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Motion of the Vortex Rings near the Boundary Plane of the Fluid

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Experimental studies were made on the motion of circular vortex rings approaching a fluid boundary. The vortex rings were made visible by the electrolysis method. The trajectories of the core center and the velocities of the vortex rings were determined by the photographs which were taken successively during the motion of the rings. As the vortex ring approaches the boundary, its radius increases and the traveling velocity decreases, and these processes go quite similarly independent of the size of the ring. Experimental results were compared with the theoretical analysis.

§1. Introduction

A vortex ring, which is produced in a uniform fluid, proceeds perpendicularly to its plane at a uniform velocity. When the vortex ring approaches a rigid boundary or a free surface of the fluid, the radius of the vortex ring increases and its travling velocity decreases rapidly.

The theory of the motion of the vortex ring in an inviscid fluid is described in many text books, for example, those of Lamb[1] and of Batchelor.[2] The behaviors of the vortex ring near the fluid boundary were analyzed by Dyson[3] at the end of the last century. Recently the experimental studies on the motion of vortex rings ejected from circular and lenticular orifices were made in a water without the effect of the boundaries by the author.[4] Further experimental studies were carried out on the behaviors of vortex rings near a rigid boundary plane or a free suface of water. The electrolysis method was used in order to visualize the vortex ring and many photographs were taken successively during its motion. Changes in the traveling velocity and the radius of the vortex ring near the fluid boundary were determined by those photographs.

These were compared with the theoretical calculations for the motion of the vortex ring in existance of the image ring due to the boundary. The motion of a pair of vortex filaments of infinite length near a rigid boundary is also referred to for comparison. The results of experiment and theory are in accord in general but do not agree well quantitatively. Some discussions are made about these results.

§2. Results of the Theoretical Analysis

Dyson has analyzed the motion of the vortex ring approaching a fixed plane in an ideal fluid and its results are as follows. We consider the case that a vortex ring approaching a rigid plane along z-axis. Fig. 1 shows the meridian section of the vortex ring and the boundary plane, here a and ε are the radius of the vortex ring and that of its core, respectively, when the distance of the ring from the boundary is z. As the ring approaches the boundary, its radius increases according to the following equation.

$$a\left(\log\frac{8a}{\varepsilon}-\frac{7}{4}\right)-2a\operatorname{cosec}\theta\,(F-E)+a\sin\theta\,F=a_0\left(\log\frac{8a_0}{\varepsilon_0}-\frac{7}{4}\right),^{*}$$

where a_0 and ε_0 are the radius of the ring and its core, respectively, when it is infinitely distant from the boundary ($z=\infty$), and θ is a half angle which the ring subtends at the origin O in the meridian plane. (cf. Fig. 1) F and E are the complete elliptic integrals of the first and the second kind, respectively, whose modulus $k=\sin\theta$.

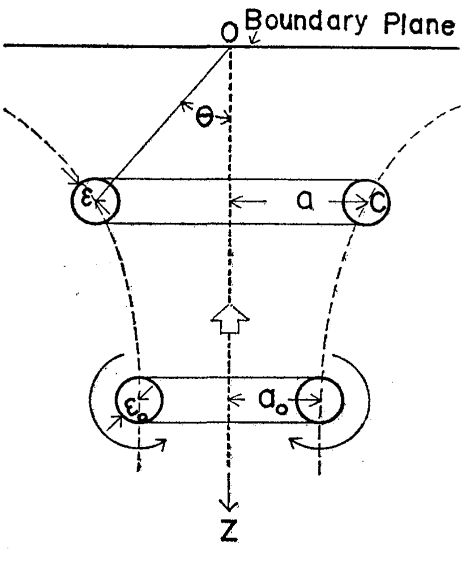

Fig. 1. The motion of a vortex ring approaching the boundary plane. a and ε are the radius of the vortex ring and that of its core, θ is a half angle which the ring subtends at the origin O in the meridian plane. C is the center of the core of the vortex ring.

In order to determine the relations a vs. z, the condition that the volume of the ring core containing the vorticity is kept constant or $a\varepsilon^2=a_0\varepsilon_0^2$ is used.

The traveling velocity of the ring w and the rate of growth of its radius u are also given as follows:

* In this equation some notations of Dyson's paper are changed into those used in Fig. 1.

$$w=\frac{dz}{dt}=\frac{\kappa}{\pi a}\left[\frac{1}{2}\left(\log\frac{8a}{\varepsilon}-\frac{1}{4}\right)-\operatorname{cosec}\theta\,(2-\sec^2\theta+\tan^2\theta)(F-E)\right.$$

$$\left.+\sin\theta\,(-2+\sec^2\theta-3\tan^2\theta)F+\tan^2\theta\sin\theta\,E\right],$$

and

$$u=\frac{da}{dt}=\frac{\kappa}{\pi a}[\tan\theta\sin\theta\,(E-2F)+2\sec\theta\,(F-E)],$$

where κ is the strength of the vortex ring.

Changes in the radius, its rate of growth and the traveling velocity of the vortex ring are calculated for the case $a_0/\varepsilon_0=100$, 10 and 5. The results are shown in Figs. 5, 6 and 7 together with experimental data. In these figures, the distance from the boundary, the radius and the traveling velocity of the vortex ring are all expressed in non-dimensional form, that is, z/a_0, a/a_0, w/w_0 and u/w_0, where a_0 is the radius and w_0 is the traveling velocity of the vortex ring when it is far from the boundary. The theory of the vortex ring in ideal fluid shows[1)]

$$w_0=\frac{\kappa}{4\pi a}\left(\log\frac{8a_0}{\varepsilon_0}-\frac{1}{4}\right).$$

As the ring approaches the boundary, its radius increases indefinitely and the traveling velocity becomes vanishingly small.

Now we consider the motion of a pair of parallel linear vortex filaments, with the equal strength κ and with the opposite direction, approaching a fixed plane parallel to the filaments. This is the two-dimensional version of the motion of the vortex ring near the boundary, and is also the motion of a vortex filament near the rectangular corner. Denote by x and z the coordinates of one of the pair of filaments in plane perpendicular to the filaments.*

Induced velocities at the vortex filament by the other vortex filament and the images due to the boundary plane are

$$w=\frac{dz}{dt}=-\frac{\kappa}{4\pi}\frac{x^2}{z(x^2+z^2)},$$

$$u=\frac{da}{dt}=\frac{\kappa}{4\pi}\frac{z^2}{x(x^2+z^2)}.$$

The trajectory of the vortex filament is given by the equation:

$$\frac{dx}{x^3}+\frac{dz}{z^3}=0,$$

whence

$$a_0^2(x^2+z^2)=4x^2z^2,$$

* Fig. 1 is also useful for this case, if C is regarded as the position of a vortex filament and a as x.

where a_0 is the half distance between the vortex filaments when they are infinitely distant from the boundary.

The trajectory and the velocities of the vortex filament are also shown in Figs. 5, 6 and 7 together with those of vortex ring. It is interesting that the vortex filament does not approach the boundary in the distance nearer than a.

§ 3. Experimental Apparatus and Method

Experiments were carried out in an acrylic tank with the size of 20 cm×20cm×55 cm, and the water is filled 20 cm in depth. The electrolysis method was adopted to visualize the motion of vortex rings, in which the orifice made of tin was used as the anode. Schematic diagram is shown in Fig. 2. A general description of the apparatus and an acount of the experimental techniques —formation of vortex rings and electrolysis method— were given in the previous paper.[4)]

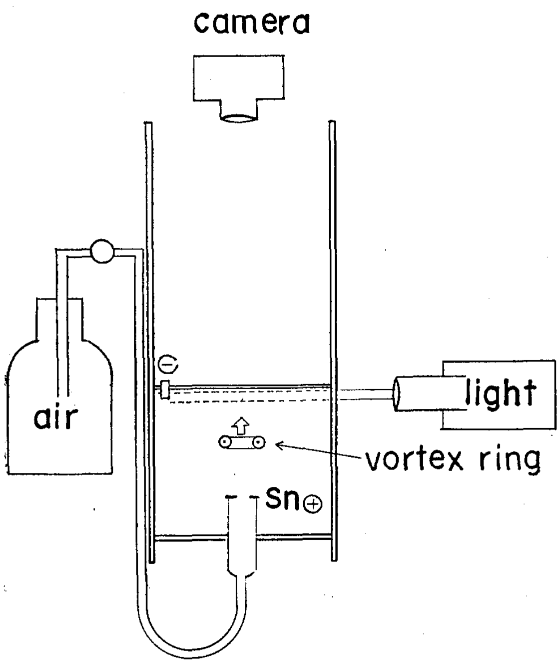

Fig. 2. Schematic diagram of measuring apparatus.

Two cases were tested, one was for the free surface of water and the other was for the rigid plane boundary where the surface of water was covered with the acrylic plate of 1 mm thick. Camera was set with a small angle upward from the horizontal plane in order to make both real and reflected images in the same picture. Photographs were taken in every 0.5 sec using the motor-drive camera as the vortex ring approached the boundary plane.

Three kinds of circular orifice, of 1.9 cm, 2.3 cm and 2.7 cm in diameter, respectively, were used. The radii and the traveling distances of the vortex rings were measured in these photographs and their changes in time were examined.

§ 4. Experimental Results and Discussions

Immediately after the ejection from a circular orifice, the traveling velocity of the vortex ring decreases rapidly and its radius increases a little. After this period the vortex ring moves perpendicularly to its plane with the almost constant velocity and radius. When it approaches the boundary plane, its radius increases and traveling velocity decreases both fairly rapidly.

In the preliminary experiments there have been found no appreciable difference in the quantitative measurements for the two cases, that is, for a free surface and for a rigid boundary. Therefore all the following measurements were carried out for the case with the free surface.

An example of a series of photographs of the meridian section of the ring is shown in Fig. 3. These photographs are taken in every

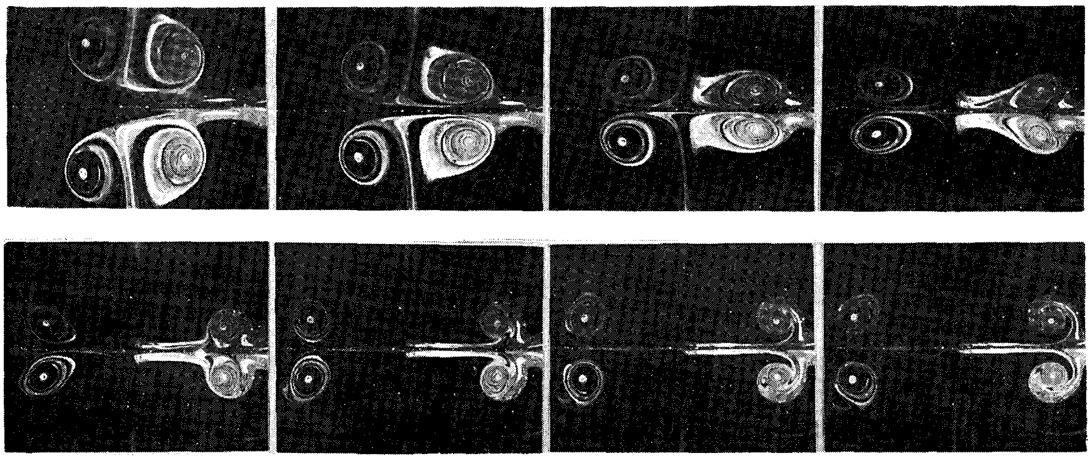

Fig. 3. A series of a vortex motion near the boundary plane. (side views) Diameter of the orifice $D=23$ mm. These are taken in each 0.5 sec successively.

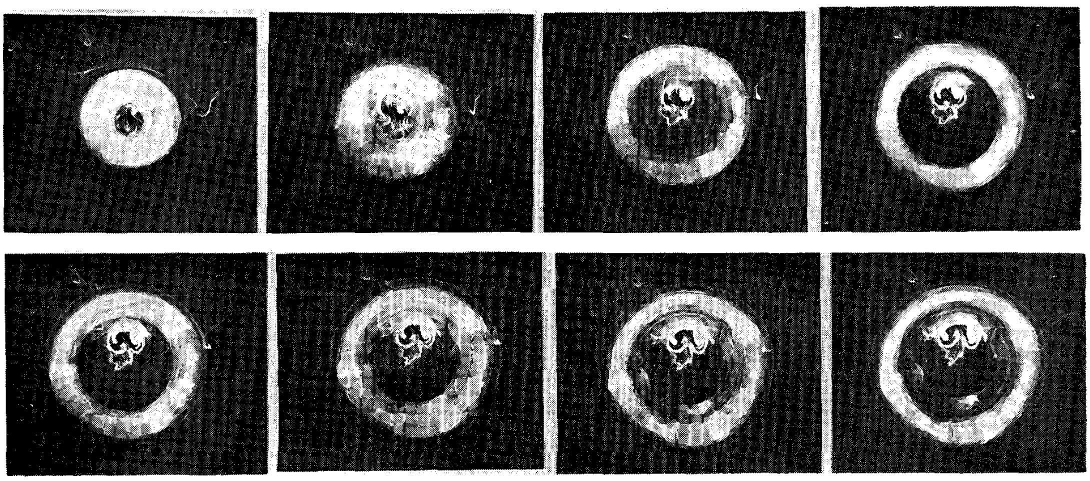

Fig. 4. A series of the front views of the vortex ring. $D=23$ mm.

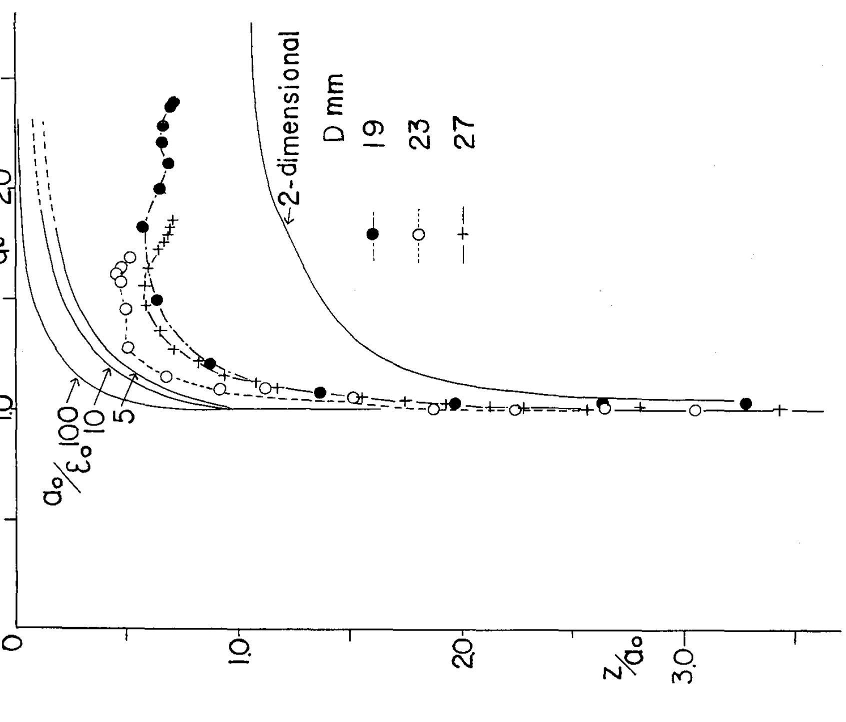

Fig. 5. Trajectries of the core center of vortex ring, z vs. a.

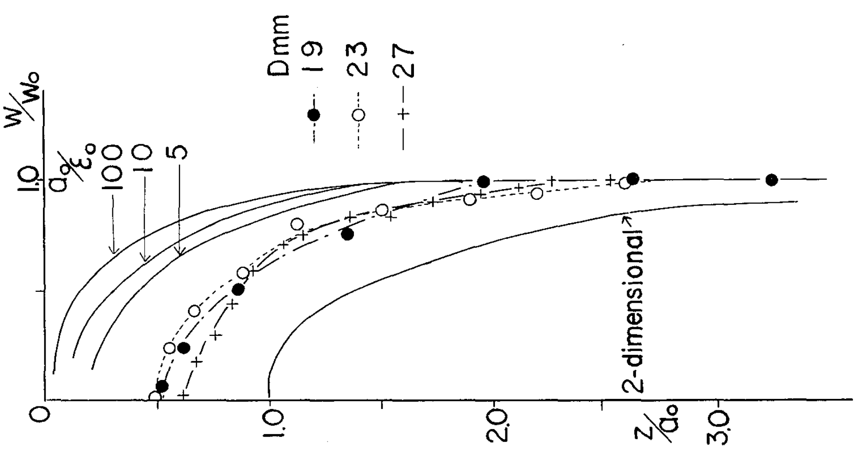

Fig. 6. Traveling velocity of the vortex ring, w/w_0 vs. z/a_0.

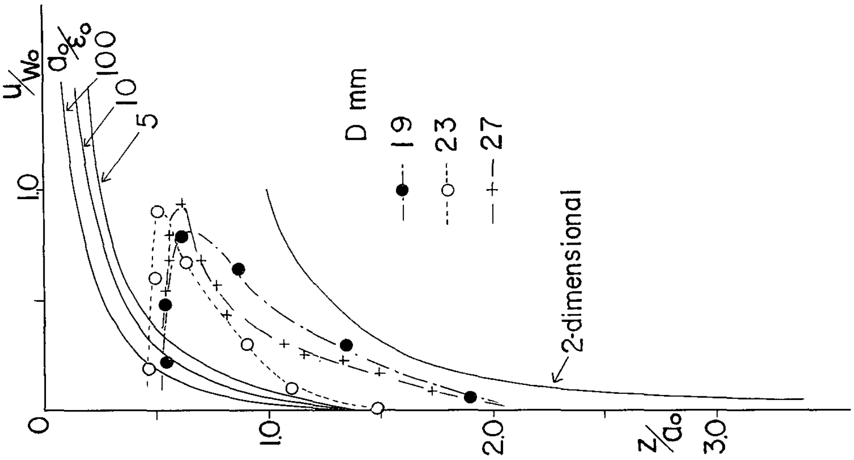

Fig. 7. Rate of growth of the radius, u/w_0 vs. z/a_0.

0.5 sec. The upper half of each photograph shows the reflected image due to the water surface and the lower half is the real image of the vortex ring in the water. An example of a series of the front views of one ring approaching the free surface is shown in Fig. 4.

By use of a great number of photographs similar to Fig. 3, the position of the core center in the meridian plane z and the radius of the vortex ring a were determined for each time. Further, from differences of these data for successive time, the velocities of the core center, $w=dz/dt$ and $u=da/dt$ were obtained. The former is the traveling velocity and the latter is the rate of growth of the radius of the ring. The trajectories and velocities of the core center are plotted in Figs. 5, 6 and 7 together with the theoretical results. In these figures all quantities are expressed in the non-dimensional form, that is, z/a_0, a/a_0, w/w_0, and u/w_0. As for experimental data, a_0 and w_0 are the radius and the traveling velocity of the vortex ring in the region where the effect of the boundary does not reach.

It was found generally that the radius of the vortex ring became larger and its traveling velocity smaller as the diameter of the orifice from which the ring was ejected became larger. Further almost all the experimental data expressed in the non-dimensional form fall in the very narrow region about a single curve in each figure.* This fact suggests that there exists a kind of similarity law independent of the scale of the orifice or the vortex ring. Generally vortex ring moves with the constant velocity w_0 and constant radius a_0 up to the distance of about $2a_0$ from the boundary. When the ring approaches near the boundary, its radius increases and the traveling velocity decreases rapidly, and at the distance about $0.5a_0$ from the boundary the ring almost ceases to proceed on further and its radius increases up to about $1.5a_0$ or more.

In the final stage the ring oscillates up and down several times. The smaller ring oscillates faster and more times than the larger ring. Then the ring deforms from the circle, and the core diffuses and the vortex ring decays out eventually.

As shown in Fig. 5, theoretical analysis suggests that the vortex ring approaches nearer the boundary and its radius increases indefinitely irrespective of the ratio a_0/ε_0. As mentioned above actual vortex rings did not approach the boundary nearer than $0.5a_0$. The radius of the core ε_0 or strength of the vortex ring κ was not able to determine experimentally, but the value of a_0/ε_0 for actual vortex ring must be about 80 or more because its shape was not a sphere but a torus.[2] Some discrepancies between the results of theory and

* Among many results only a set of data for each orifice are shown in Figs. 5, 6 and 7.

experiment are evident.

Here it is reminded that the theory assumes the ideal fluid. In the final period of approaching, actual ring deforms and its core diffuses appreciably. These phenomena suggest that the effect of viscosity is remarkable in this stage and it causes the discrepancy between theory and experiments. The up-and-down motion of the ring near the boundary may be attributed to the deformation of the vortex core.

Acknowledgement

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