10}+a_2}}\right)\right.$$
$$\left.-\operatorname{erf}\left(\frac{u_2t\cos\theta-l_1}{\sqrt{a_{10}+a_2}}\right)\right] \tag{14}$$

and for eq. 10,

$$C_{4,0}=C_4(x,y,0,t,)=\frac{q}{u_1\sqrt{a_{10}+a_2}\,(b_{10}+b_2)\sqrt{\pi}}\,e^{-\frac{1}{a_{10}+a_2}(l_2-u_2t\sin\theta)^2}$$
$$\times\, e^{-\frac{h}{b_{10}+b_2}}\frac{1}{2}\left[\operatorname{erf}\left(\frac{u_1T+u_2t\cos\theta-l_1}{\sqrt{a_{10}+a_2}}\right)-\operatorname{erf}\left(\frac{u_2t\cos\theta-l_1}{\sqrt{a_{10}+a_2}}\right)\right] \tag{15}$$

In these equations, $\operatorname{erf}(\phi)$ is the error integral which is defined by

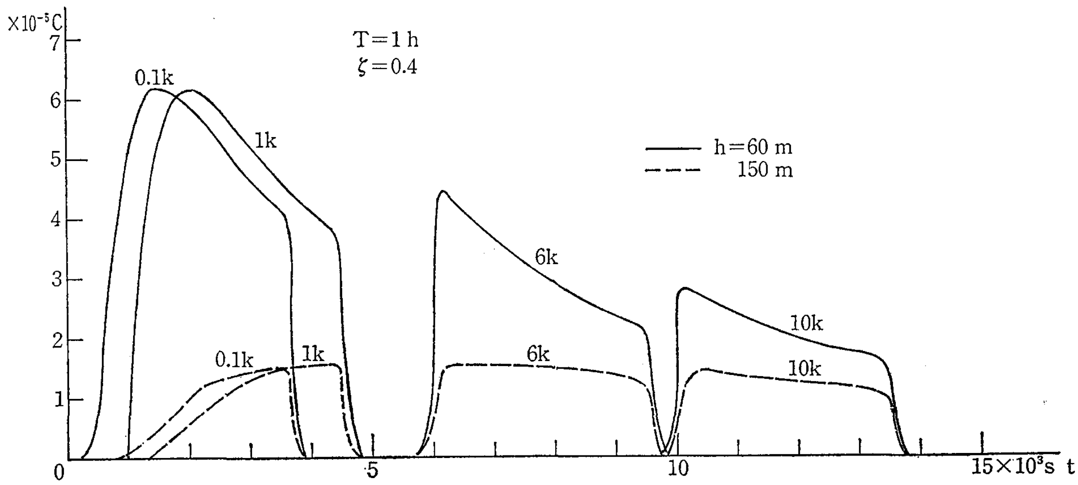
$$\operatorname{erf}(\phi)=\frac{2}{\sqrt{\pi}}\int_0^{\phi}e^{-u^2}du \tag{16}$$

Results

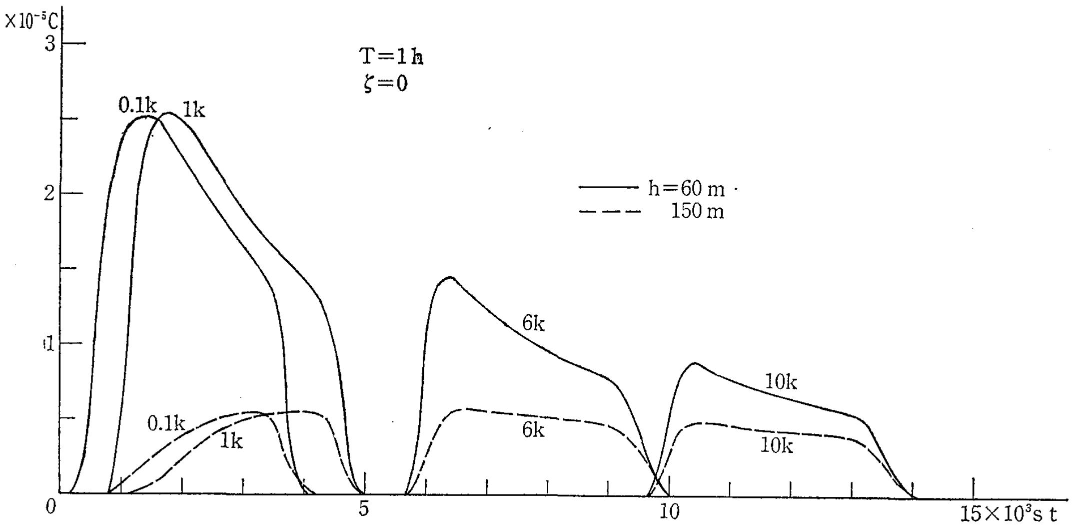
1) $\theta=180°$: We calculated the ground level concentrations in the cases of $h=60$, 100, 150 m, $T=1$, 2, 3 h, stability parameter $\zeta=0.4$ (stable), 0 (neutral), -0.2 (unstable), $q/u_1=1$ and $u_1=u_2=1$ m/s.

The time changes of concentration are shown in Fig. 3 and 4. Contrary to the concentration in the case of continuous plume, it changes remarkably with time, and it becomes noticeable only during a certain period, exposed interval λ. So, for the air pollution problems, the quantities, dosage D,

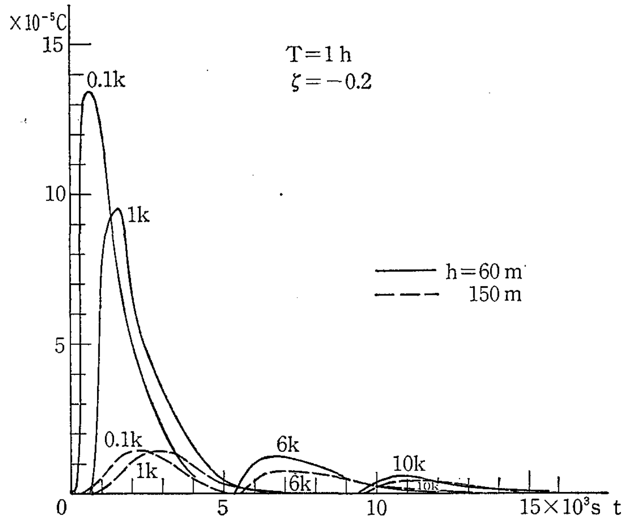
$$D=\int_0^{\infty}Cdt\ c.h=\frac{1}{3600}\int_0^{\infty}Cdt\ c.s$$



(a) $\zeta=0.4$

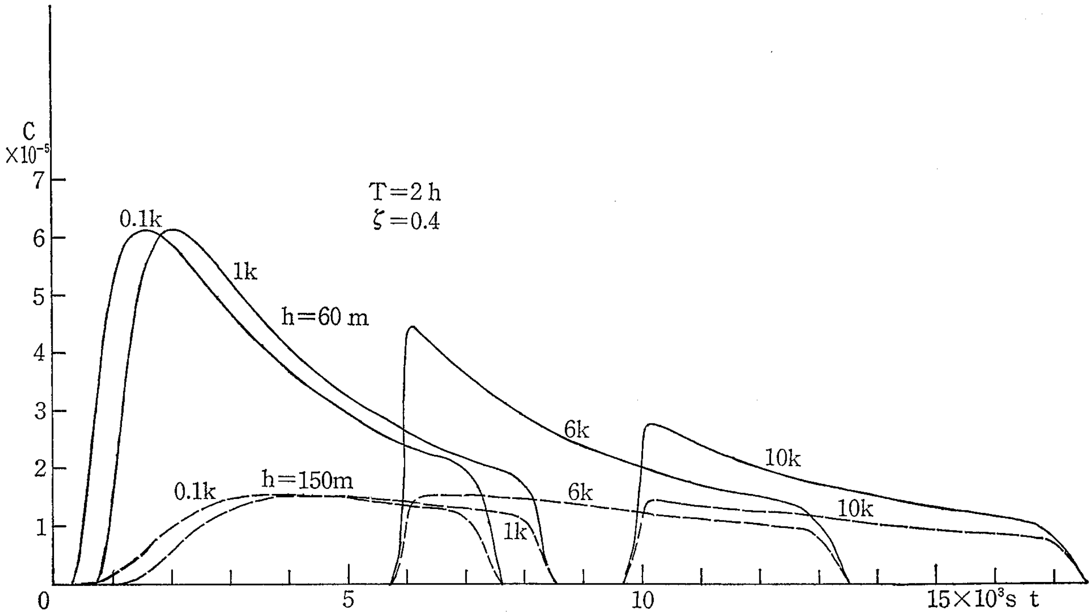


(b) $\zeta=0$

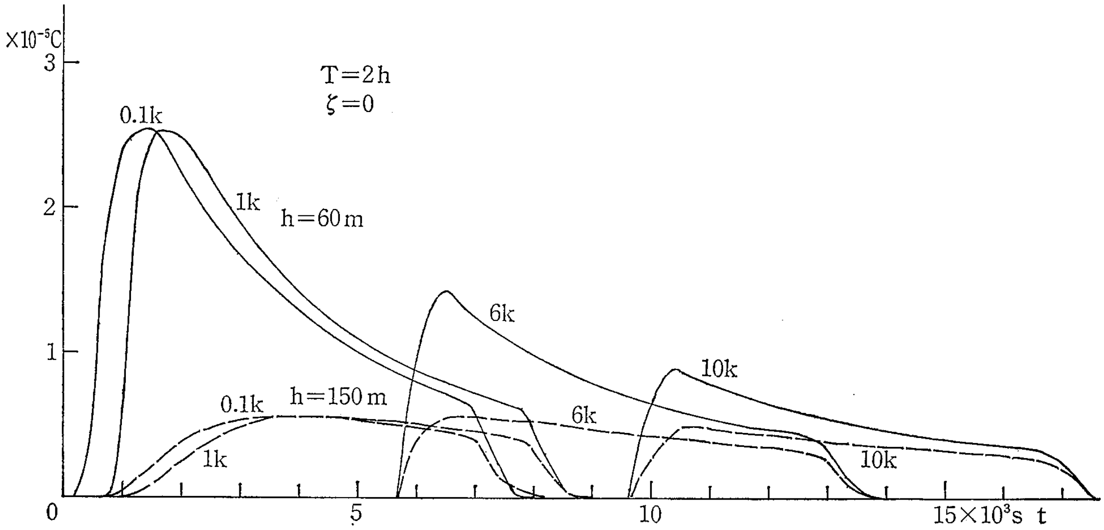


(c) $\zeta=-0.2$

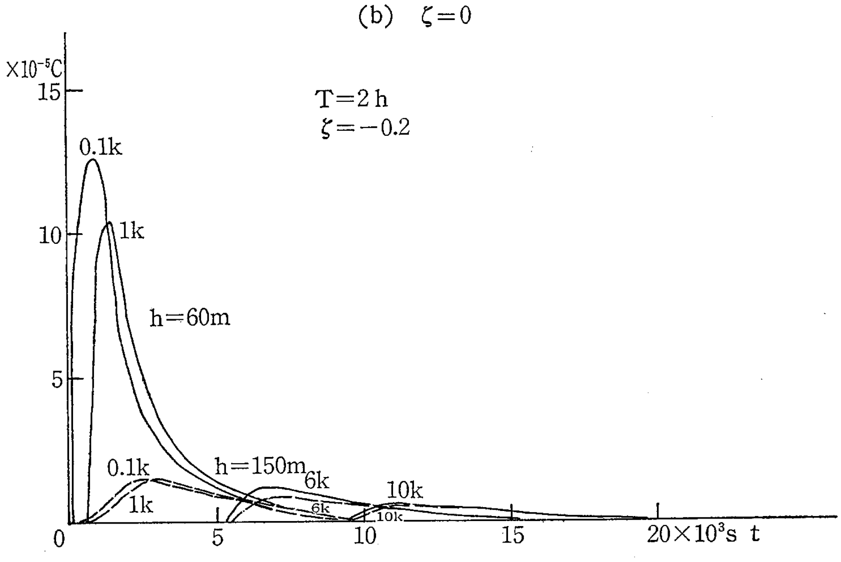
Fig. 3. Time changes of concentration. T=1 h, h=60 and 150 m.



(a) ζ=0.4



(b) ζ=0



(c) ζ=−0.2

Fig. 4. Time changes of concentration T=2 h, h=60 and 150 m.

Table 2. Relations between C_{max}, D, $\bar{C}$, C_c, λ and x

T = 1 h (3600 s) h = 60 m

ζ	X (m)	C_{max} (10^{-5})	D (10^{-5})	$\bar{C}$ (10^{-5})	λ (s) (10^3)	C_c (10^{-5})
0.4	100	6.154	4.422	4.33	3.68	
	200	6.158	4.477	4.32	3.73	
	400	6.156	4.586	4.07	4.06	0.0041
	600	6.156	4.690	4.16	4.06	0.1435
	800	6.156	4.784	4.24	4.06	0.6377
	1000	6.155	4.857	4.30	4.07	1.479
	2000	6.158	4.830	4.26	4.08	5.290
	4000	5.669	3.970	3.49	4.10	5.842
	6000	4.463	3.163	2.76	4.13	4.637
	8000	3.497	2.566	2.22	4.16	3.634
	10000	2.800	2.127	1.84	4.17	2.911
	20000	1.233	1.053	0.888	4.27	1.289
	40000	0.4960	0.4532	0.369	4.42	0.5098
	60000	0.2808	0.2647	0.210	4.53	0.2881
	100000	0.1353	0.1303	0.0998	4.70	0.1379
0	100	2.545	1.774	1.57	4.06	
	200	2.551	1.792	1.56	4.13	0.0000
	400	2.544	1.829	1.53	4.31	0.0101
	600	2.555	1.863	1.54	4.36	0.1640
	800	2.555	1.889	1.55	4.38	0.5490
	1000	2.548	1.903	1.55	4.41	1.033
	2000	2.524	1.798	1.44	4.49	2.437
	4000	1.961	1.381	1.08	4.60	2.185
	6000	1.456	1.064	0.817	4.69	1.618
	8000	1.113	0.8453	0.641	4.75	1.225
	10000	0.8775	0.6911	0.517	4.81	0.9608
	20000	0.3772	0.3308	0.235	5.07	0.4080
	40000	0.1489	0.1391	0.0916	5.47	0.1581
	60000	0.0845	0.08029	0.0502	5.76	0.0887
	100000	0.0407	0.03896	0.0225	6.23	0.0422
-0.2	100	1.383	0.6103	0.259	8.49	
	200	1.358	0.6096	0.256	8.56	0.0006
	400	1.190	0.6008	0.241	8.99	0.2147
	600	1.134	0.5808	0.231	9.06	0.7036
	800	1.042	0.5501	0.211	9.37	1.012
	1000	0.9543	0.5132	0.194	9.54	1.118
	2000	C.5637	0.3427	0.120	10.31	0.8088
	4000	0.2275	0.1744	0.0539	11.66	0.3459
	6000	0.1213	0.1058	0.0295	12.93	0.1872
	8000	0.0752	0.07117	0.0184	13.95	0.1179
	10000	0.0507	0.05116	0.0123	14.96	0.0816
	20000	0.0127	0.00168	0.00346	17.51	0.0216
	40000	0.0030	0.00530	0.00075	25.44	0.0083
	60000	0.0013	0.00274	0.00032	30.06	0.0043
	100000	0.0004	0.00121	0.00011	37.08	0.0019

1-a

T = 2 h (7200 s) h = 60 m

ζ	X (m)	C_{max} (10^{-5})	D (10^{-5})	$\bar{C}$ (10^{-5})	λ (s) (10^3)	C_c (10^{-5})
0.4	100	6.138	7.175	3.51	7.36	
	200	6.118	7.202	3.49	7.42	
	400	6.158	7.255	3.38	7.73	0.0041
	600	6.153	7.306	3.40	7.74	0.1435
	800	6.114	7.349	3.42	7.74	0.6377
	1000	6.127	7.372	3.43	7.74	1.479
	2000	6.161	7.117	3.31	7.75	5.290
	4000	5.696	5.888	2.72	7.78	5.842
	6000	4.474	4.801	2.21	7.81	4.637
	8000	3.486	3.985	1.83	7.82	3.634
	10000	2.772	3.371	1.55	7.84	2.911
	20000	1.209	1.789	0.813	7.92	1.289
	40000	0.4926	0.8193	0.366	8.05	0.5098
	60000	0.2811	0.4925	0.218	8.15	0.2881
	100000	0.1352	0.2492	0.0913	9.83	0.1379
0	100	2.550	2.692	1.23	7.89	
	200	2.549	2.701	1.23	7.94	0.00000
	400	2.556	2.717	1.21	8.12	0.01016
	600	2.544	2.732	1.20	8.18	0.1640
	800	2.521	2.740	1.20	8.19	0.5490
	1000	2.528	2.736	1.20	8.22	1.033
	2000	2.467	2.549	1.11	8.30	2.437
	4000	1.878	2.005	0.861	8.39	2.185
	6000	1.410	1.593	0.678	8.46	1.618
	8000	1.086	1.301	0.550	8.51	1.225
	10000	0.8652	1.089	0.458	8.57	0.9608
	20000	0.3776	0.5629	0.230	8.80	0.4080
	40000	0.1431	0.2532	0.0997	9.15	0.1581
	60000	0.0845	0.1511	0.0577	9.43	0.08871
	100000	0.0407	0.007575	0.0276	9.89	0.04225
-0.2	100	1.268	0.7189	0.169	15.3	
	200	1.269	0.7164	0.168	15.4	0.00066
	400	1.299	0.7040	0.164	15.5	0.2147
	600	1.222	0.6806	0.154	15.9	0.7036
	800	1.135	0.6466	0.145	16.1	1.012
	1000	1.039	0.6066	0.135	16.2	1.118
	2000	0.5703	0.4225	0.0900	16.9	0.8088
	4000	0.2162	0.2347	0.0477	17.7	0.3459
	6000	0.1171	0.1530	0.0293	18.8	0.1872
	8000	0.0755	0.1093	0.0199	19.8	0.1179
	10000	0.0531	0.08277	0.0145	20.5	0.08160
	20000	0.0168	0.03210	0.00479	24.1	0.02169
	40000	0.0048	0.01132	0.00139	29.3	0.00830
	60000	0.0022	0.006019	0.000639	33.9	0.00437
	100000	0.0008	0.002698	0.000240	40.4	0.00198

2-a

T = 3 h (10800 s) h = 60 m

ζ	X (m)	C_{max} (10^{-5})	D (10^{-5})	$\bar{C}$ (10^{-5})	λ (s) (10^3)	C_c (10^{-5})
0.4	100	6.027	8.665	2.836	11.0	
	200	6.021	8.680	2.815	11.1	
	400	6.134	8.714	2.752	11.4	0.0041
	600	6.078	8.744	2.761	11.4	0.1435
	800	6.004	8.770	2.756	11.4	0.6377
	1000	6.110	8.770	2.756	11.4	1.479
	2000	6.043	8.426	2.661	11.4	5.290
	4000	5.486	7.044	2.224	11.4	5.842
	6000	4.304	5.831	1.841	11.4	4.637
	8000	3.392	4.910	1.551	11.4	3.634
	10000	2.736	4.209	1.329	11.4	2.911
	20000	1.241	2.339	0.7385	11.4	1.289
	40000	0.4942	1.122	0.3513	11.5	0.5098
	60000	0.2765	0.6906	0.2162	11.5	0.2881
	100000	0.1345	0.3577	0.1120	11.5	0.1379
0	100	2.554	3.171	0.9758	11.7	
	200	2.553	3.276	0.9774	11.7	
	400	2.557	3.185	0.9803	11.7	0.00000
	600	2.551	3.195	0.9665	11.9	0.01016
	800	2.533	3.196	0.9668	11.9	0.1640
	1000	2.504	3.185	0.9556	12.0	0.5490
	2000	2.546	2.969	0.8906	12.0	1.033
	4000	2.008	2.375	0.7065	12.1	2.437
	6000	1.475	1.921	0.5715	12.1	2.185
	8000	1.107	1.594	0.4705	12.2	1.618
	10000	0.8559	1.354	0.3964	12.3	0.9608
	20000	0.3688	0.7357	0.2119	12.5	0.1080
	40000	0.1492	0.3478	0.0978	12.8	0.1581
	60000	0.0842	0.2125	0.0524	13.1	0.08871
	100000	0.0406	0.1093	0.0289	13.6	0.04225
-0.2	100	0.9908	0.7617	0.1281	21.4	
	200	0.9909	0.7585	0.1264	21.6	0.00066
	400	1.047	0.7454	0.1237	21.7	0.2147
	600	0.9986	0.7213	0.1186	21.9	0.7036
	800	0.9390	0.6864	0.1123	22.0	1.012
	1000	0.8677	0.6457	0.1042	22.3	1.118
	2000	0.5529	0.4581	0.07266	22.7	0.8088
	4000	0.2275	0.2646	0.04053	23.5	0.3459
	6000	0.1203	0.1784	0.02643	24.3	0.1872
	8000	0.0747	0.1313	0.01882	25.1	0.1179
	10000	0.0513	0.1019	0.01422	25.8	0.08160
	20000	0.0173	0.04324	0.00538	28.9	0.02169
	40000	0.0056	0.01671	0.00179	33.6	0.00830
	60000	0.0028	0.00925	0.00088	37.7	0.00437
	100000	0.0011	0.00427	0.00034	44.2	0.00198

3-a

Table 2. Continued.

T = 1 h (3600 s) h = 100 m

ζ	X (m)	C_{max} (10^{-5})	D (10^{-5})	$\overline{C}$ (10^{-5})	λ(s) (10^3)	C_c (10^{-5})
0.4	100	2.834	1.905	1.92	3.57	
	200	2.833	1.942	1.93	3.62	
	400	2.834	2.014	1.94	3.73	
	600	2.834	2.084	1.93	3.88	0.0005
	800	2.834	2.155	1.91	4.07	0.0108
	1000	2.831	2.223	1.97	4.07	0.0579
	2000	2.834	2.488	2.18	4.10	1.090
	4000	2.792	2.510	2.19	4.12	2.690
	6000	2.750	2.245	1.95	4.15	2.784
	8000	2.373	1.855	1.69	4.16	2.495
	10000	2.099	1.698	1.46	4.18	2.163
	20000	1.080	0.9372	0.790	4.27	1.123
	40000	0.4683	0.4300	0.350	4.42	0.4808
	60000	0.2720	0.2570	0.204	4.53	0.2789
	100000	0.1338	0.1290	0.0988	4.70	0.1364
0	100	1.079	0.7535	0.676	4.01	
	200	1.79	0.7664	0.680	4.06	
	400	1.079	0.7919	0.684	4.17	
	600	1.079	0.8172	0.684	4.30	
	800	1.078	0.8420	0.684	4.43	0.0099
	1000	1.079	0.8660	0.698	4.47	0.0431
	2000	1.079	0.9468	0.752	4.53	0.5238
	4000	1.076	0.9067	0.705	4.63	1.052
	6000	0.9677	0.7850	0.601	4.70	1.000
	8000	0.8212	0.6691	0.504	4.78	0.8794
	10000	0.6922	0.5731	0.426	4.84	0.7470
	20000	0.3435	0.3052	0.215	5.10	0.3720
	40000	0.1458	0.1365	0.0897	5.48	0.1560
	60000	0.0848	0.08056	0.0501	5.79	0.0899
	100000	0.0416	0.03982	0.0228	6.28	0.0437
-0.2	100	0.4083	0.2633	0.105	9.16	
	200	0.4088	0.2682	0.103	9.34	
	400	0.3981	0.2707	0.100	9.74	0.00022
	600	0.3887	0.2722	0.986	9.94	0.00896
	800	0.3865	0.2724	0.958	10.24	0.04340
	1000	0.3815	0.2701	0.936	10.39	0.09816
	2000	0.3126	0.2315	0.758	10.99	0.2893
	4000	0.1772	0.1477	0.414	12.53	0.2452
	6000	0.1085	0.09844	0.257	13.79	0.1657
	8000	0.07170	0.06958	0.166	15.06	0.1161
	10000	0.04965	0.05157	0.114	16.31	0.08556
	20000	0.01340	0.01785	0.301	21.33	0.03012
	40000	0.00312	0.00571	0.719	28.61	0.01011
	60000	0.00132	0.00296	0.315	33.81	0.00537
	100000	0.00455	0.00132	0.112	42.39	0.00267

1-b

T = 2 h (7200 s) h = 100 m

ζ	X (m)	C_{max} (10^{-5})	D (10^{-5})	$\overline{C}$ (10^{-5})	λ(s) (10^3)	C_c (10^{-5})
0.4	100	2.830	3.958	1.97	7.25	
	200	2.831	3.979	1.97	7.29	
	400	2.833	4.024	1.96	7.40	
	600	2.825	4.066	1.89	7.76	0.0005
	800	2.830	4.108	1.91	7.76	0.0108
	1000	2.834	4.149	1.92	7.76	0.0575
	2000	2.831	4.283	1.99	7.76	1.090
	4000	2.833	4.075	1.88	7.79	2.690
	6000	2.744	3.620	1.67	7.81	2.784
	8000	2.420	3.172	1.46	7.82	2.495
	10000	2.063	2.786	1.28	7.84	2.163
	20000	1.061	1.614	0.733	7.92	1.123
	40000	0.465	0.7809	0.349	8.06	0.4808
	60000	0.2723	0.4793	0.212	8.15	0.2789
	100000	0.1337	0.2469	0.107	8.32	0.1364
0	100	1.078	1.464	0.673	7.83	
	200	1.079	1.471	0.672	7.88	
	400	1.079	1.486	0.670	7.98	
	600	1.077	1.501	0.657	8.22	
	800	1.079	1.515	0.662	8.24	0.0099
	1000	1.079	1.528	0.666	8.26	0.0431
	2000	1.077	1.559	0.674	8.33	0.5238
	4000	1.070	1.436	0.614	8.41	1.052
	6000	0.9510	1.246	0.528	8.49	1.000
	8000	0.8109	1.075	0.453	8.55	0.8794
	10000	0.6866	0.9338	0.390	8.61	0.7470
	20000	0.3434	0.5265	0.215	8.83	0.3720
	40000	0.1458	0.2500	0.0980	9.18	0.1560
	60000	0.0848	0.1521	0.0578	9.47	0.0899
	100000	0.0716	0.0776	0.0281	9.96	0.0437
-0.2	100	0.3918	0.3717	0.0826	16.2	
	200	0.3912	0.3719	0.0821	16.3	
	400	0.3792	0.3715	0.0811	16.5	0.00022
	600	0.3682	0.3702	0.0789	16.9	0.00896
	800	0.3593	0.3676	0.0783	16.9	0.04340
	1000	0.3496	0.3627	0.0759	17.2	0.09816
	2000	0.2956	0.3122	0.0631	17.8	0.2893
	4000	0.1685	0.2103	0.0403	18.8	0.2452
	6000	0.1051	0.1484	0.0268	19.9	0.1657
	8000	0.07259	0.1106	0.0191	20.9	0.1161
	10000	0.05311	0.08588	0.0141	21.9	0.08556
	20000	0.01797	0.03474	0.00479	26.1	0.03012
	40000	0.00802	0.01230	0.00135	32.7	0.01011
	60000	0.00231	0.06523	0.000623	37.7	0.00537
	100000	0.00854	0.02925	0.000233	45.2	0.00267

2-b

T = 3 h (10800 s) h = 100 m

ζ	X (m)	C_{max} (10^{-5})	D (10^{-5})	$\overline{C}$ (10^{-5})	λ(s) (10^3)	C_c (10^{-5})
0.4	100	2.8530	5.226	1.726	10.9	
	200	2.831	5.241	1.715	11.0	
	400	2.832	5.269	1.709	11.1	
	600	2.810	5.298	1.673	11.4	0.00055
	800	2.826	5.326	1.682	11.4	0.01081
	1000	2.833	5.352	1.690	11.4	0.05794
	2000	2.829	5.419	1.711	11.4	1.090
	4000	2.810	5.094	1.609	11.4	2.690
	6000	2.714	4.540	1.421	11.5	2.784
	8000	2.393	4.006	1.254	11.5	2.495
	10000	2.072	3.547	1.110	11.5	2.163
	20000	1.086	2.130	0.6611	11.6	1.123
	40000	0.4665	1.074	0.3305	11.7	0.4808
	60000	0.2681	0.6734	0.2055	11.8	0.2789
	100000	0.1330	0.3547	0.1073	11.9	0.1364
0	100	1.069	1.888	5.861	11.6	
	200	1.086	1.893	5.824	11.7	
	400	1.066	1.902	5.804	11.8	
	600	1.072	1.921	5.736	12.0	
	800	1.066	1.921	5.762	12.0	0.00998
	1000	1.072	1.929	5.788	12.0	0.04319
	2000	1.070	1.937	5.764	12.1	0.5238
	4000	1.073	1.773	5.233	12.2	1.052
	6000	0.9646	1.550	4.573	12.2	1.000
	8000	0.8110	1.350	3.951	12.3	0.8794
	10000	0.6747	1.184	3.466	12.3	0.7470
	20000	0.3368	0.6946	1.985	12.6	0.3720
	40000	0.1461	0.3445	0.9615	12.9	0.1560
	60000	0.0844	0.2145	0.5895	13.1	0.08899
	100000	0.0416	0.1121	0.2968	13.6	0.04374
-0.2	100	0.4127	0.4176	0.06681	22.5	
	200	0.4118	0.4175	0.06650	22.6	
	400	0.4069	0.4162	0.06486	23.1	0.00022
	600	0.6017	0.4140	0.06424	23.2	0.00896
	800	0.3949	0.4107	0.06373	23.2	0.04340
	1000	0.3859	0.4050	0.06204	23.5	0.09816
	2000	0.3123	0.3509	0.05242	24.1	0.2893
	4000	0.1770	0.2432	0.03516	24.9	0.2452
	6000	0.1083	0.1766	0.02455	25.9	0.1657
	8000	0.07258	0.1350	0.01820	16.7	0.1161
	10000	0.05216	0.1073	0.01410	27.4	0.08556
	20000	0.01868	0.04740	0.00552	30.9	0.03012
	40000	0.00607	0.01835	0.00178	37.0	0.01011
	60000	0.00297	0.01011	0.00087	41.5	0.00537
	100000	0.00115	0.00465	0.00034	48.9	0.00267

3-b

Table 2. Continued.

T = 1 h (3600 s) h = 150 m

ζ	X (m)	C_{max} (10^{-5})	D (10^{-5})	$\bar{c}$ (10^{-5})	λ(s) (10^3)	c_c (10^{-5})
0.4	100	1.517	0.7965	0.829	3.46	
	200	1.521	0.8179	0.839	3.51	
	400	1.528	0.8606	0.861	3.60	
	600	1.534	0.9032	0.879	3.70	
	800	1.540	0.9468	0.895	3.81	
	1000	1.540	0.9896	0.918	4.09	0.0011
	2000	1.541	1.196	1.05	4.11	0.1552
	4000	1.541	1.441	1.26	4.12	1.021
	6000	1.541	1.458	1.27	4.15	1.463
	8000	1.530	1.376	1.19	4.16	1.540
	10000	1.442	1.264	1.09	4.17	1.472
	20000	0.8973	0.7949	0.670	4.27	0.9269
	40000	0.4267	0.3940	0.321	4.42	0.4372
	60000	0.2556	0.2423	0.193	4.53	0.2620
	100000	0.1291	0.1244	0.0953	4.71	0.1366
0	100	0.5434	0.3003	0.276	3.92	
	200	0.5449	0.3080	0.279	3.98	
	400	0.5476	0.3233	0.286	4.07	
	600	0.5500	0.3387	0.292	4.17	
	800	0.5524	0.3541	0.299	4.27	
	1000	0.5528	0.3695	0.306	4.35	0.0006
	4000	0.5541	0.5203	0.402	4.66	0.0006
	6000	0.5544	0.5164	0.392	4.74	0.3880
	8000	0.5363	0.4805	0.360	4.81	0.5322
	10000	0.4936	0.4370	0.324	4.86	0.5480
	20000	0.2964	0.2676	0.188	5.12	0.5167
	40000	0.1384	0.1300	0.0850	5.51	0.3170
	60000	0.08321	0.07908	0.0489	5.82	0.8804
	100000	0.04199	0.04007	0.0229	6.30	0.04403
-0.2	100	0.1427	0.1007	0.0336	10.8	
	200	0.1428	0.1022	0.0341	10.8	
	400	0.1450	0.1051	0.0347	10.9	
	600	0.1416	0.1075	0.0343	11.3	
	800	0.1433	0.1110	0.0345	11.6	0.00024
	1000	0.1418	0.1134	0.0352	11.6	0.00163
	2000	0.1346	0.1162	0.0337	12.4	0.04223
	4000	0.1040	0.9713	0.0255	13.7	0.1040
	6000	0.07728	0.7470	0.0177	15.2	0.09978
	8000	0.05607	0.5746	0.0125	16.5	0.08317
	10000	0.04094	0.4498	0.00915	17.7	0.06788
	20000	0.01256	0.1743	0.00265	23.7	0.02907
	40000	0.00302	0.5884	0.000666	31.8	0.01062
	60000	0.00128	0.3104	0.000292	38.3	0.00615
	100000	0.00044	0.1407	0.000107	47.3	0.00268

1–c

T = 2 h (7200 s) h = 150 m

ζ	X (m)	C_{max} (10^{-5})	D (10^{-5})	$\bar{c}$ (10^{-5})	(s) (10^3)	c_c (10^{-5})
0.4	100	1.541	2.205	1.12	7.12	
	200	1.540	2.222	1.12	7.17	
	400	1.539	2.256	1.12	7.27	
	600	1.540	2.288	1.12	7.37	
	800	1.540	2.322	1.10	7.46	
	1000	1.540	2.354	1.09	7.76	0.0011
	2000	1.542	2.505	1.16	7.78	0.1552
	4000	1.539	2.635	1.22	7.79	1.021
	6000	1.542	2.544	1.77	7.81	1.463
	8000	1.515	2.363	1.09	7.82	1.540
	10000	1.416	2.164	0.993	7.85	1.472
	20000	0.8843	1.392	0.633	7.92	0.9269
	40000	0.6241	0.7197	0.322	8.05	0.4372
	60000	0.2561	0.4530	0.200	8.15	0.2620
	100000	0.1289	0.2385	0.103	8.32	0.1366
0	100	0.5533	0.7963	0.372	7.71	
	200	0.5536	0.8021	0.372	7.76	
	400	0.5540	0.8136	0.373	7.86	
	600	0.5543	0.8249	0.373	7.97	
	800	0.5536	0.8364	0.374	8.06	
	1000	0.5543	0.8476	0.368	8.30	
	2000	0.5545	0.8997	0.387	8.38	0.00064
	4000	0.5538	0.9346	0.398	8.45	0.00067
	6000	0.5538	0.8910	0.377	8.51	0.3880
	8000	0.5364	0.8199	0.344	8.58	0.5322
	10000	0.4943	0.7454	0.311	8.63	0.5480
	20000	0.2955	0.4703	0.191	8.85	0.5167
	40000	0.1384	0.2397	0.0938	9.20	0.3170
	60000	0.0831	0.1499	0.0569	9.49	0.8804
	100000	0.0419	0.0782	0.0282	9.99	0.04403
-0.2	100	0.1442	0.1841	0.0377	17.6	
	200	0.1441	0.1848	0.0378	17.6	
	400	0.1441	0.1860	0.0372	18.0	
	600	0.1397	0.1874	0.0373	18.1	
	800	0.1387	0.1886	0.0369	18.4	0.00024
	1000	0.1397	0.1894	0.0365	18.7	0.00163
	2000	0.1346	0.1846	0.0346	19.2	0.04223
	4000	0.1050	0.1529	0.0268	20.5	0.1040
	6000	0.07814	0.1209	0.0202	21.5	0.09978
	8000	0.05913	0.09634	0.0154	22.5	0.08317
	10000	0.04597	0.07815	0.0120	23.5	0.06788
	20000	0.01728	0.03459	0.00438	28.5	0.02907
	40000	0.00507	0.01274	0.00127	36.0	0.01062
	60000	0.00232	0.00682	0.00059	41.6	0.00615
	100000	0.00084	0.00309	0.00021	51.1	0.00268

2–c

T = 3 h (10800 s) h = 150 m

ζ	X (m)	C_{max} (10^{-5})	D (10^{-5})	$\bar{c}$ (10^{-5})	λ(s) (10^3)	c_c (10^{-5})
0.4	100	1.538	3.226	1.075	20.8	
	200	1.539	3.238	1.073	10.8	
	400	1.539	3.262	1.077	10.9	
	600	1.540	3.286	1.075	11.0	
	800	1.541	0.3308	1.073	11.1	
	1000	1.538	3.303	1.052	11.4	0.0011
	2000	1.535	3.438	1.086	11.4	0.1552
	4000	1.540	3.487	1.101	11.4	1.021
	6000	1.542	3.326	1.041	11.5	1.463
	8000	1.529	3.082	0.9649	11.5	1.540
	10000	1.440	2.828	0.8854	11.5	1.472
	20000	0.9014	1.859	0.5769	11.6	0.9269
	40000	0.4248	0.9940	0.3058	11.7	0.4372
	60000	0.2522	0.6381	0.1947	11.8	0.2620
	100000	0.1283	0.3430	0.1038	11.9	0.1366
0	100	0.5536	1.148	0.3592	11.5	
	200	0.5534	1.152	0.3606	11.5	
	400	0.5531	1.160	0.3599	11.6	
	600	0.5526	1.168	0.3593	11.7	
	800	0.5521	1.175	0.3586	11.8	
	1000	0.5530	1.183	0.3521	12.1	
	2000	0.5542	1.220	0.3630	12.1	0.00064
	4000	0.5539	1.226	0.3618	12.2	0.00067
	6000	0.5529	1.158	0.3389	12.2	0.3880
	8000	0.5231	1.065	0.3116	12.3	0.5322
	10000	0.4787	0.9713	0.2820	12.4	0.5480
	20000	0.2918	0.6282	0.1795	12.6	0.5167
	40000	0.1387	0.3320	0.0926	12.9	0.3170
	60000	0.0828	0.2120	0.0578	13.2	0.8804
	100000	0.0419	0.1133	0.0297	13.7	0.04403
-0.2	100	0.1418	0.2276	0.03400	24.1	
	200	0.1416	0.2279	0.03391	24.2	
	400	0.1411	0.2284	0.03383	24.3	
	600	0.1401	0.2286	0.03345	24.6	
	800	0.1418	0.2297	0.03281	25.2	0.00024
	1000	0.1401	0.2299	0.03284	25.2	0.00163
	2000	0.1270	0.2221	0.03173	25.8	0.04223
	4000	0.1033	0.1858	0.02489	26.8	0.1040
	6000	0.07698	0.1491	0.01930	27.8	0.09978
	8000	0.05767	0.1210	0.01524	28.6	0.08317
	10000	0.04480	0.1000	0.01229	29.3	0.06788
	20000	0.01838	0.04788	0.00517	33.3	0.02907
	40000	0.00616	0.01921	0.00171	41.3	0.01062
	60000	0.00298	0.01066	0.00084	45.9	0.00615
	100000	0.00116	0.00493	0.00032	54.4	0.00268

3–c

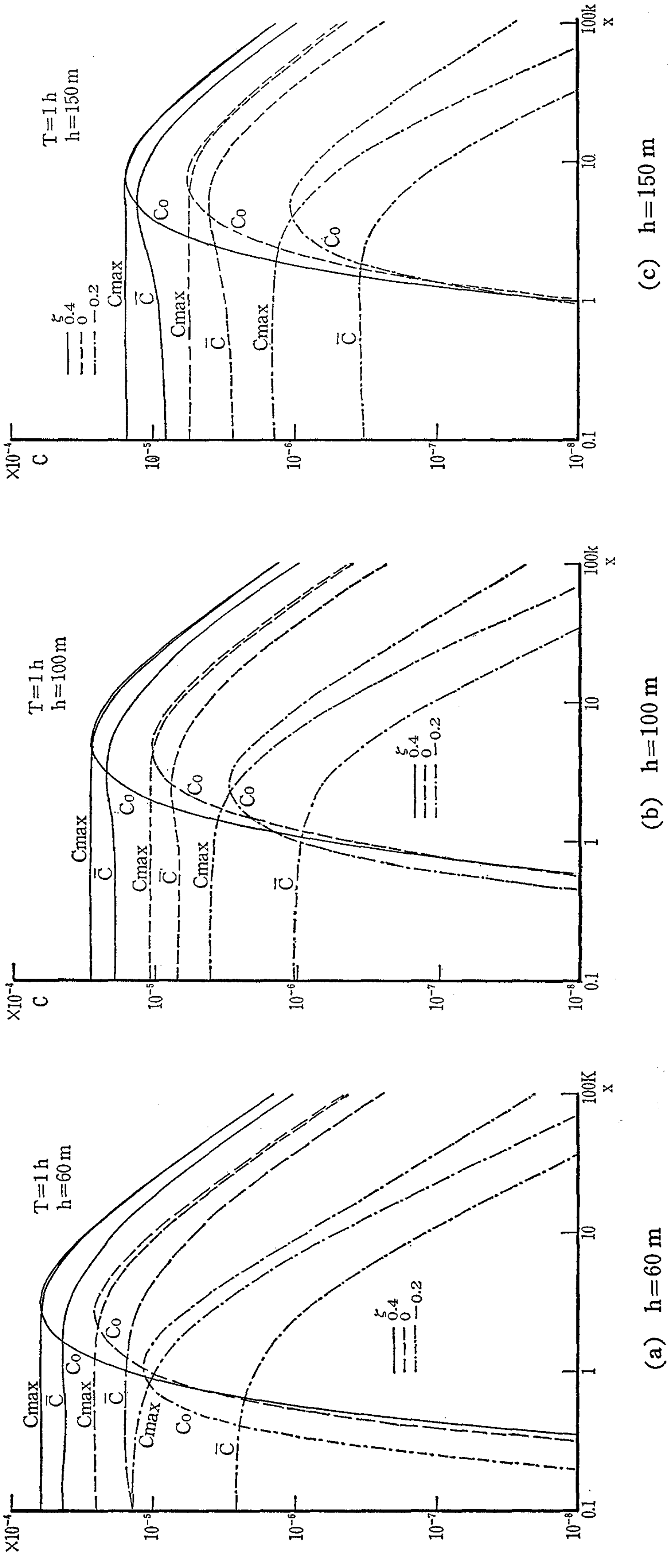


Fig. 5. Relations between C_{max}, D, $\bar{C}$, C_c and x for T=1 h.

or mean concentration $\bar{C}$

$$\bar{C}=D/\lambda=\int_0^\infty Cdt/\lambda$$

are essential. However, though λ is infinite theoretically, it is finite in practical sense, and there may be some ambuguities in determining the both ends of the interval. We assumed that the ends are the instants when the concentrations are 10^{-4} of the maximum concentration.

We show the relations between $C_{\max}$, $\bar{C}$, λ and x, together with C_c which is the concentraion of a continuous plume. (Table 2).

Results of the cases $T=1\ h$, $\zeta=0$: $h=60$, 100 and 150 m are shown in Fig. 5 a, b and c.

As a representative case, time changes of concentration for $T=1$, 2 and 3 h are shown in Fig. 6. When T's become longer, the curves donot change in the main part and only extend themselves their latter parts.

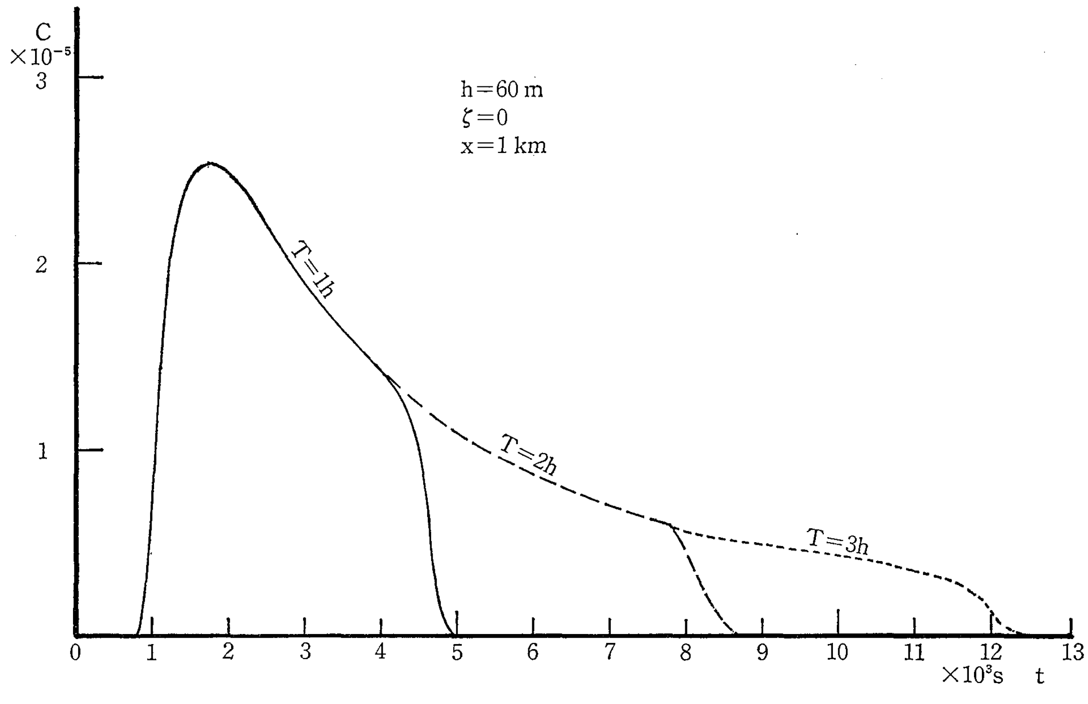


Fig. 6. Time changes of concentration for T=1, 2 and 3 h.

2) $\theta=175°$: The ground level concentrations were calculated in the cases of $h=60\ m$, $T=1\ h$, $\zeta=0$, $u_1=u_2=1\ m/s$; $x=100$, $y=0$ ($l_1=-99.62$, $l_2=8.716$) m and $x=1000$, $y=0$ ($l_1=-996.2$, $l_2=87.16$) m.

The results are shown in Fig. 7. The graphs differ considerably with those in the case $\theta=180°$. The values of $\bar{C}$ are shown in Table 3.

3) $\theta=90°$: The calculation was carried out in the cases $h=60\ m$, $T=1\ h$, $\zeta=0$, $u_1=u_2=1\ m/s$: $x=100$, $y=100$ ($l_1=100$, $l_2=100$) ; $x=100$,

$y=1000$ ($l_1=1000$, $l_2=100$); $x=1000$, $y=100$ ($l_1=100$, $l_2=1000$) and $x=1000$, $y=1000$ ($l_1=1000$, $l_2=1000$) m.

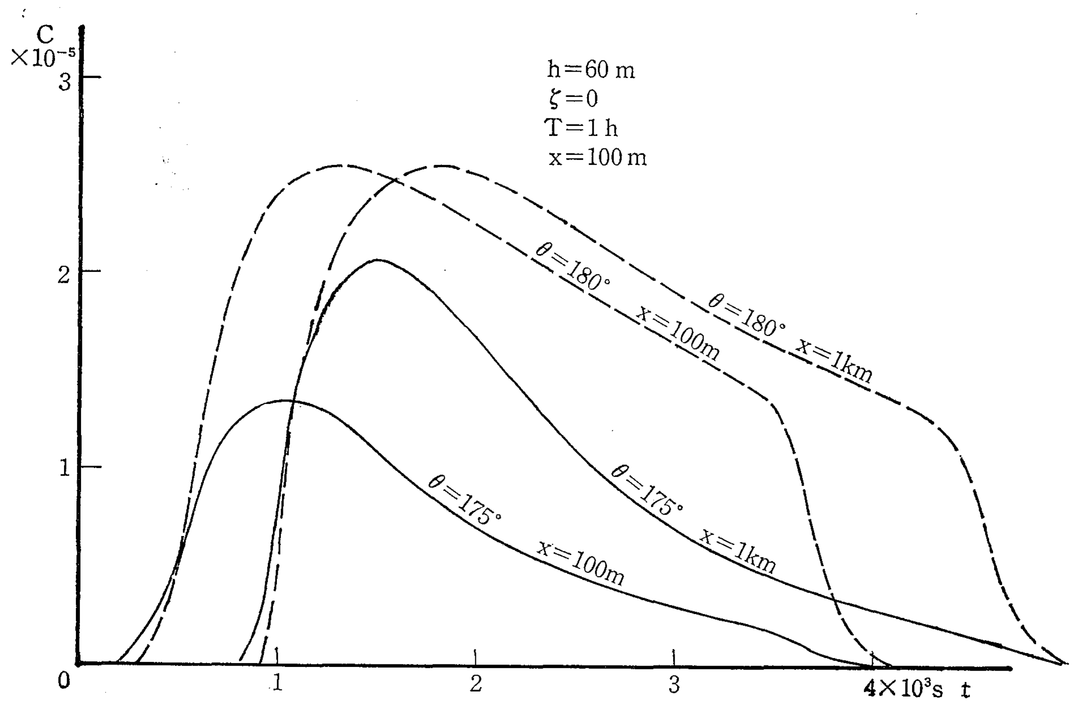


Fig. 7. Time changes of concentration for $\theta=175°$ and $180°$ in the case of h=60 m, $\zeta=0$, T=1 h and x=100 m.

Table 3. Relations between C_{max}, D, $\bar{C}$, λ and $\theta°$.

T = 1 h (3600 s), h = 60 m, ζ = 0

θ (o)	x (m)	y (m)	C_{max}	D (ch)	$\bar{C}$	λ (s)
180	100	0	2.55(-5)	1.47(-5)	1.57(-5)	3700
175	100	0	1.34(-5)	6.36(-6)	6.18(-6)	3700
	1000	0	2.07(-5)	9.94(-6)	8.73(-6)	4100
90	100	100	1.27(-12)	1.65(-12)	4.94(-11)	130
	100	1000	1.20(-12)	4.69(-5)	5.76(-6)	260
	1000	100	1.00(-5)	3.75(-4)	3.99(-6)	330
	1000	1000	1.16(-5)	4.42(-4)	4.65(-6)	350

The results are shown in Fig. 8 and 9. The exposed intervals become very short compared with above two cases and the values of $\bar{C}$ become consierably small (Table 3).

Consideration

1) The value of u_1 affects the concentration and the length of the plume in the first stage, and that of u_2 affects the duration of the exposed interval and on the value of dosage, but does not affect $\bar{C}$.

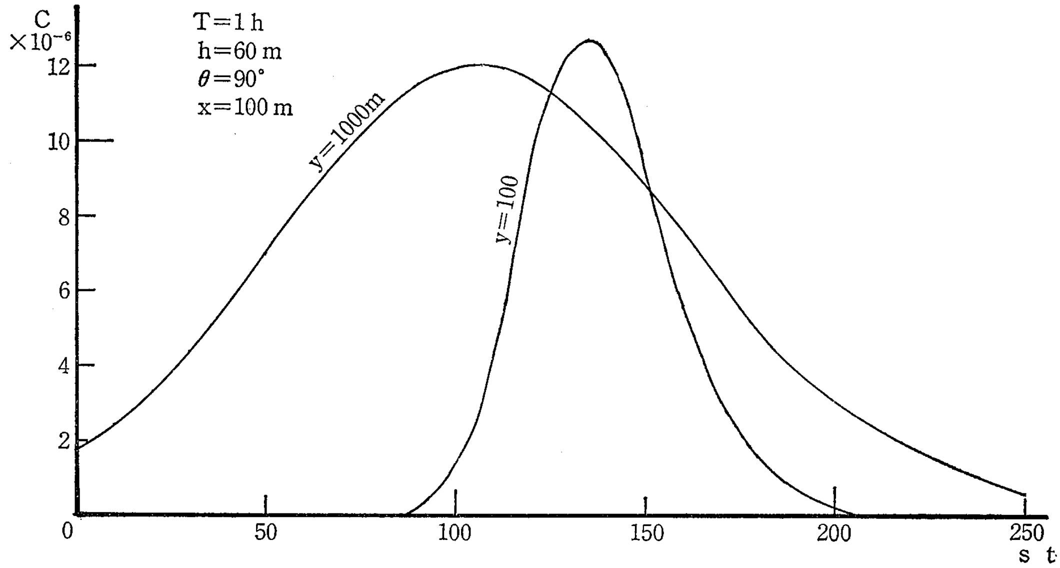


Fig. 8. Time changes of concentration in the cases of $\theta=90°$, $h=60$ m, $\zeta=0$, $T=1$ h, $x=100$ m, $y=100$ m and 1000 m.

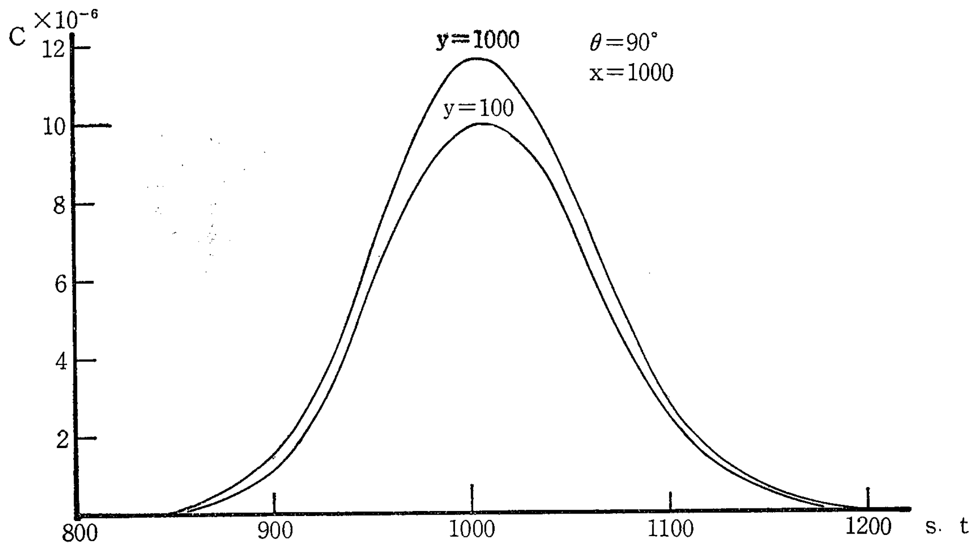


Fig. 9. Time changes of concentration in the cases of $\theta=90°$, $h=60$ m, $\zeta=0$, $T=1$ h, $x=1000$ m, $y=100$ and 1000 m.

2) In the case when θ is nearly 180°, the concentrations of continuous source C_c are very low in the region near the source, but those resulted in the second stage are considerably high.

3) The values of $\bar{C}$ in the case of 175° are considerably less than those in the case of 180°. Only the difference of 5° affects remarkably.

4) When $\theta=$ is nearly 90°, the exposed intervals become very short, and the concentrations become much lower.

Literature

(1) Sakagami, J.: 1954, On the Turbulent Diffusion in the Atmosphere, Natural Science Rep., Ochanomizu Univ., 5 (1), pp. 79-91.
Sakagami, J.: 1960, On the Relations between the Diffusion Parameters and Meterological Conditions, ditto, 11 (1), pp. 7-27.