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Representation-free Conservation Laws

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§ 1. Introduction

Field invariants[1] such as the energy momentum tensor, the angular momentum tensor and the charge current vector associated to a field have been derived from the scalar Lagrangian density of the field so as to satisfy their respective conservation laws by virtue of the Noether theorem[1,2]. These field invariants are expressed as functions varying from a point in space-time to another. In other words they are regarded as functions in space-time and written in the space-time representation or the x-representation.

As in the preceding papers,[3,4] we postulate that the fundamental laws of physics should be expressed free from representation frames in the Hilbert space where the norm of a wave function $u(x^0, x^1, x^2, x^3)$ is defined by

$$\iiiint_{-\infty}^{\infty} |u(x^0, x^1, x^2, x^3)|^2 \, dx^0 dx^1 dx^2 dx^3$$

that is, the square of the absolute value of u integrated over the entire space-time.

We derive here field invariants free from representations such as the x-representation or the p-representation, by establishing an analogue of the Noether theorem, for the inhomogeneous Lorentz transformation and for the gauge transformation of the first kind.

In § 2, we summarize the Noether theorem after Fonda and Ghirardi.[2] In § 3, we introduce the representative of the Lagrangian density by way of the action. In § 4, we formulate the Noether theorem free from representations. In § 5, we define the energy momentum tensor, the angular momentum tensor and the charge current vector free from representations with the aid of the Noether theorem formulated above. In § 6, we note some additional remarks concerning conservation laws and compare conventional field invariants and those in our formulation.

§ 2. The Noether Theorem

Conservation laws in the field theory are founded on the basis of the Noether theorem. The Noether theorem is derived from the assumption that the Lagrangian density is invariant under an infinitesimal inhomogeneous Lorentz transformation or an infinitesimal gauge transformation. We denote a typical wave function by $\phi^A(x)$ and the Lagrangian density by $L(\phi^A(x), \partial_i\phi^A(x))$, $(\partial_i\phi(x)\equiv\partial\phi(x)/\partial x^i)$, the index A enumerating all independent wave functions. We consider an infinitesimal Lorentz transformation

$$x^k\to\bar{x}^k=x^k+a^k+\varepsilon^k_{\cdot j}x^j\equiv x^k+\Delta x^k, \quad (\varepsilon_{ij}=-\varepsilon_{ji})$$

and denote the wave field referred to the coordinate system $\bar{x}^k$ by $\bar{\phi}^A(\bar{x})$. If we introduce two types of variation for wave functions, the local variation δ_L

$$\delta_L\phi^A(x)\equiv\bar{\phi}^A(\bar{x})-\phi^A(x)$$

and the total variation δ_T

$$\delta_T\phi^A(x)\equiv\bar{\phi}^A(x)-\phi^A(x),$$

there are the following relations

$$\delta_L\phi^A(x)=\Delta x^k\partial_k\phi^A(x)+\delta_T\phi^A(x)$$

and

$$\delta_L[\partial_j\phi^A(x)]=\Delta x^k\partial_k\partial_j\phi^A(x)+\delta_T[\partial_j\phi^A(x)]$$

among them.

From the invariance of the Lagrangian density we get

$$\partial_k\Big\{\frac{\partial L}{\partial(\partial_k\phi^A(x))}\delta_T\phi^A(x)\Big\}+\Delta x^k\partial_k L=0$$

or

$$\partial_k\Big\{\frac{\partial L}{\partial(\partial_k\phi^A(x))}\delta_L\phi^A(x)+\Big[-\frac{\partial L}{\partial(\partial_k\phi^A(x))}\partial_j\phi^A(x)+Lg_j{}^k\Big]\Delta x^j\Big\}=0. \tag{1}$$

This is the Noether theorem. The variation $\delta_L\phi^A(x)$ may be written as

$$\delta_L\phi^A(x)=\frac{1}{2}\varepsilon_{ij}C^{ijAB}\phi^B(x) \tag{2}$$

$$(C^{ijAB}=-C^{jiAB})$$

From the arbitrariness of ten independent parameters a^k and ε_{ij} we get the conservation laws of the energy momentum tensor $T^k{}_j$ and the angular momentum tensor M^{kij}

$$\partial_k T^k{}_j=0$$

$$\partial_k M^{kij}=0$$

where

$$T^k{}_j=\frac{\partial L}{\partial(\partial_k\phi^A(x))}\partial_j\phi^A(x)-Lg_j{}^k \tag{3}$$

$$M^{kij}=T^{ki}x^j-T^{kj}x^i+\frac{\partial L}{\partial(\partial_k\phi^A(x))}C^{ijAB}\phi^B(x).$$

If $\phi^A(x)$ admits the gauge transformation of the first kind, we have

$$\Delta x^k=0, \qquad \delta_L\phi^A(x)=\delta_T\phi^A(x)=i\alpha\phi^A(x)$$

where α is an arbitrary real parameter. We get then conservation of the charge current vector J^k

$$\partial_k J^k=0$$

where

$$J^k=i\frac{\partial L}{\partial(\partial_k\phi^A)}\phi^A. \tag{4}$$

§ 3. The Lagrangian Density

We try to formulate the Noether theorem conforming to the postulate of representation invariance. Since the Lagrangian density depends on wave functions and their first derivatives at a point in space-time, we introduce the representative of the Lagrangian density $\langle x|L|x'\rangle$ by factorizing the action $\mathcal{L}$ in some way. For example, the action

$$\mathcal{L}=\langle U|x\rangle\langle x|p_kp^k-m^2|x'\rangle\langle x'|U\rangle$$

of the scalar field $U(x)\equiv\langle x|U\rangle$, $U^*(x)\equiv\langle U|x\rangle$, is factorized to give the representative of the Lagrangian density

$$\begin{aligned}\langle x|L|x'\rangle&=\langle U|x''\rangle\langle x''|p_k|x'\rangle\langle x|p^k|x'''\rangle\langle x'''|U\rangle-m^2\langle U|x'\rangle\langle x|U\rangle\\&=\langle Up_k|x'\rangle\langle x|p^kU\rangle-m^2\langle U|x'\rangle\langle x|U\rangle\end{aligned}$$

where

$$\langle Up_k|x'\rangle\equiv\langle U|x''\rangle\langle x''|p_k|x'\rangle$$

$$\langle x|p^kU\rangle\equiv\langle x|p^k|x'''\rangle\langle x'''|U\rangle$$

the signs of integration with respect to dummy suffices being dropped. In general we make the requirement that the representative $\langle x|L|x'\rangle$ of the Lagrangian density is hermitian and bilinear in $\langle\phi^A|x'\rangle$, $\langle\phi^Ap_k|x'\rangle$ and $\langle x|\phi^A\rangle$, $\langle x|p_k\phi^A\rangle$ and that the trace of $\langle x|L|x'\rangle$ is equal to the action, or

$$\mathcal{L}=\langle x|L|x\rangle.$$

We assign to the vector and Dirac fields the following Lagrangian densities respectively

$$\langle x|L|x'\rangle=\langle\phi^ip_k|x'\rangle\langle x|p^k\phi_i\rangle-m^2\langle\phi^i|x'\rangle\langle x|\phi_i\rangle$$

the vector field

$$\begin{aligned}\langle x|L|x'\rangle=&\frac{1}{2}\{\langle\bar\phi_\alpha\gamma_{\alpha\beta}{}^kp_k|x'\rangle+m\langle\bar\phi_\beta|x'\rangle\}\langle x|\phi_\beta\rangle\\&+\frac{1}{2}\langle\bar\phi_\alpha|x'\rangle\{\langle x|\gamma_{\alpha\beta}{}^kp_k\phi_\beta\rangle+m\langle x|\phi_\alpha\rangle\}\end{aligned}$$

$(\bar{\phi}=\phi^{\dagger}\gamma^{4})$

the Dirac field.

There are certain relations useful in the following,

$$\left.\begin{array}{ll}\dfrac{\partial\langle x|L|x'\rangle}{\partial\langle\phi^A|x'\rangle}=\dfrac{\partial\mathcal{L}}{\partial\langle\phi^A|x\rangle}, & \dfrac{\partial\langle x|L|x'\rangle}{\partial\langle\phi^A p_i|x'\rangle}=\dfrac{\partial\mathcal{L}}{\partial\langle\phi^A p_i|x\rangle}, \\ \dfrac{\partial\langle x|L|x'\rangle}{\partial\langle x|\phi^A\rangle}=\dfrac{\partial\mathcal{L}}{\partial\langle x'|\phi^A\rangle}, & \dfrac{\partial\langle x|L|x'\rangle}{\partial\langle x|p_i\phi^A\rangle}=\dfrac{\partial\mathcal{L}}{\partial\langle x'|p_i\phi^A\rangle},\end{array}\right\} \tag{5}$$

where $\langle\phi^A|x\rangle$, $\langle\phi^A p_i|x\rangle$, $\langle x|\phi^A\rangle$ and $\langle x|p_i\phi^A\rangle$ are regarded as independent variables. The variation of the action

$$\delta\mathcal{L}=\frac{\partial\mathcal{L}}{\partial\langle\phi^A|x\rangle}\delta\langle\phi^A|x\rangle+\frac{\partial\mathcal{L}}{\partial\langle\phi^A p_i|x\rangle}\delta\langle\phi^A p_i|x\rangle$$
$$+\frac{\partial\mathcal{L}}{\partial\langle x|\phi^A\rangle}\delta\langle x|\phi^A\rangle+\frac{\partial\mathcal{L}}{\partial\langle x|p_i\phi^A\rangle}\delta\langle x|p_i\phi^A\rangle$$

gives field equations

$$\frac{\partial\mathcal{L}}{\partial\langle\phi^A|x\rangle}+\frac{\partial\mathcal{L}}{\partial\langle\phi^A p_i|x'\rangle}\langle x|p_i|x'\rangle=0$$

and

$$\frac{\partial\mathcal{L}}{\partial\langle x|\phi^A\rangle}+\frac{\partial\mathcal{L}}{\partial\langle x'|p_i\phi^A\rangle}\langle x'|p_i|x\rangle=0$$

as the coefficients of $\delta\langle\phi^A|x\rangle$ and $\delta\langle x|\phi^A\rangle$ respectively.

To avoid clumsy use of suffices and make clear the independence from the representation we use sometimes the following symbols[5]

$$\frac{\partial\mathcal{L}}{\partial\langle\phi^A}\equiv N^A\rangle \quad \text{instead of} \quad \frac{\partial\mathcal{L}}{\partial\langle\phi^A|x\rangle}\equiv\langle x|N^A\rangle$$

$$\frac{\partial\mathcal{L}}{\partial\langle\phi^A p_i}\equiv M^{Ai}\rangle \quad \text{instead of} \quad \frac{\partial\mathcal{L}}{\partial\langle\phi^A p_i|x\rangle}\equiv\langle x|M^{Ai}\rangle$$

$$\frac{\partial\mathcal{L}}{\partial\phi^A\rangle}\equiv\langle N^A \quad \text{instead of} \quad \frac{\partial\mathcal{L}}{\partial\langle x|\phi^A\rangle}\equiv\langle N^A|x\rangle$$

and

$$\frac{\partial\mathcal{L}}{\partial p_i\phi^A\rangle}\equiv\langle M^{Ai} \quad \text{instead of} \quad \frac{\partial\mathcal{L}}{\partial\langle x|p_i\phi^A\rangle}\equiv\langle M^{Ai}|x\rangle.$$

Then field equations will be

$$\left.\begin{array}{l} N^A\rangle+p_k M^{Ak}\rangle=0 \\ \langle N^A+\langle M^{Ak}p_k=0.\end{array}\right\} \tag{6}$$

§ 4. A Reformulated Noether Theorem

To derive the Noether theorem we assume that the representative of the Lagrangian density is invariant under the infinitesimal inhomogeneous Lorentz transformation. We compute now the variation of

$\langle x|L|x'\rangle$

$$\delta_L\langle x|L|x'\rangle \equiv \langle \bar{x}|\bar{L}|\bar{x}'\rangle - \langle x|L|x'\rangle$$
$$= \langle \bar{x}|\bar{L}|\bar{x}'\rangle - \langle x|\bar{L}|\bar{x}'\rangle + \langle x|\bar{L}|\bar{x}'\rangle - \langle x|\bar{L}|x'\rangle + \delta_T\langle x|L|x'\rangle$$

where

$$\delta_T\langle x|L|x'\rangle \equiv \langle x|\bar{L}|x'\rangle - \langle x|L|x'\rangle.$$

Neglecting higher orders we have

$$\delta_L\langle x|L|x'\rangle = \varDelta x^k\partial_k\langle x|L|x'\rangle + \varDelta x'^k\partial_k'\langle x|L|x'\rangle + \delta_T\langle x|L|x'\rangle. \tag{7}$$

We see here that

$$\varDelta x^k\partial_k\langle x|L|x'\rangle = \varDelta x^k\langle x|ip_kL|x'\rangle = \langle x|JL|x'\rangle \qquad (J = i\varDelta x^k p_k)$$

and

$$\varDelta x'^k\partial_k'\langle x|L|x'\rangle = -\langle x|Lip_k|x'\rangle\varDelta x'^k = -\langle x|Lip_k\varDelta x^k|x'\rangle = -\langle x|LJ|x'\rangle$$

because of the relation

$$[p_k, \varDelta x^k] = [p_k, a^k + \varepsilon^{k}_{\cdot j}x^j] = \varepsilon^{k}_{\cdot j}(-i)g^j{}_k = -i\varepsilon^{k}_{\cdot k} = 0.$$

Therefore we have

$$\delta_L\langle x|L|x'\rangle = \langle x|[J, L]|x'\rangle + \delta_T\langle x|L|x'\rangle$$

or, in the representation-free form

$$\delta_L L = [J, L] + \delta_T L. \tag{8}$$

We compute next the variation $\delta_T L$ regarding $<\phi^A$, $<\phi^A p_i$, $\phi^A>$ and $p_i\phi^A>$ as independent variables, and we have

$$\delta_T\langle x|L|x'\rangle = \delta_T\langle\phi^A|x'\rangle\frac{\partial\langle x|L|x'\rangle}{\partial\langle\phi^A|x'\rangle} + \delta_T\langle\phi^A p_k|x'\rangle\frac{\partial\langle x|L|x'\rangle}{\partial\langle\phi^A p_k|x'\rangle} + \delta_T\langle x|\phi^A\rangle\frac{\partial\langle x|L|x'\rangle}{\partial\langle x|\phi^A\rangle} + \delta_T\langle x|\phi^A p_k\rangle\frac{\partial\langle x|L|x'\rangle}{\partial\langle x|p_k\phi^A\rangle}$$
$$= \delta_T\langle\phi^A|x'\rangle\langle x|N^A\rangle + \delta_T\langle\phi^A p_k|x'\rangle\langle x|M^{Ak}\rangle + \delta_T\langle x|\phi^A\rangle\langle N^A|x'\rangle + \delta_T\langle x|p_k\phi^A\rangle\langle M^{Ak}|x'\rangle.$$

We note here that $\delta_T p_k = 0$, because, analogously to (8)

$$\delta_L p_k = [J, p_k] + \delta_T p_k = \varepsilon_k{}^l p_l + \delta_T p_k$$

and

$$\delta_L p_k = \varepsilon_k{}^l p_l$$

from the transformation of a covariant vector.

Hence we have, with the aid of wave equations (6)

$$\delta_T\langle x|L|x'\rangle = -\delta_T\langle \phi^A|x'\rangle\langle x|p_k M^{Ak}\rangle + \delta_T\langle \phi^A|x''\rangle\langle x''|p_k|x'\rangle\langle x|M^{Ak}\rangle$$
$$-\delta_T\langle x|\phi^A\rangle\langle M^{Ak}p_k|x'\rangle + \delta_T\langle x''|\phi^A\rangle\langle x|p_k|x''\rangle\langle M^{Ak}|x'\rangle$$

or

$$\delta_T L = -p_k M^{Ak}\rangle\langle\delta_T\phi^A + M^{Ak}\rangle\langle\delta\phi^A p_k$$
$$-\delta_T\phi^A\rangle\langle M^{Ak}p_k + p_k\delta_T\phi^A\rangle\langle M^{Ak}$$
$$=[M^{Ak}\rangle\langle\delta_T\phi^A - \delta_T\phi^A\rangle\langle M^{Ak}, p_k].$$

We use here the relations

$$\langle\delta_T\phi^A = \langle\delta_L\phi^A + \langle\phi^A J$$
$$\delta_T\phi^A\rangle = \delta_L\phi^A\rangle - J\phi^A\rangle$$

analogous to (8).

The final result is

$$\delta_L L = [M^{Ak}\rangle\langle\delta_L\phi^A - \delta_L\phi^A\rangle\langle M^{Ak} + M^{Ak}\rangle\langle\phi^A J$$
$$+J\phi^A\rangle\langle M^{Ak}, p_k] + [J, L] = 0. \tag{9}$$

This is the equivalent to the Noether theorem (1).

§ 5. Field Invariants

However, the $\delta_L L$ does not have a divergence form. So we modify it by use of the Jacobi identity and the relation

$$L = [Q^k, p_k] \tag{10}$$

where

$$Q^k = \frac{1}{2}M^{Ak}\rangle\langle\phi^A - \frac{1}{2}\phi^A\rangle\langle M^{Ak}. \tag{11}$$

The relation (10) is proved as follows. Since $\langle x|L|x'\rangle$ is linear in $\langle\phi^A|x'\rangle$, $\langle\phi^A p_k|x'\rangle$, we have

$$\langle x|L|x'\rangle = \langle x|\phi^A\rangle\frac{\partial\langle x|L|x'\rangle}{\partial\langle x|\phi^A\rangle} + \langle x|p_k\phi^A\rangle\frac{\partial\langle x|L|x'\rangle}{\partial\langle x|p_k\phi^A\rangle}$$
$$=\langle x|\phi^A\rangle\langle N^A|x'\rangle + \langle x|p_k\phi^A\rangle\langle M^{Ak}|x'\rangle$$

or

$$L = \phi^A\rangle\langle N^A + p_k\phi^A\rangle\langle M^{Ak}$$
$$=[p_k, \phi^A\rangle\langle M^{Ak}]$$

by virtue of (6). We have similarly

$$L = N^A\rangle\langle\phi^A + M^{Ak}\rangle\langle\phi^A p_k = [M^{Ak}\rangle\langle\phi^A, p_k].$$

From these relations we get (10) and

$$[R^k, p_k] = 0 \tag{12}$$

where

$$R^k = \frac{1}{2}M^{Ak}\rangle\langle\phi^A + \frac{1}{2}\phi^A\rangle\langle M^{Ak}. \tag{13}$$

Use of the Jacobi identity and (10) leads to the expression

$$[J, L]=[J, [Q^k, p_k]]=[Q^k, [J, p_k]]+[[J, Q^k], p_k]$$
$$=[[J, Q^k]-\varepsilon^k{}_l Q^l, p_k]$$

and a modified Noether theorem

$$\delta_L L=[M^{Ak}\rangle\langle\delta_L\phi^A-\delta_L\phi^A\rangle\langle M^{Ak}+M^{Ak}\rangle\langle\phi^A J$$
$$+J\phi^A\rangle\langle M^{Ak}+[J, Q^k]-\varepsilon^k{}_l Q^l, p_k]=0. \tag{14}$$

If we pick up the coefficients of a^l, we have conservation of the energy momentum tensor

$$[T^k{}_l, p_k]=0 \tag{15}$$

where

$$T^k{}_l=M^{Ak}\rangle\langle\phi^A p_l+p_l\phi^A\rangle\langle M^{Ak}+[p_l, Q^k]$$
$$=R^k p_l+p_l R^k=\{R^k, p_l\} \tag{16}$$
$$(\{A, B\}=AB+BA).$$

Direct verification of (15) is easy because of the relation (12).

If we pick up the coefficients of ε_{ij}, we get conservation of the angular momentum tensor

$$[M^{kij}, p_k]=0 \tag{17}$$

where

$$M^{kij}=\{T^{ki}, x^j\}-\{T^{kj}, x^i\}+S^{kij} \tag{18}$$
$$S^{kij}=M^{Ak}\rangle\langle\phi^B C^{ijAB}-C^{ijAB}\phi^B\rangle\langle M^{Ak}$$
$$+p^j R^k x^i+x^i R^k p^j-p^i R^k x^j-x^j R^k p^i+ig^{ki}Q^j-ig^{kj}Q^i.$$

For the scalar field, the divergence $[S^{kij}, p_k]$ vanishes. So the term S^{kij} may be dropped from the left member of (18).

A symmetric energy momentum tensor Θ^{ij} may be given by

$$\Theta^{ij}=T^{ij}+[p_k, S^{kij}+S^{ijk}-S^{jki}]/4i$$

following the standard procedure.[6]

If we start directly from (9), we have another energy momentum tensor

$$'T^k{}_l=M^{Ak}\rangle\langle\phi^A p_l+p_l\phi^A\rangle\langle M^{Ak}-g^k{}_l L$$

and another angular momentum tensor

$$'M^{kij}=\{'T^{ki}, x^j\}-\{'T^{kj}, x^i\}+'S^{kij}$$
$$'S^{kij}=S^{kij}-2ig^{ki}Q^j+2ig^{kj}Q^i.$$

For the infinitesimal gauge transformation, we have

$$J=0, \qquad \delta_L\phi\rangle=i\alpha\phi\rangle, \qquad \langle\delta_L\phi=-i\alpha\langle\phi$$

and conservation of the charge current vector J^k

$$[J^k, p_k]=0 \tag{19}$$

where

$$J^k = 2R^k$$
$$= M^{Ak}\rangle\langle\phi^A + \phi^A\rangle\langle M^{Ak}. \tag{20}$$

§ 6. Some Remarks

In classical field theory, the conservation of the charge current

$$\frac{\partial J^k(x)}{\partial x^k} = 0$$

yields the constancy of the space integral of $J^0(x)$. This results from integrating the above equation over the entire space

$$\frac{\partial}{\partial x^0}\int J^0(x)dx^1dx^2dx^3 = -\int\left(\frac{\partial J^1}{\partial x^1} + \frac{\partial J^2}{\partial x^2} + \frac{\partial J^3}{\partial x^3}\right)dx^1dx^2dx^3 = 0$$

under a suitable boundary condition.

An analoguous consequence holds in our case. Taking the trace of the conservation equation

$$[J^0, p_0] = -[J^1, p_1] - [J^2, p_2] - [J^3, p_3]$$

over the entire space, we get

$$[\mathrm{tr}\, J^0, p_0] = -\mathrm{tr}\{[J^1, p_1] + [J^2, p_2] + [J^3, p_3]\} = 0$$

where $\mathrm{tr}\, A$ is defined by the integral of diagonal part of A over the entire space, that is,

$$\mathrm{tr}\, A = \langle x^0, x^1, x^2, x^3 | A | x'^0, x^1, x^2, x^3\rangle.$$

The $\mathrm{tr}[J^1, p_1]$ vanishes since $\mathrm{tr}\, AB = \mathrm{tr}\, BA$, under suitable boundary condition at infinity.

Therefore $\mathrm{tr}\, J^0$ commutes with p_0, so $\mathrm{tr}\, J^0$ is a constant.

In the same way, $\mathrm{tr}\, T^0{}_l$ is proved to be a constant vector, that is, the energy momentum vector.

In classical field theory, the energy momentum tensor plays no important role except for the provider of the energy momentum vector. If we construct the energy momentum tensor of a scalar field with a solution to the field equation, we meet a queer situation. The scalar field equations

$$(\partial^k\partial_k + m^2)\phi(x) = 0$$
$$(\partial^k\partial_k + m^2)\phi^*(x) = 0$$
$$(L(x) = \partial_k\phi^*\partial^k\phi - m^2\phi^*\phi)$$

have a solution

$$\phi(x) = c\,\exp(iq_kx^k), \qquad \phi^*(x) = c^*\,\exp(-iq_kx^k)$$

q_k being a constant momentum vector satisfying $q_kq^k = m^2$. The solution leads us to

and
$$\left.\begin{aligned} T^{kj}(x) &= 2q^k q^j cc^* \\ J^k(x) &= 2q^k cc^*. \end{aligned}\right\} \tag{21}$$

The tensor $T^{kj}(x)$ and the vector $J^k(x)$ are constant throughout the entire space time. Their Fourier transforms vanish everywhere except for $p_0=p_1=p_2=p_3=0$. The consequence is very queer.

In our formulation we have
$$\langle x|\phi\rangle=\phi(x), \qquad \langle\phi|x\rangle=\phi^*(x)$$
$$\left.\begin{aligned} \langle x|T^{kl}|x'\rangle &= 2q^k q^l cc^* \exp[iq_k(x^k-x'^k)] \\ \langle x|J^k|x'\rangle &= 2q^k cc^* \exp[iq_k(x^k-x'^k)] \end{aligned}\right\} \tag{22}$$
or, in the momentum representation,
$$\langle p|\phi\rangle=c'\delta(p-q), \qquad \langle\phi|p\rangle=c'^*\delta(p-q)$$
$$\langle p|T^{kl}|p'\rangle=2q^k q^l c' c'^*\delta(p-q)\delta(p'-q)$$
$$\langle p|J^k|p'\rangle=2q^k c' c'^*\delta(p-q)\delta(p'-q)$$
where
$$c'=c(2\pi)^2, \qquad \delta(p)=\delta(p_0)\delta(p_1)\delta(p_2)\delta(p_3).$$

These expressions describe that these field invariants are diagonal in the p-representation and vanish except for $p=p'=q$, in agreement with the physical picture.

Comparison of two expressions (21) and (22) of energy-momentum tensor or charge-current vector seems to make evident the failure of the conventional formulation.

In conclusion we assert that field invariants discussed above should be regarded not as functions depending on a position in space-time but as observables depending on two positions in space-time, as is exemplified by the density matrix.

When field variables $\phi^A\rangle$, $\langle\phi^A$ are quantized after the formula of the second quantization,[4] the expectation value of the charge current vector and that of the energy momentum tensor introduced in this paper satisfy their respective conservation laws because field equations still hold in their expectation values. A more detailed description will appear elsewhere.

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