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On the Vertical Concentration Distribution of the Atmospheric Diffusion

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For the atmospheric diffusion, the vertical concentration distribution is one of the most important factors. Almost all diffusion theories hitherto assumed that the vertical concentration profile has the normal (Gaussian) form. We have criticized that assumption and have published a series of papers.[1] However, there have been scarcely any reliable data on the vertical profiles up to some tens meters high, so it was difficult to investigate them without any ambiguities.

The report "Green Glow Diffusion Program"[2] contains very valuable data on vertical concentration profiles, so we analysed the data in order to make clear the vertical diffusion phenomena in the atmosphere.

The concentration distributions can be quantitatively determined only by solving the Fickian equation which has the following form:

$$u\frac{\partial C}{\partial x}=\frac{\partial}{\partial z}\left(K_z\frac{\partial C}{\partial z}\right)=\frac{\partial}{\partial z}\left(kz^{\alpha}\frac{\partial C}{\partial z}\right) \quad (1),$$

where C is concentration, u is wind speed, x and z are coordinates leeward and vertically upward respectively: K_z is the vertical diffusion coefficient which is function of z, namely kz^{α}, in which k and α are constants.

CASE 1: When K_z is constatnt, namely $\alpha=0$. The solution of eq. (1) for instantaneous point source at the height h is

$$C=\frac{1}{\sqrt{B_1\pi}}\left\{\exp\left(\frac{(h+z)^2}{B_1}\right)+\exp\left(\frac{(h-z)^2}{B_1}\right)\right\}$$

$$B_1=\frac{4kx}{u} \quad (2).$$

When the source is on the ground, namely $h=0$, eq. (2) takes the following form:

$$C=\frac{2}{\sqrt{B_1\pi}}e^{-\frac{z^2}{B_1}} \quad (3).$$

Even if h is not zero, when z is large compared with h, eq. (2) becomes

$$C=\frac{1}{\sqrt{B_1\pi}}\exp\left\{-\frac{z^2}{B_1}\left(1+\frac{h}{z}\right)^2\right\}\left\{1+\exp\left(-\frac{4hz}{B_1}\right)\right\}\sim\frac{1}{\sqrt{B_1\pi}}e^{-\frac{z^2}{B_1}} \tag{4}$$

CASE 2: When K_z is proportional to z, namely $\alpha=1$, the solution is given by[3]

$$\left.\begin{aligned} C&=\frac{1}{B_2}e^{-\frac{h+z}{B_2}}J_0\left(i\frac{2\sqrt{hz}}{B_2}\right)\\ B_2&=\frac{kx}{u}\end{aligned}\right\} \tag{5}$$

where $J_0(i\xi)$ is the 1st kind Bessel function of the order zero with imaginary argument.

When the source is on the ground, eq. (4) takes the following form:

$$C=\frac{1}{B_2}e^{-\frac{z}{B_2}} \tag{6}$$

Even if h is not zero, when z is large compared with h, eq. (5) becomes

$$C=\frac{1}{B_2}e^{-\frac{h+z}{B_2}}J_0\left(i\frac{2\sqrt{hz}}{B_1}\right)=\frac{1}{B_2}e^{-\frac{z}{B_2}\left(1+\frac{h}{z}\right)}J_0\left(i\frac{2\sqrt{hz}}{B_2}\right) \tag{7}$$

When $2\sqrt{hz}/B_2$ is small, eq. (7) becomes

$$C=\frac{1}{B_2}e^{-\frac{z}{B_2}\left(1+\frac{h}{z}\right)}\left(1+\frac{hz}{B_2^2}\right)\sim\frac{1}{B_2}e^{-\frac{z}{B_2}} \tag{8}$$

and if $2\sqrt{hz}/B_2$ is large, eq. (7) becomes

$$C=\frac{1}{B_2}e^{-\frac{z}{B_2}\left(1+\frac{h}{z}\right)}\frac{e^{\frac{2\sqrt{hz}}{B_2}}}{\sqrt{2\pi\frac{2\sqrt{hz}}{B_2}}}\left(1+\frac{B_2}{16\sqrt{hz}}\right)=\frac{1}{B_2}e^{\frac{-\frac{z}{B_2}\left(1-\sqrt{\frac{h}{z}}\right)^2\left(1+\frac{B_2}{16\sqrt{hz}}\right)}{2\sqrt{\frac{\pi}{B_2}}(hz)^{1/4}}}$$

$$\sim\frac{1}{B_2}e^{-\frac{z}{B_2}} \tag{9}$$

When we plot logC against z from the data, and if the vertical diffusion proceeds as the case 1, the points cannot arrange themselves on straight lines, and when the diffusion proceeds as the case 2, these points will show straight lines. On the other hand, if the case 1 is real, the plotted points for logC against z^2 can show straight lines.

One of the results, Experiment No. 11, is shown in Fig. 1,* and marks ○ show the observed results. These marks for larger values of z arrange themselves on a straight line (full line), but the marks × plotted against z^2 do not show any straight line (broken line).

* In the figures, φ means the azimuthal angle of the observing post.

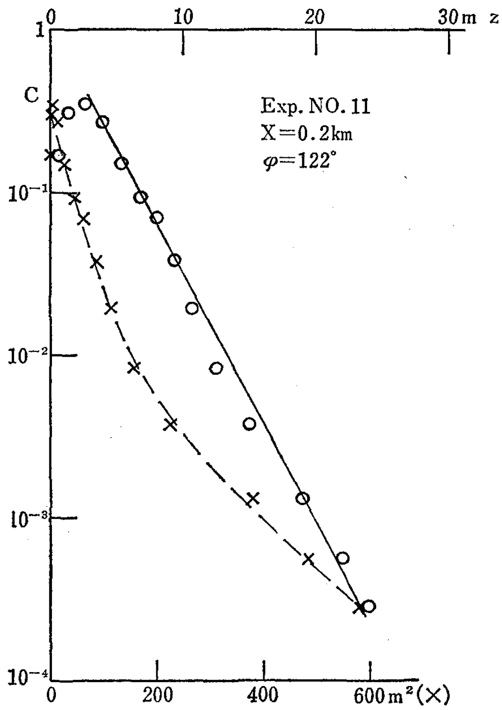

Fig. 1. Vertical concentration profile
Exp. No. 11, x=0.2 km, φ=122°

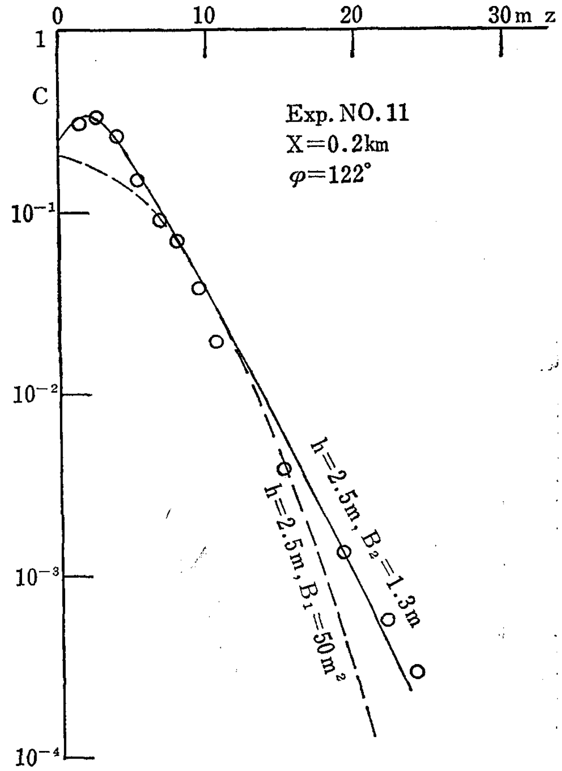

Fig. 2. Vertical concentration profile
Exp. No. 11, x=0.2 km, φ=122°

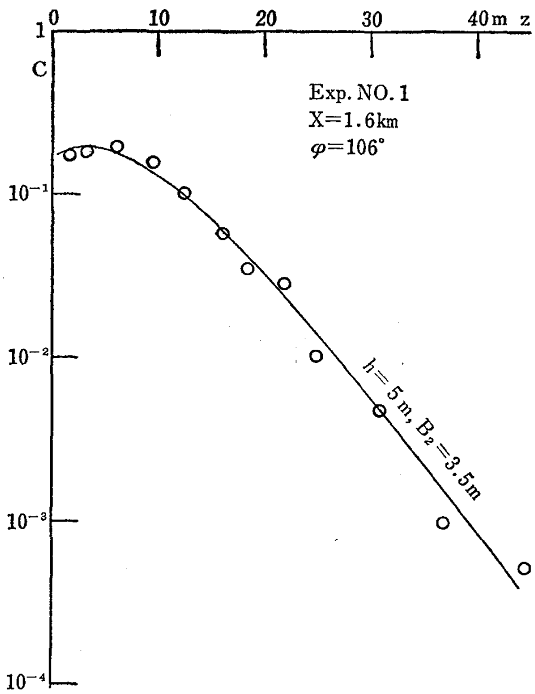

Fig. 3. Vertical concentration profile
Exp. No. 1, x=1.6 km, φ=106°

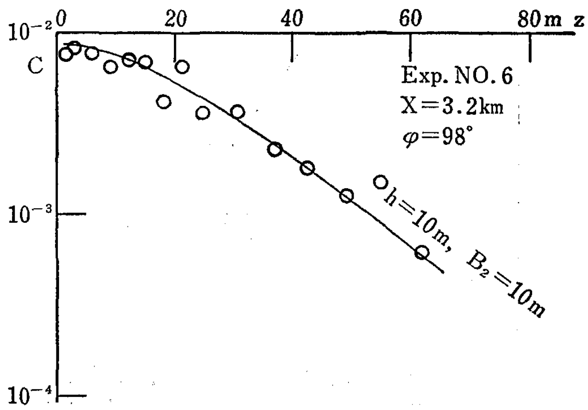

Fig. 4. Vertical concentration profile
Exp. No. 6, x=3.2 km, φ=98°

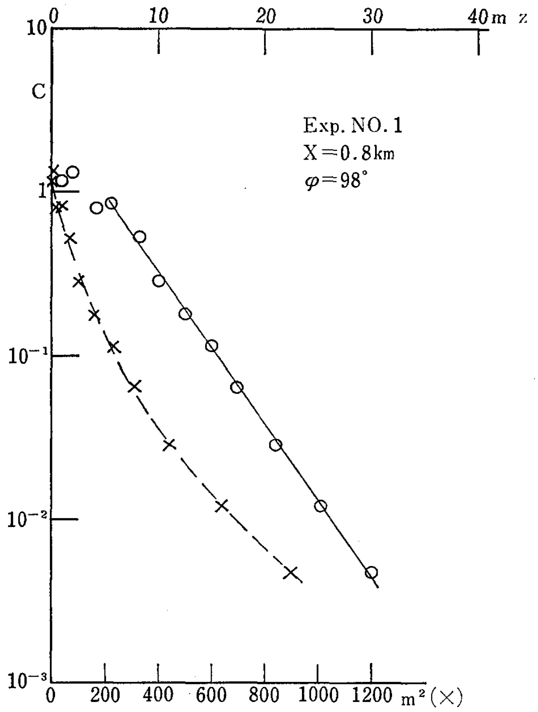

Fig. 5. Vertical concentration profile Exp. No. 1, x=0.8 km, φ=98°

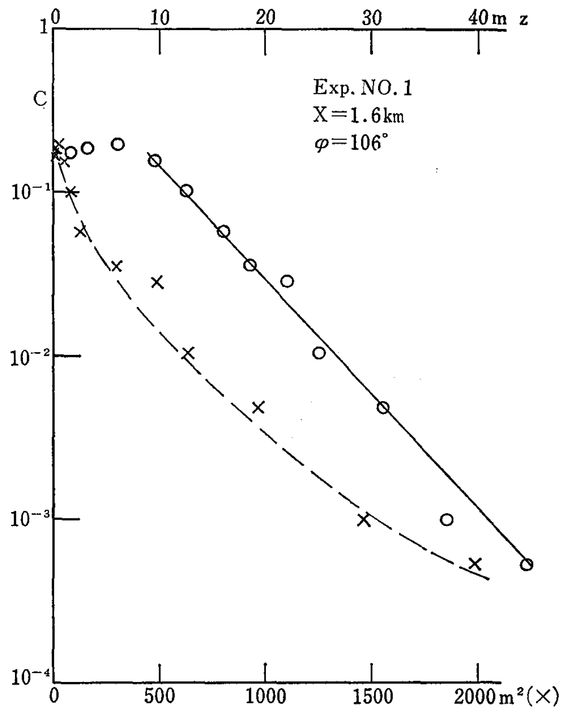

Fig. 6. Vertical concentration profile Exp. No. 1, x=1.6 km, φ=106°

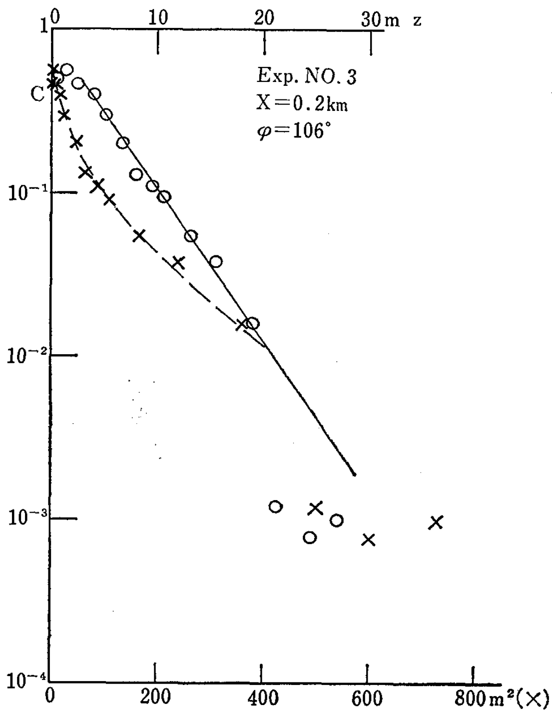

Fig. 7. Vertical concentration profile Exp. No. 3, x=0.2 km, φ=106°

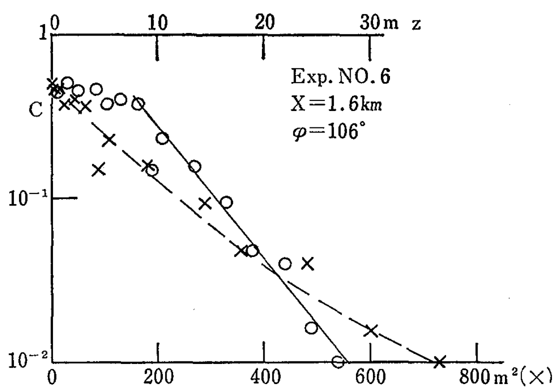

Fig. 8. Vertical concentration profile Exp. No. 6, x=1.6 km, φ=106°

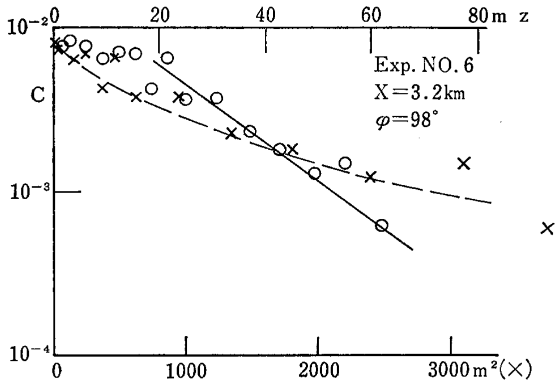

Fig. 9. Vertical concentration profile
Exp. No. 6, x=3.2 km, φ=98°

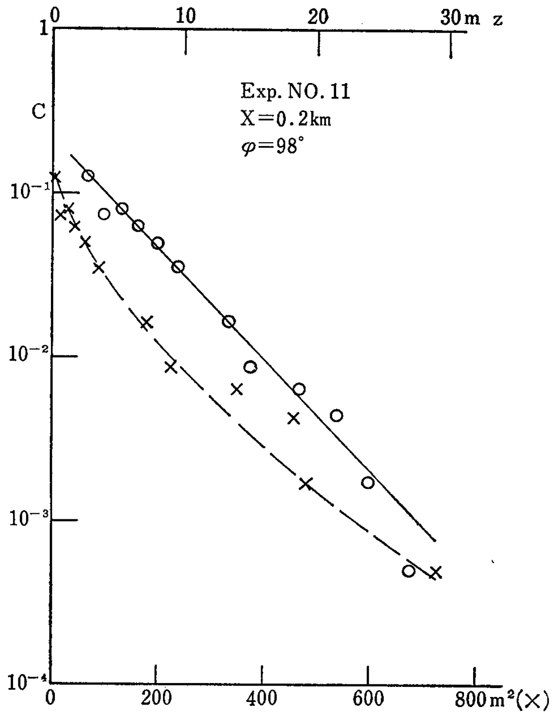

Fig. 10. Vertical concentration profile
Exp. No. 11, x=0.2 km, φ=98°

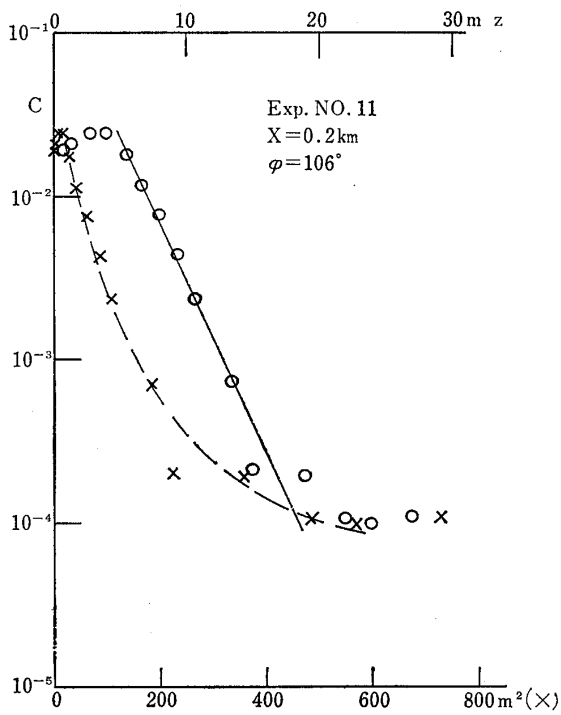

Fig. 11. Vertical concentration profile
Exp. No. 11, x=0.2 km, φ=106°

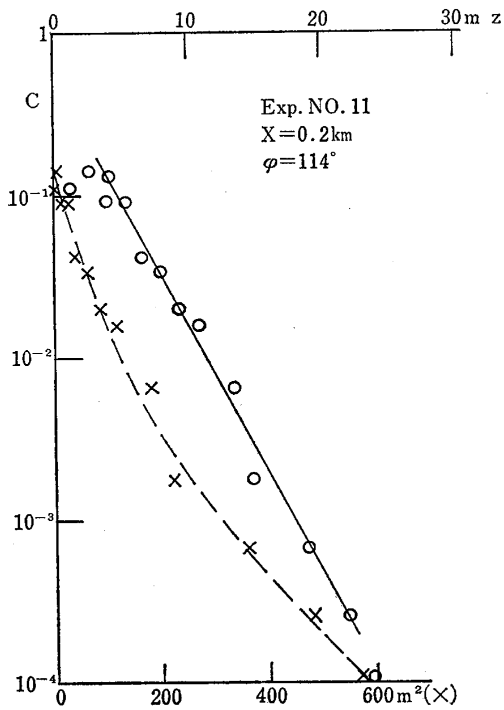

Fig. 12. Vertical concentration profile
Exp. No. 11, x=0.2 km, φ=114°

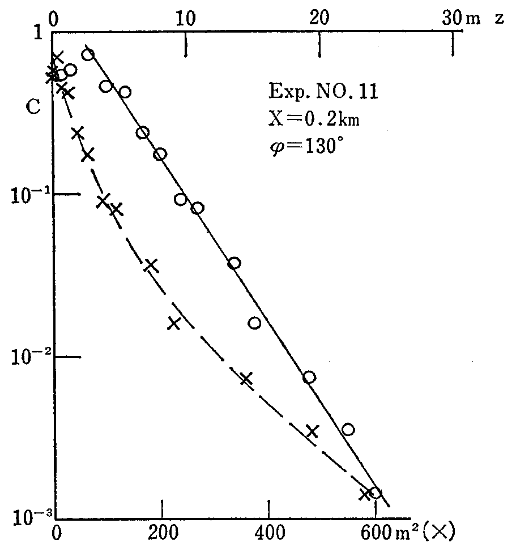

Fig. 13. Vertical concentration profile
Exp. No. 11, x=0.2 km, φ=130°

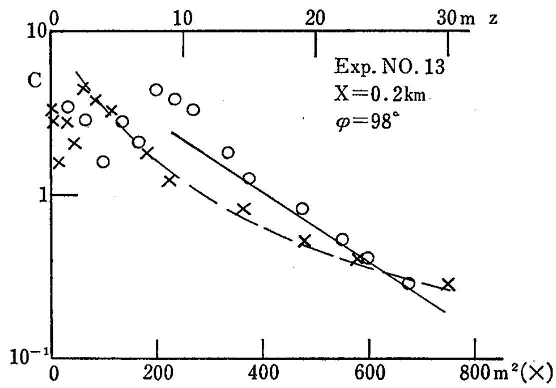

Fig. 14. Vertical concentration profile
Exp. No. 13, x=0.2 km, φ=98°

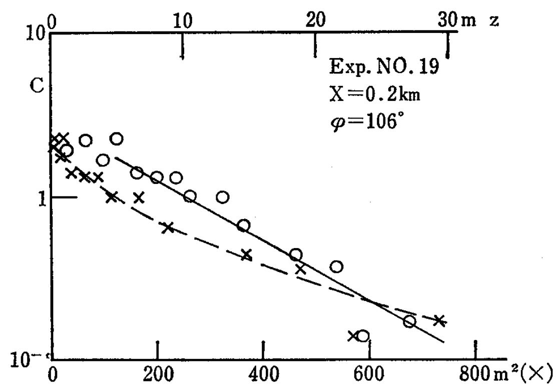

Fig. 16. Vertical concentration profile
Exp. No. 19, x=0.2 km, φ=106°

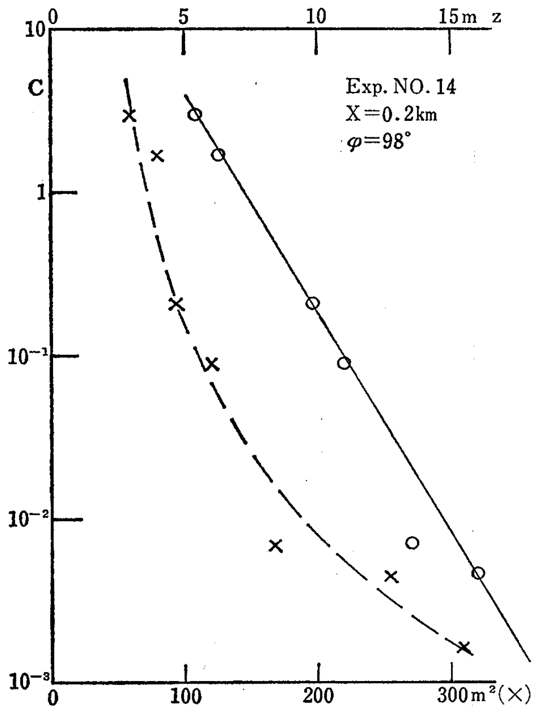

Fig. 15. Vertical concentration profile
Exp. No. 14, x=0.2 km, φ=98°

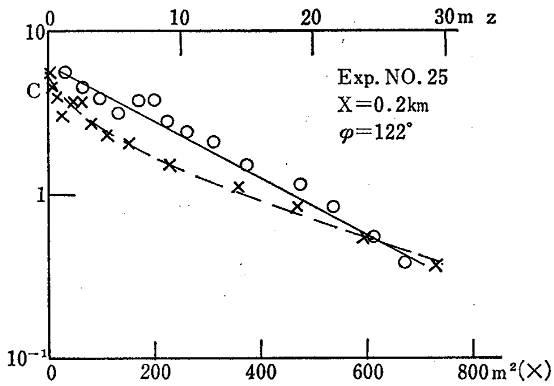

Fig. 17. Vertical concentration profile
Exp. No. 25, x=0.2 km, φ=122°

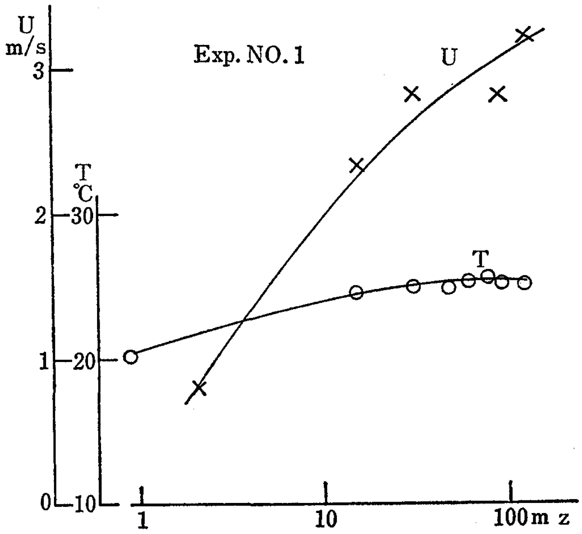

Fig. 18. Vertical profiles of wind speed and temperature Exp. No. 1

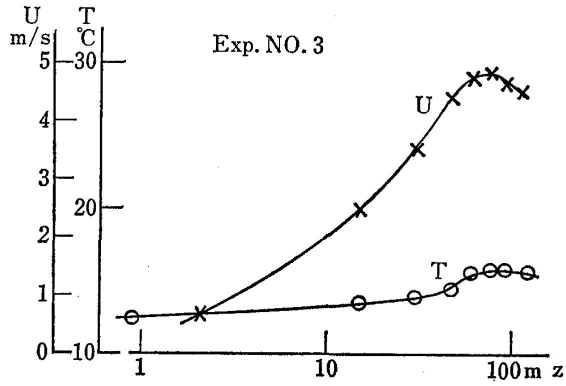

Fig. 19. Vertical profiles of wind speed and temperature Exp. No. 3

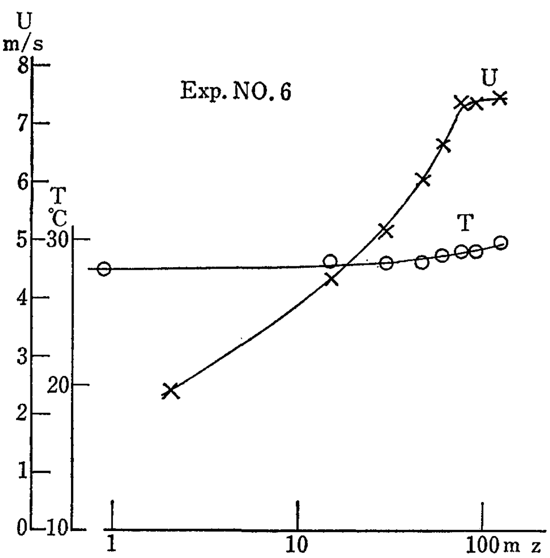

Fig. 20. Vertical profiles of wind speed and temperature Exp. No. 6

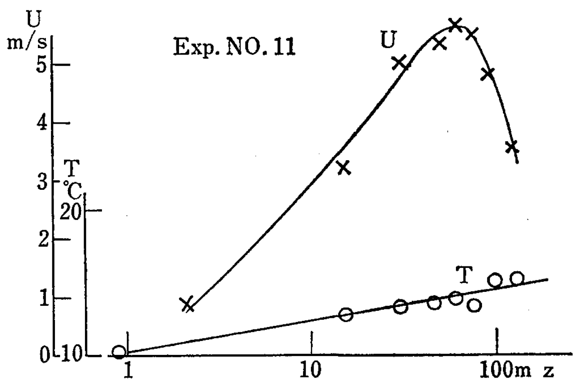

Fig. 21. Vertical profiles of wind speed and temperature Exp. No. 11

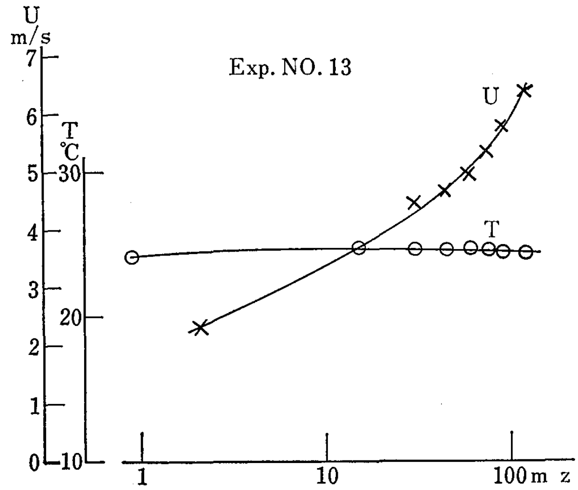

Fig. 22. Vertical profiles of wind speed and temperature Exp. No. 13

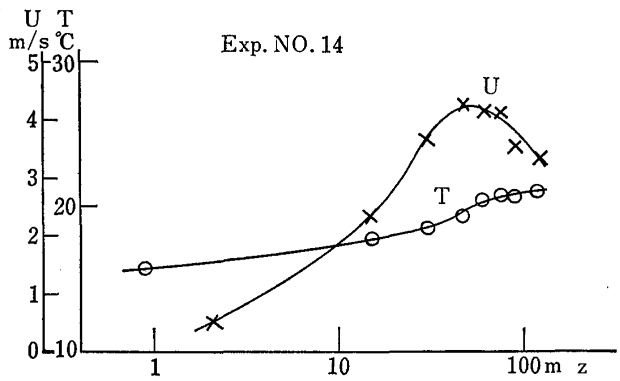

Fig. 23. Vertical profiles of wind speed and temperature Exp. No. 14

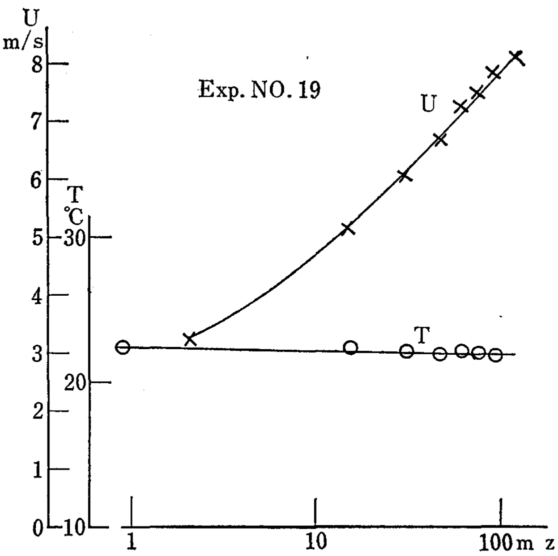

Fig. 24. Vertical profiles of wind speed and temperature Exp. No. 19

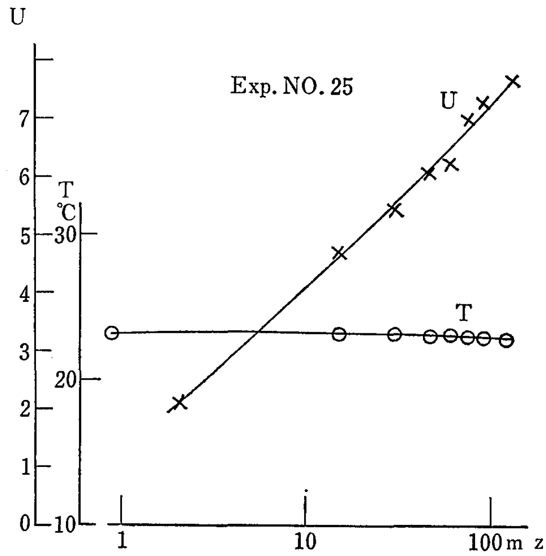

Fig. 25. Vertical profiles of wind speed and temperature Exp. No. 25

In the Green Glow Program, the source height was not exactly zero, but 2 to 8 m above the ground, so the points for smaller values of z deviate somewhat from straight line. Applying eq. (5) to the data of the Experiment No. 11, we obtain the full line in Fig. 2 and it agrees well to the observed data, but the result calculated from eq. (2) with the most adequate values of h and B_1 (broken line) does not fit the data. Even for the results at far leeward distances, 1.6 and 3.2 km, eq. (5) agrees well to the distributions (Fig. 3 and 4). Therefore, we can disregard the deviation for smaller values of z to criticize the true nature of the vertical diffusion.

Figures 5~17 are similar to Fig. 1, and Fig. 18~25 show the vertical profiles of wind speed and temperature for each experiments as the fundamental meteorological data. All the results of vertical concentration distributions indicate that the vertical diffusion coefficient is not constant, but it is proportional to height. So we can conclude that the normal (Gaussian) form is not adequate to express the vertical concentration profiles in the atmosphere.

Literature

1) Sakagami, J., 1954: Natural Science Rep., Ochanomizu University, **5** (1), pp. 79–91: 1956, ditto, **7** (1) pp. 25–61: 1960, ditto, **11** (2), pp. 127–259: 1961, ditto, **12** (1), pp. 7–27: 1962, ditto **12** (2), pp. 1–24: 1962, ditto, **13** (1), pp. 11–21: 1962, ditto, **13** (2), pp. 33–45: 1963, ditto, **14** (1), pp. 17–36 (Collaborators:—Kimura, Y. and Kato, M.): 1965, **16** (1), pp. 21–36: 1966, ditto, **17** (2), pp. 33–43, (Collaborator:—Kato, M.): 1968, ditto **19** (1), pp. 1–21 (Collaborator:—Kato, M.): 1970, Proc. 16th National Congr. Appl. Mech., 1968, pp. 135–144.

2) 1962: Geophysical Res. Papers No. 73, Geophy. Res. Directorate, Air Force Cambridge Res. Lab., Vol. I and II.

3) Sakagami, J.: 1954, Natural Sci. Rep., Ochanomizu Univ., **5** (1), pp. 79–91.