On the Concentrations of Matter Emitted from a Source in the Atmosphere when a 'Flux-Zero' Level Exists above the Source

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Introduction

In the case when there appears a level at which flux of matter becomes zero, in other words, the atmosphere is covered with a lid in some sense, the concentration of matter emitted from a source in the lower atmophere is expected to be higher than the concentration when there is no such a level. However, there have been scarcely any papers which treat quantitatively the relations between the concentration and the height of the level. Furthermore, though the conditions in which such a level appears are closely related to the vertical distributions of air temperature and intensity of turbulence, the quantitative relation between them are not clear yet.

Therefore, in this paper, we calculated the concentration distribution in the region below such level, assuming that such level appeared in a certain height by some cause.

A) Sakagami's formula

A-1) *Instantaneous point source.* We adopt the differential equation for the diffusion of matter in the following type:

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}=k_0\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)+\frac{\partial}{\partial z}\left(kz\frac{\partial C}{\partial z}\right)$$

$$-\infty<x,\ y<+\infty, \qquad z\geqq 0 \qquad 1\text{-}1),$$

where C is the concentration, t is time, u is the wind speed at the source, which is at the position $(0, 0, h)$, x, y and z are coordinates leeward, cross wind ward and vertically upward respectively, and k and k_0 are constants for the diffusion. Putting that

$$x-ut=x',\ t=t' \qquad 1\text{-}2),$$

we get

$$\frac{\partial C}{\partial t'}=k_0\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)+\frac{\partial}{\partial z}kz\frac{\partial C}{\partial z} \qquad 1\text{-}3).$$

The solution of this equation for an instanteneous point source which is at (0, 0, h) has been obtained already by one of the authors,[1] and it is given by

$$C=\frac{M}{4k_0t'kt'\pi}\exp\left\{-\left(\frac{x'^2+y^2}{4k_0t'}+\frac{z+h}{kt'}\right)\right\}I_0\left(\frac{2\sqrt{hz}}{kt'}\right) \qquad 1\text{-}4),$$

where M is the total amount of matter emitted instantaneously, and I_0 is the modified Bessel function of the order zero. With the former variables, we have

$$C=\frac{M}{4k_0tkt\pi}\exp\left\{-\left(\frac{(x-ut)^2+y^2}{4k_0t}+\frac{z+h}{kt}\right)\right\}I_0\left(\frac{2\sqrt{hz}}{kt}\right) \qquad 1\text{-}5).$$

A-2) *The case when the flux-zero level exists at* $z=H$

A-2-1) *Fundamental solution.* At first, we consider only z and t;

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}=\frac{\partial}{\partial z}\left(kz\frac{\partial C}{\partial z}\right) \qquad 2\text{-}1).$$

Transforming the variables as 1-2), we have

$$\frac{\partial C}{\partial t'}=\frac{\partial}{\partial z}\left(kz\frac{\partial C}{\partial z}\right) \qquad 2\text{-}2).$$

The boundary conditions are

$$z=0 \qquad kz\frac{\partial C}{\partial z}=0 \qquad 2\text{-}3),$$

$$z=H \qquad kz\frac{\partial C}{\partial z}=0 \qquad 2\text{-}4),$$

and the initial condition is

$$t=t'=0 \qquad C=Q_2\delta(z-h) \qquad 2\text{-}5).$$

If we put

$$C=e^{-at'}\varphi(z) \qquad 2\text{-}6),$$

2-2) becomes

$$\frac{d}{dz}\left(kz\frac{d\varphi}{\partial z}\right)=-a\varphi \qquad 2\text{-}7).$$

Changing the variable z to s by $z=s^2$, 2-7) becomes

$$\frac{1}{s}\frac{d}{ds}\left(s\frac{d\varphi}{ds}\right)+\frac{4a}{k}\varphi=0 \qquad 2\text{-}8).$$

The particular solutions of this equation are

$$\varphi=J_0\left(2\sqrt{\frac{a}{k}}s\right)=J_0\left(2\sqrt{\frac{a}{k}z}\right)$$

or 2-9),

$$Y_0\left(2\sqrt{\frac{az}{k}}\right)$$

where J_0 and Y_0 are the first kind Bessel function of the order zero, but Y_0 is not adoptable.

According to one of the boundary conditions $\left(\frac{d\varphi}{dz}\right)_{z=H}=0$, we have

$$J_1\left(2\sqrt{\frac{aH}{k}}\right)=0 \qquad 2\text{–}10).$$

Denoting the zero points of J_1 by j_ν, namely $j_0=0.0$, $j_1=3.83$, $j_2=7.016$, $\cdots$, we get

$$2\sqrt{\frac{aH}{k}}=j_\nu, \qquad \nu=0,\ 1,\ 2,\ \cdots$$

and

$$a=\frac{k}{H}\left(\frac{j_\nu}{2}\right)^2 \qquad 2\text{–}11).$$

So the solution which satisfies the boundary conditions is

$$C=\sum_{\nu=1}^{\infty} e^{-\frac{k}{H}\left(\frac{j_\nu}{2}\right)^2 t'} J_0\left(j_\nu\sqrt{\frac{z}{H}}\right)A_\nu \qquad 2\text{–}12).$$

From the initial condition, we get

$$Q_2\delta(z-h)=\sum_{\nu=1}^{\infty} J_0\left(j_\nu\sqrt{\frac{z}{H}}\right)A_\nu \qquad 2\text{–}13).$$

On the other hand, by putting $\sqrt{\frac{z}{H}}=s$, we have

$$\int_0^H J_0\left(j_\nu\sqrt{\frac{z}{H}}\right)J_0\left(j_\nu\sqrt{\frac{z}{H}}\right)dz$$

$$=\int_0^1 J_0(j_\nu s)\,J_0\left(j_\mu\sqrt{\frac{z}{H}}\right)H2sds$$

$$=2H\frac{j_\mu sJ_0(j_\mu s)J_{-1}(j_\mu s)-j_\nu sJ_{-1}(j_\nu s)J_0(j_\mu s)}{j_\nu{}^2-j_\mu{}^2}\Bigg|_0^1=0 \qquad \nu\neq\mu \qquad 2\text{–}14)$$

and

$$\int_0^1 J_0\left(j_\nu\sqrt{\frac{z}{H}}\right)J_0\left(j_\mu\sqrt{\frac{z}{H}}\right)dz=2H\int_0^1[J(j_\nu s)]^2ds$$

$$=2H\frac{s^2}{2}\left\{(J_0(j_\nu s))^2-J_{-1}(j_\nu s)J_1(j_\nu s)\right\}\Bigg|_0^1=H[J_0(j_\nu)]^2 \qquad \nu=\mu \qquad 2\text{–}15).$$

Multiplying $J_0\left(j_\nu\sqrt{\frac{z}{H}}\right)$ on both sides of 2-13) and integrating by z, we have:

Left hand side=

$$\int_0^H Q_2\delta(z-h)J_0\left(j_\mu\sqrt{\frac{z}{H}}\right)dz=Q_2J_0\left(j_\mu\sqrt{\frac{h}{H}}\right) \tag{2-16}$$

Right hand side=

$$A_\mu H[J_0(j_\mu)]^2 \tag{2-17}$$

So we obtain

$$A_\mu=\frac{Q_2}{H}\frac{J_0\left(j_\mu\sqrt{\frac{h}{H}}\right)}{[J_0(j_\mu]^2} \tag{2-18}$$

Then we have

$$C=\frac{Q_2}{H}\sum_{\mu=0}^{\infty}e^{-\frac{k}{H}\left(\frac{j_\nu}{2}\right)^2t'}\frac{J_0\left(j_\mu\sqrt{\frac{h}{H}}\right)}{[J_0(j_\mu)]^2}J_0\left(j_\mu\sqrt{\frac{z}{H}}\right) \tag{2-19}$$

A-2-2) *Three dimensional diffusion for instantaneous point source.* In this case the source is assumed to be at $(0, 0, h)$, the concentration is given by

$$C=\frac{Q_3}{H}\sum_{\nu=0}^{\infty}\frac{e^{-\frac{x'^2+y^2}{4k_0t'}}}{\pi 4k_0t'}e^{-\frac{kt'}{4}\frac{1}{H}j_\nu^2}\frac{J_0\left(j_\nu\sqrt{\frac{z}{H}}\right)J_0\left(j_\nu\sqrt{\frac{h}{H}}\right)}{[J_0(j_\nu)]^2} \tag{3-1}$$

Recurring to the first variables, 3-1) becomes

$$C=\frac{Q_3}{H\pi}\sum_{\nu=0}^{\infty}\frac{e^{-\frac{(x-ut)^2+y^2}{4k_0t}-\frac{kt}{4H}j_\nu^2}}{4k_0t}\frac{J_0\left(j_\nu\sqrt{\frac{z}{H}}\right)J_0\left(j_\nu\sqrt{\frac{h}{H}}\right)}{[J_0(j_\nu)]^2} \tag{3-2}$$

A-2-3) *Instantaneous line source.* Assuming that the source lies at $x=0$ and between $y=-L$ and $y=L$, the concentration is given by

$$C=\frac{Q_4}{H\pi}\sum_{\nu=0}^{\infty}\int_{-L}^{L}\frac{e^{-\frac{(y-\eta)^2}{4k_0t}}}{4k_0t}d\eta$$

$$\times e^{-\frac{(x-ut)^2}{4k_0t}-\frac{kt}{4H}j_\nu^2}\frac{J_0\left(j_\nu\sqrt{\frac{z}{H}}\right)J_0\left(j_\nu\sqrt{\frac{h}{H}}\right)}{[J_0(j_\nu)]^2}$$

$$=\frac{Q_4}{H\sqrt{\pi}}\sum_{\nu=0}^{\infty}\frac{1}{\sqrt{4k_0t}}\frac{\Phi\left(\frac{y+L}{\sqrt{4k_0t}}\right)-\Phi\left(\frac{y-L}{\sqrt{4k_0t}}\right)}{2}$$

$$\times \exp\left\{-\left(\frac{(x-ut)^2}{4k_0t}+\frac{kt}{4H}j_\nu{}^2\right)\right\}\frac{J_0\left(j_\nu\sqrt{\frac{z}{H}}\right)J_0\left(j_\nu\sqrt{\frac{h}{H}}\right)}{[J_0(j_\nu)]^2} \qquad 4\text{-}1).$$

When $|L|\rightarrow\infty$, this equation becomes

$$C=\frac{Q_4}{H\sqrt{\pi}}\sum_{\nu=0}^{\infty}\frac{1}{\sqrt{4k_0t}}e^{-\left\{\frac{(x-ut)^2}{4k_0t}+\frac{kt}{4H}j_\nu{}^2\right\}}\frac{J_0\left(j_\nu\sqrt{\frac{z}{H}}\right)J_0\left(j_\nu\sqrt{\frac{h}{H}}\right)}{[J_0(j_\nu)]^2} \qquad 4\text{-}2).$$

A-3) *Continuous source.* We put 1-5), 2-19), 3-2) and 4-2) as

$$C=Q_if(x-ut,\ t) \qquad 5\text{-}1).$$

For the continuous source, 5-1) becomes

$$C=\int_0^t Q_j(\tau)\,f\{x-u(t-\tau),\ t-\tau\}\,d\tau \qquad 5\text{-}2).$$

Putting $t-\tau=\xi$, we have

$$C=\int_t^0 Q_i(t-\xi)\,f(x-u\xi,\ \xi)(-d\xi)=\int_0^t Q_i(t-\xi)\,f(x-u\xi,\ \xi)\,d\xi \qquad 5\text{-}3).$$

If

$$\left.\begin{array}{ll} Q_i(\tau)\neq 0 & 0\leqq\tau\leqq\varLambda \\ Q_i(\tau)=0 & \varLambda\leqq\tau \end{array}\right\} \qquad 5\text{-}4),$$

where $\varLambda$ is the duration of the source,

$$C=\int_0^t Q_if(x-u\xi,\ \xi)\,d\xi \qquad 0\leqq\tau\leqq\varLambda \qquad 5\text{-}5),$$

$$C=\int_0^{t-\varLambda} Q_if(x-u\xi,\ \xi)\,d\xi \qquad \varLambda\leqq\tau \qquad 5\text{-}6).$$

When $\varLambda$ is sufficiently large, so $t<\varLambda$,

$$C=\int_0^t Q_if(x-u\xi,\ \xi)\,d\xi \qquad 5\text{-}7).$$

We have from 3-2)

$$C=\frac{Q_1}{\pi H}\int_0^t\sum_{\nu=0}^{\infty}\frac{\exp\left[-\frac{\{x-u(t-\tau)\}^2+y^2}{4k_0(t-\tau)}-\frac{k(t-\tau)}{4H}j_\nu{}^2\right]}{4k_0(t-\tau)}$$

$$\times\frac{J_0\left(j_\nu\sqrt{\frac{z}{H}}\right)J_0\left(j_\nu\sqrt{\frac{h}{H}}\right)}{[J_0(j_\nu)]^2}\,d\tau$$

$$=\frac{Q_1}{\pi H}\sum_{\nu=0}^{\infty}\frac{J_0\left(j_\nu\sqrt{\frac{z}{H}}\right)J_0\left(j_\nu\sqrt{\frac{h}{H}}\right)}{[J_0(j_\nu)]^2}$$

$$\times\int_0^t\frac{\exp\left\{-\frac{(x-u\xi)^2+y^2}{4k_0\xi}-\frac{k\xi}{4H}j_\nu^2\right\}}{4k_0\xi}d\xi \qquad 5\text{-}8).$$

On the other hand, when $t\to\infty$,

$$\int_0^\infty e^{-at-\frac{b}{t}}\frac{dt}{t}=2K_0(\sqrt{ab}) \qquad 5\text{-}9),$$

where K_0 is the modified Bessel function of the order zero. As

$$\frac{(x-u\xi)^2+y^2}{4k_0\xi}+\frac{k\xi}{4H}j_\nu^2=\frac{x^2-2ux\xi+u^2\xi^2+y^2}{4k_0\xi}+\frac{kj_\nu^2}{4H}\xi$$

$$=\left(\frac{u^2}{4k_0}+\frac{kj_\nu^2}{4H}\right)\xi+\frac{x^2+y^2}{4k_0\xi}-\frac{ux}{2k_0},$$

5-8) becomes

$$C=\frac{Q_1}{\pi H}\sum_{\nu=0}^{\infty}\frac{J_0\left(j_\nu\sqrt{\frac{z}{H}}\right)J_0\left(j_\nu\sqrt{\frac{h}{H}}\right)}{[J_0(j_\nu)]^2}$$

$$\times e^{\frac{ux}{2k_0}}\int_0^t\frac{\exp\left\{-\left(\frac{u^2}{4k_0}+\frac{kj_\nu^2}{4H}\right)\xi+\frac{x^2+y^2}{4k_0\xi}\right\}}{4k_0\xi}d\xi \qquad 5\text{-}10).$$

So we have

$$C=\frac{Q_1e^{\frac{ux}{2k_0}}}{2k_0\pi H}\sum_{\nu=0}^{\infty}K_0\left(\sqrt{\left(\frac{u^2}{4k_0}+\frac{kj_\nu^2}{4H}\right)\frac{x^2+y^2}{4k_0}}\right)$$

$$\times\frac{J_0\left(j_\nu\sqrt{\frac{z}{H}}\right)J_0\left(j_\nu\sqrt{\frac{h}{H}}\right)}{[J_0(j_\nu)]^2} \qquad 5\text{-}11).$$

A-4) *Continuous line source when the flux-zero level exists.* When the flux-zero level exists at $z=H$, the concentration for a continuous infinite line source is given by

$$C=\frac{Q_4}{H\sqrt{\pi}}\sum_{\nu=0}^{\infty}\int_0^\infty\frac{\exp\left[-\left\{\frac{(x-u\xi)^2}{4k_0\xi}+\frac{k\xi}{4H}j_\nu^2\right\}\right]}{4\sqrt{k_0\xi}}$$

$$\times\frac{J_0\left(j_\nu\sqrt{\frac{z}{H}}\right)J_0\left(j_\nu\sqrt{\frac{h}{H}}\right)}{[J_0(j_\nu)]^2}d\xi \qquad 6\text{-}1).$$

As

$$\int_0^\infty \frac{e^{-at-\frac{b}{t}}}{\sqrt{t}}\,dt=\sqrt{\frac{\pi}{a}}\,e^{-2\sqrt{ab}} \tag{6-2}$$

we have

$$C=\frac{Q_4}{H\sqrt{\pi}}\sum_{\nu=0}^{\infty}\sqrt{\frac{\pi}{\left(\dfrac{u^2}{4k_0}+\dfrac{kj_\nu^2}{4H}\right)}}\;\frac{\exp\left\{-2\sqrt{\dfrac{x^2}{4k_0}\left(\dfrac{u^2}{4k_0}+\dfrac{kj_\nu^2}{4H}\right)}+\dfrac{ux}{2k_0}\right\}}{\sqrt{4k_0}}$$

$$\times\frac{J_0\left(j_\nu\sqrt{\dfrac{z}{H}}\right)J_0\left(j_\nu\sqrt{\dfrac{h}{H}}\right)}{[J_0(j_\nu)]^2}$$

$$=\frac{Q_4}{H}\sum_{\nu=0}^{\infty}\frac{1}{\sqrt{u^2+\dfrac{k_0kj_\nu^2}{H}}}\exp\left\{-\frac{ux}{2k_0}\left(\sqrt{1+\frac{k_0kj_\nu^2}{Hu^2}}-1\right)\right\}$$

$$\times\frac{J_0\left(j_\nu\sqrt{\dfrac{z}{H}}\right)J_0\left(j_\nu\sqrt{\dfrac{h}{H}}\right)}{[J_0(j_\nu)]^2}$$

$$=\frac{Q_4}{H}\frac{1}{u}\sum_{\nu=0}^{\infty}\frac{1}{\sqrt{1+\dfrac{k_0kj_\nu^2}{Hu^2}}}\exp\left\{-\frac{ux}{2k_0}\left(\sqrt{1+\frac{k_0kj_\nu^2}{Hu^2}}-1\right)\right\}$$

$$\times\frac{J_0\left(j_\nu\sqrt{\dfrac{z}{H}}\right)J_0\left(j_\nu\sqrt{\dfrac{h}{H}}\right)}{[J_0(j_\nu)]^2} \tag{6-3}$$

A-5) *In the case when the diffusion coefficients are the functions of travelling time.* In this case, the diffusion equation is given by

$$\frac{\partial C}{\partial t}=\frac{\partial}{\partial x}\left(K_x\frac{\partial C}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y\frac{\partial C}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_z\frac{\partial C}{\partial z}\right) \tag{7-1}$$

assuming that

$$K_x=f_1(t)k_x(y,z) \tag{7-2}$$

$$K_y=f_2(t)k_y(x,z) \tag{7-3}$$

$$K_z=f_3(t)k_z(x,y,z) \tag{7-4}$$

so 7-1) becomes

$$\frac{\partial C}{\partial t}=f_1k_x\frac{\partial^2 C}{\partial x^2}+f_2k_y\frac{\partial^2 C}{\partial y^2}+f_3\frac{\partial}{\partial z}\left(k_z\frac{\partial C}{\partial z}\right)$$

$$\therefore\ \frac{1}{f_1}\frac{\partial C}{\partial t}=k_x\frac{\partial^2 C}{\partial x^2}+\frac{f_2}{f_1}k_y\frac{\partial^2 C}{\partial y^2}+\frac{f_3}{f_1}\frac{\partial}{\partial z}\left(k_z\frac{\partial C}{\partial z}\right) \tag{7-5}$$

Putting

$$\frac{\partial \xi}{\partial t}=f_1 \qquad 7\text{-}6).$$

we have

$$\frac{\partial C}{\partial \xi}=k_x\frac{\partial^2 C}{\partial x^2}+g_2(\xi)k_y\frac{\partial^2 C}{\partial y^2}+g_3(\xi)\frac{\partial}{\partial z}\left(k_z\frac{\partial C}{\partial z}\right) \qquad 7\text{-}7).$$

where

$$g_2(\xi)=\frac{f_2(\xi)}{f_1(\xi)}, \qquad g_3(\xi)=\frac{f_3(\xi)}{f_1(\xi)} \qquad 7\text{-}8).$$

Putting

$$C=F(x,\ \xi)\phi(\xi,\ y,\ z)=\frac{e^{-\frac{x^2}{4k_x\xi}}}{\sqrt{4k_x\xi}}\phi \qquad 7\text{-}9),$$

7-7) becomes

$$\frac{\partial C}{\partial \xi}-k_x\frac{\partial^2 C}{\partial x^2}=\frac{C}{\phi}\frac{\partial \phi}{\partial \xi}=F\frac{\partial \phi}{\partial \xi}=g_2(\xi)k_y\frac{\partial^2 C}{\partial y^2}+g_3(\xi)\frac{\partial}{\partial z}\left(k_z\frac{\partial C}{\partial z}\right)$$

$$=g_2(\xi)k_yF\frac{\partial^2 \phi}{\partial y^2}+g_3(\xi)F\frac{\partial}{\partial z}\left(k_z\frac{\partial \phi}{\partial z}\right) \qquad 7\text{-}10),$$

$$\frac{\partial \phi}{\partial \xi}=g_2(\xi)k_y\frac{\partial^2 \phi}{\partial y^2}+g_3(\xi)\frac{\partial}{\partial z}\left(k_z\frac{\partial \phi}{\partial z}\right) \qquad 7\text{-}11),$$

$$\frac{1}{g_2}\frac{\partial \phi}{\partial \xi}=k_y\frac{\partial^2 \phi}{\partial y^2}+\frac{g_3(\xi)}{g_2(\xi)}\frac{\partial}{\partial z}\left(k_z\frac{\partial \phi}{\partial z}\right)=k_y\frac{\partial^2 \phi}{\partial y^2}+h_3(\xi)\frac{\partial}{\partial z}\left(k_z\frac{\partial \phi}{\partial z}\right) \qquad 7\text{-}12),$$

where

$$h_3(\xi)=g_3(\xi)/g_2(\xi) \qquad 7\text{-}13),$$

Putting

$$\frac{\partial \eta}{\partial \xi}=g_2 \qquad 7\text{-}14),$$

we have

$$\frac{1}{g_2}\frac{\partial \phi}{\partial \xi}=\frac{\partial \phi}{\partial \eta}=k_y\frac{\partial^2 \phi}{\partial y^2}+h_3(\eta)\frac{\partial}{\partial z}\left(k_z\frac{\partial \phi}{\partial z}\right) \qquad 7\text{-}15).$$

Putting

$$\phi=G(\eta,\ y)\psi(\eta,\ z)=\frac{e^{-\frac{y^2}{4k_y\eta}}}{\sqrt{4k_y\eta}}\psi(\eta,\ z) \qquad 7\text{-}16),$$

7-15) becomes

$$\frac{\phi}{\psi}\frac{\partial \psi}{\partial \eta}=G\frac{\partial \psi}{\partial \eta}=h_3\frac{\partial}{\partial z}\left(k_z\frac{\partial \phi}{\partial z}\right)=h_3G\frac{\partial}{\partial z}\left(k_z\frac{\partial \psi}{\partial z}\right)$$

$$\frac{\partial \psi}{\partial \eta}=h_3\frac{\partial}{\partial z}\left(k_z\frac{\partial \psi}{\partial z}\right)$$

$$\frac{1}{h_3}\frac{\partial\psi}{\partial\eta}=\frac{\partial}{\partial z}\left(k_z\frac{\partial\psi}{\partial z}\right) \qquad 7\text{-}17).$$

Putting

$$\frac{\partial\zeta}{\partial\eta}=h_3 \qquad 7\text{-}18),$$

7-16) becomes

$$\frac{\partial\psi}{\partial\zeta}=\frac{\partial}{\partial z}\left(k_z\frac{\partial\psi}{\partial z}\right) \qquad 7\text{-}19).$$

Finally, we have the solution of 7-1):

$$C=\frac{e^{-\frac{y^2}{4k_x\xi}}}{\sqrt{4k_x\xi}}\frac{e^{-\frac{y^2}{4k_y\eta}}}{\sqrt{4k_y\eta}}\psi(\zeta, z) \qquad 7\text{-}20).$$

Therefore, by substituting the variables

$$\left.\begin{aligned}
\xi&=\int f_1(t)dt\\
\eta&=\int g_2(\xi)d\xi=\int\frac{f_2}{f_1}f_1dt=\int f_2dt\\
\zeta&=\int h_3(\eta)d\eta=\int\frac{g_3(\xi)}{g_2(\xi)}g_2(\xi)d\xi=\int g_3(\xi)d\xi=\int\frac{f_3}{f_1}f_1dt=\int f_3dt
\end{aligned}\right\} \qquad 7\text{-}21),$$

in the solution of the equation which has time-independent diffusion coefficients, we obtain the solution in general case.

In the previous paper[1], the diffusion coefficients were assumed as follows:

$$\left.\begin{aligned}
&\frac{\partial C}{\partial t}=k_0f_1(t)\left(\frac{\partial^2C}{\partial x^2}+\frac{\partial^2C}{\partial y^2}\right)+kf_3(t)\frac{\partial}{\partial z}\left(z\frac{\partial C}{\partial z}\right)\\
&k_x=k_y=k_0\,,\qquad k_z=kz\\
&k_0f_1(t)=\frac{q_A\varphi_A}{4}u(1-e^{-\varphi_Aut})\\
&kf_3(t)=q_B\varphi_Bu(1-e^{-\varphi_But})
\end{aligned}\right\} \qquad 7\text{-}22).$$

In this case,

$$\xi=\frac{q_A}{4k_0}(\varphi_Aut+e^{-\varphi_Aut}-1) \qquad 7\text{-}23),$$

$$\zeta=\frac{q_B}{k}(\varphi_But+e^{-\varphi_But}-1) \qquad 7\text{-}24),$$

so,

$$4k_0\xi = q_A(\varphi_A ut + e^{-\varphi_A ut} - 1) = q_A(\varphi_A x + e^{-\varphi_A x} - 1) \equiv A \qquad 7\text{-}25),$$

$$k\zeta = q_B(\varphi_B ut + e^{-\varphi_B ut} - 1) = q_B(\varphi_B x + e^{-\varphi_B x} - 1) \equiv B \qquad 7\text{-}26),$$

A and B are used for convenience and φ_A, φ_B, q_A, q_B are diffusion parameters.[2)]

In the cases when the diffusion coefficients are the functions of time, 1-5) becomes

$$C = \frac{M}{AB\pi} \exp\left\{-\frac{(x-ut)^2+y^2}{A} - \frac{z+h}{B}\right\} I_0\left(\frac{2\sqrt{hz}}{B}\right) \qquad 7\text{-}27),$$

2-19) becomes

$$C = \frac{Q_2}{H} \sum_{\mu=0}^{\infty} e^{-\frac{A}{4H}\left(\frac{j_\mu}{2}\right)^2} \frac{J_0\left(j_\mu\sqrt{\frac{h}{H}}\right) J_0\left(j_\mu\sqrt{\frac{z}{H}}\right)}{[J_0(j_\mu)]^2} \qquad 7\text{-}28),$$

3-2) becomes

$$C = \frac{Q_3}{H} \sum_{\nu=0}^{\infty} \frac{\exp\left\{-\frac{(x-ut)^2+y^2}{A} - \frac{B}{4H} j_\nu^2\right\}}{A} \times \frac{J_0\left(j_\nu\sqrt{\frac{z}{H}}\right) J_0\left(j_\nu\sqrt{\frac{h}{H}}\right)}{[J_0(j_\nu)]^2} \qquad 7\text{-}29),$$

4-2) becomes

$$C = \frac{Q_4}{H\sqrt{\pi}} \sum_{\nu=0}^{\infty} \frac{1}{\sqrt{A}} \exp\left\{-\left(\frac{(x-ut)^2}{A} + \frac{B}{4H} j_\nu^2\right)\right\} \times \frac{J_0\left(j_\nu\sqrt{\frac{z}{H}}\right) J_0\left(j_\nu\sqrt{\frac{h}{H}}\right)}{[J_0(j_\nu)]^2} \qquad 7\text{-}30),$$

5-11) becomes

$$C = \frac{Q_1}{H} \frac{1}{\pi} \frac{2x}{Au} e^{-\frac{2x^2}{A}} \sum_{\nu=0}^{\infty} K_0\left(\frac{x}{A}\sqrt{\left(1+\frac{ABj_\nu^2}{4Hx^2}\right)(x^2+y^2)}\right) \times \frac{J_0\left(j_\nu\sqrt{\frac{z}{H}}\right) J_0\left(j_\nu\sqrt{\frac{h}{H}}\right)}{[J_0(j_\nu)]^2} \qquad 7\text{-}31),$$

and 6-3) becomes

$$C = \frac{Q_3}{Hu} \sum_{\nu=0}^{\infty} \frac{1}{\sqrt{1+\frac{ABj_\nu^2}{4Hx^2}}} \exp\left\{-\frac{2x^2}{A}\left(\sqrt{1+\frac{ABj_\nu^2}{4Hx^2}} - 1\right)\right\} \times \frac{J_0\left(j_\nu\sqrt{\frac{z}{H}}\right) J_0\left(j_\nu\sqrt{\frac{h}{H}}\right)}{[J_0(j_\nu)]^2} \qquad 7\text{-}32),$$

respectively.

A-6) *Results of the calculation.* In order to obtain the quantitative relations between the source height, the height of the flux-zero level and the concentration, we calculated the concentration for continuous line sources by using equation 7-32) in the scheme given in Table 1. The calculations were carried out by OKI-MINITAC 7000 in

Table 1. Scheme of calculation.

h (m)	H (m)	ζ		
		0.4	0.0	−0.2
50	50 100 150 200	0.4, 0.8, 1.0, 2.0, 4.0, 8.0, 10.0	0.4, 0.8, 1.0, 2.0, 4.0, 8.0, 10.0	0.2, 0.4, 0.8, 1.0, 2.0, 4.0
100	100 150 200 300	1.0, 2.0, 4.0, 8.0, 10.0, 20.0	0.8, 1.0, 2.0, 4.0, 8.0, 10.0	0.8, 1.0, 2.0, 4.0, 8.0, 10.0, 20.0
150	150 200 300 400	1.0, 2.0, 4.0, 8.0, 10.0, 20.0, 40.0	1.0, 2.0, 4.0, 8.0, 10.0, 20.0	1.0, 2.0, 4.0, 8.0, 10.0, 20.0
200	200 300 400 500	2.0, 4.0, 8.0, 10.0, 20.0	2.0, 4.0, 8.0, 10.0, 20.0	2.0, 4.0, 8.0, 10.0, 20.0
300	300 400 500 600 800	2.0, 4.0, 8.0, 10.0, 20.0, 40.0	4.0, 8.0, 10.0, 20.0	4.0, 8.0, 10.0, 20.0

Ochanomizu University. In the table, ζ denotes the grades of thermal stabilities, and $\zeta=0.4$ corresponds to stable condition, $\zeta=0$ to neutral one and $\zeta=-0.2$ to unstable conditions. Fig. 1 shows all the calculated results for $\zeta=0$, and Fig. 2 shows three stability conditions for $h=50$ m, $x=0.8$ km ; $h=150$ m, $x=1$ km ; and $h=300$ m, $x=4$ km.

We can see that the effect of H for the ground level concentrations is not so remarkable when x is not so large, but as x becomes larger, the vertical profiles become uniform and the values become larger.

The relation between H, h and the ratio C_{x_m}/C_{H_∞} are shown in

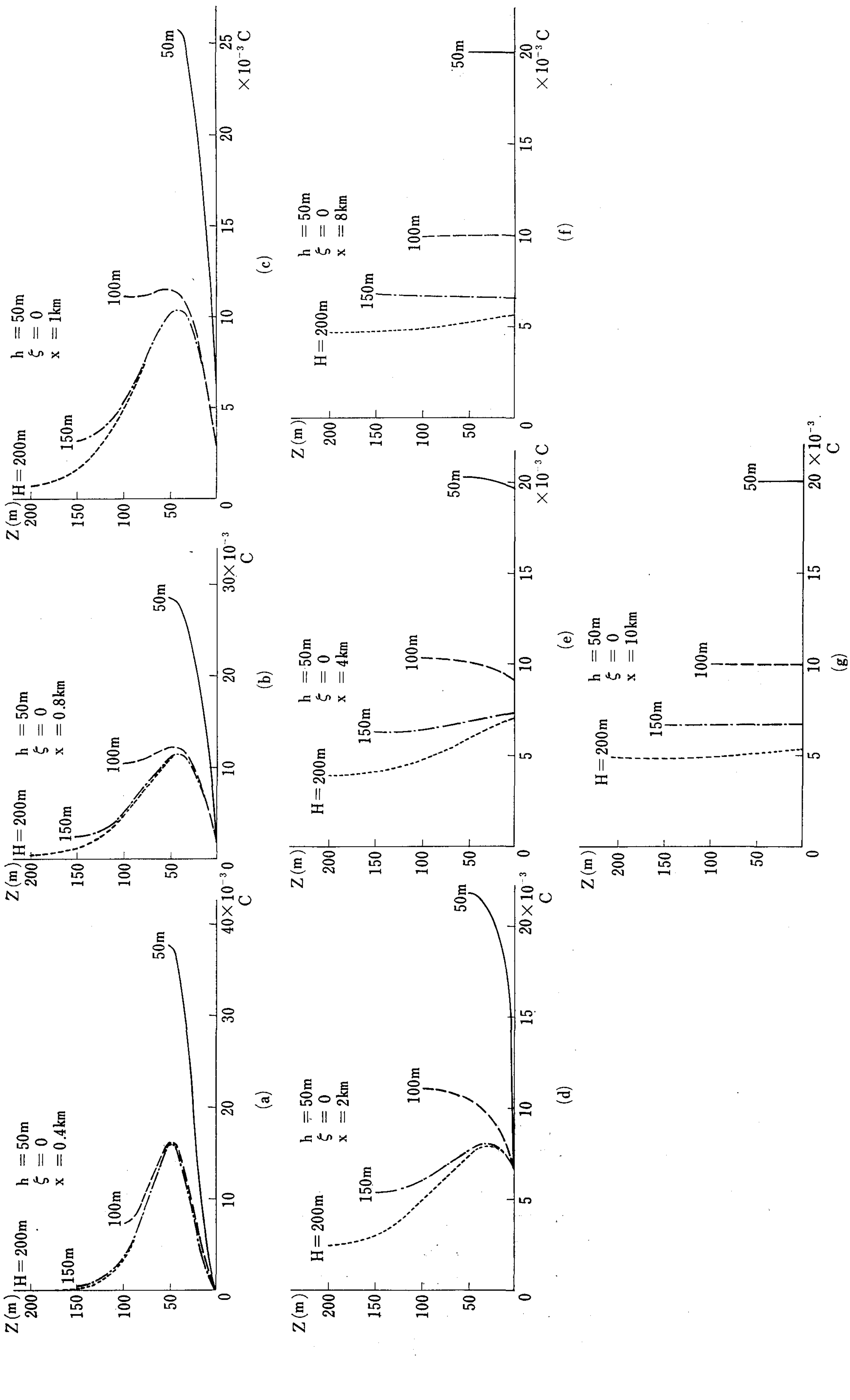

Fig. 1A. Concentration profiles : $\zeta=0$, $h=50$ m.

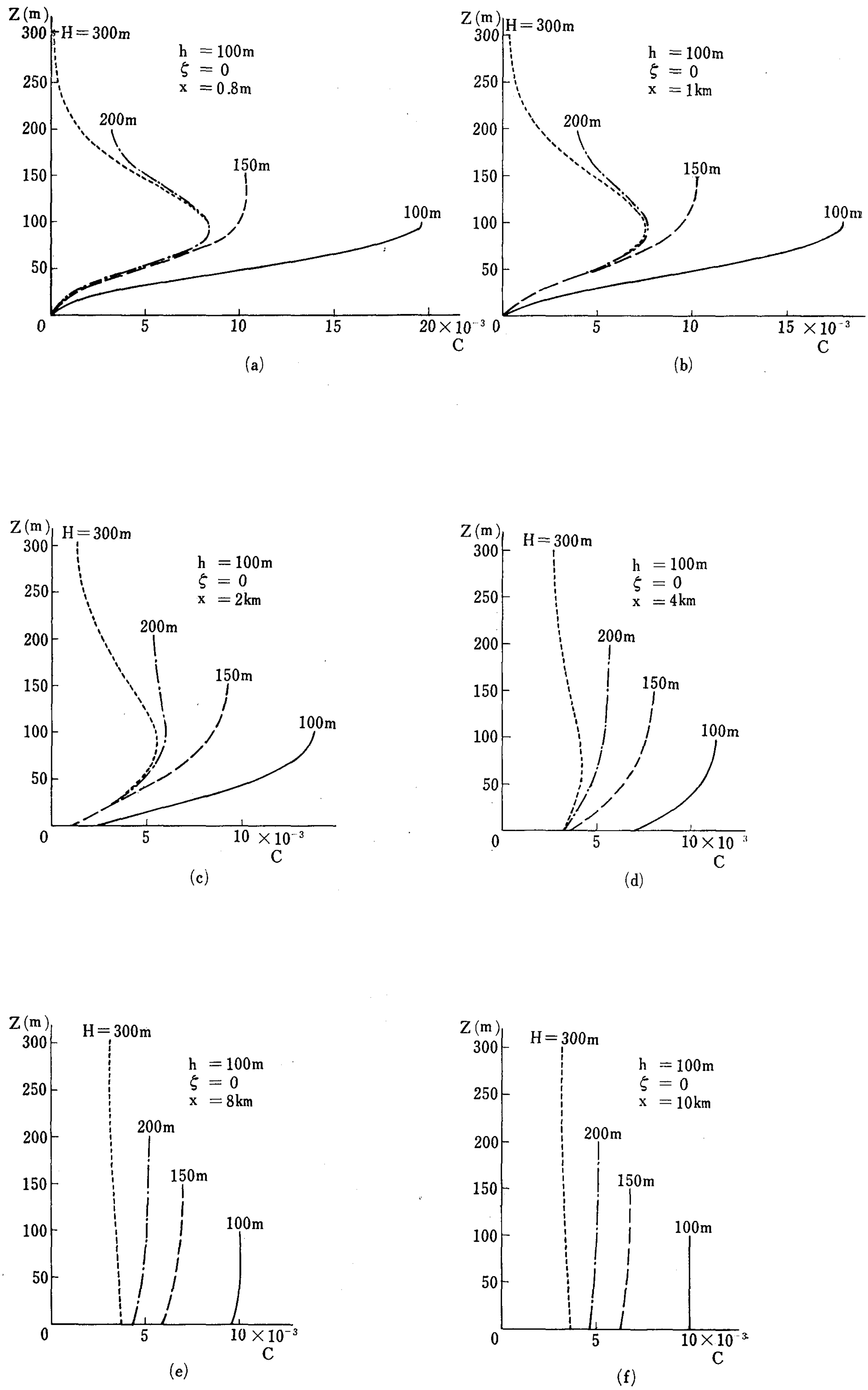

Fig. 1B. Concentration profiles: $\zeta=0$, $h=100$ m.

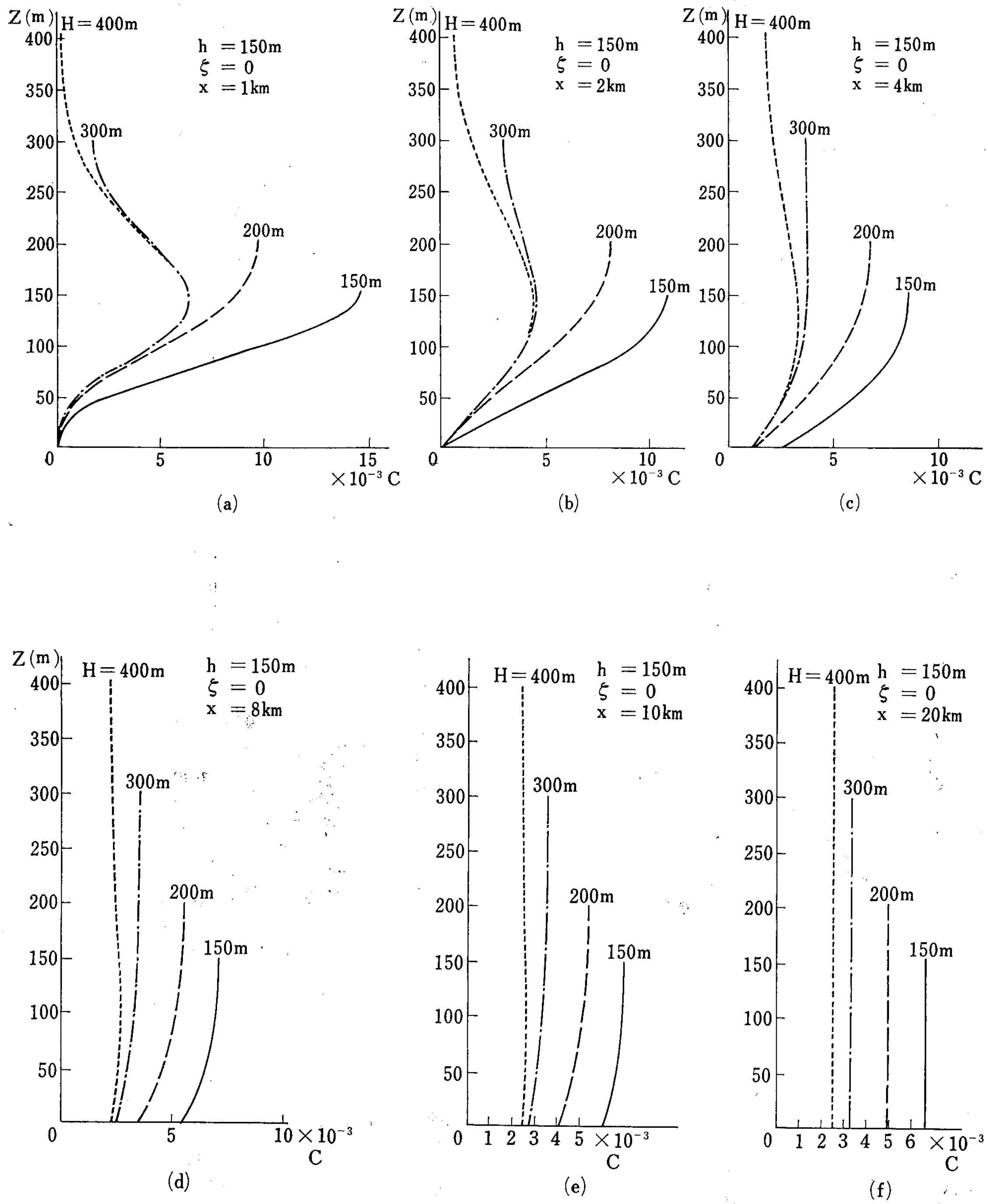

Fig. 1C. Concentration profiles: $\zeta=0$, $h=150$ m.

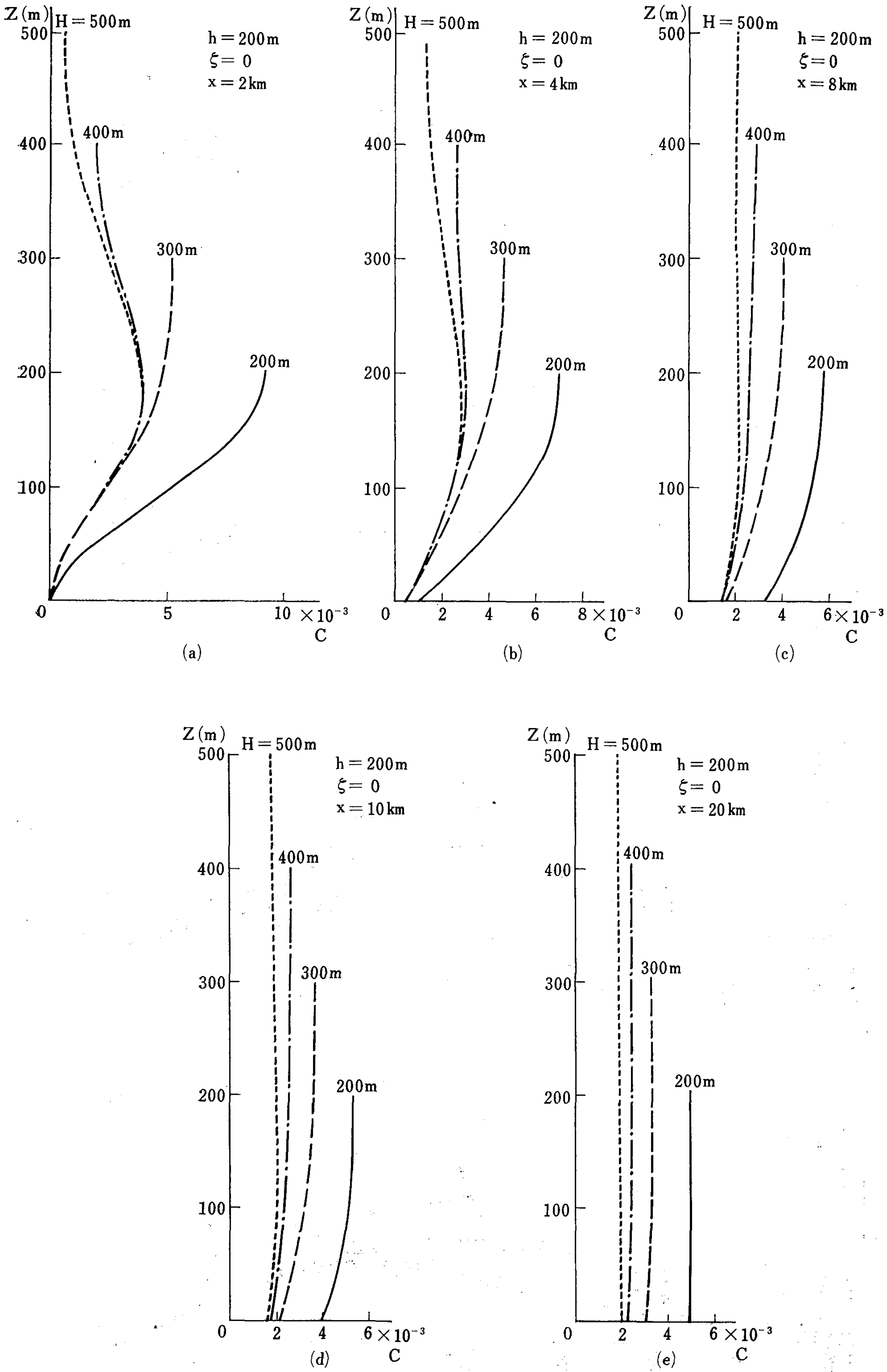

Fig. 1D. Concentration profiles: $\zeta=0$, $h=200$ m.

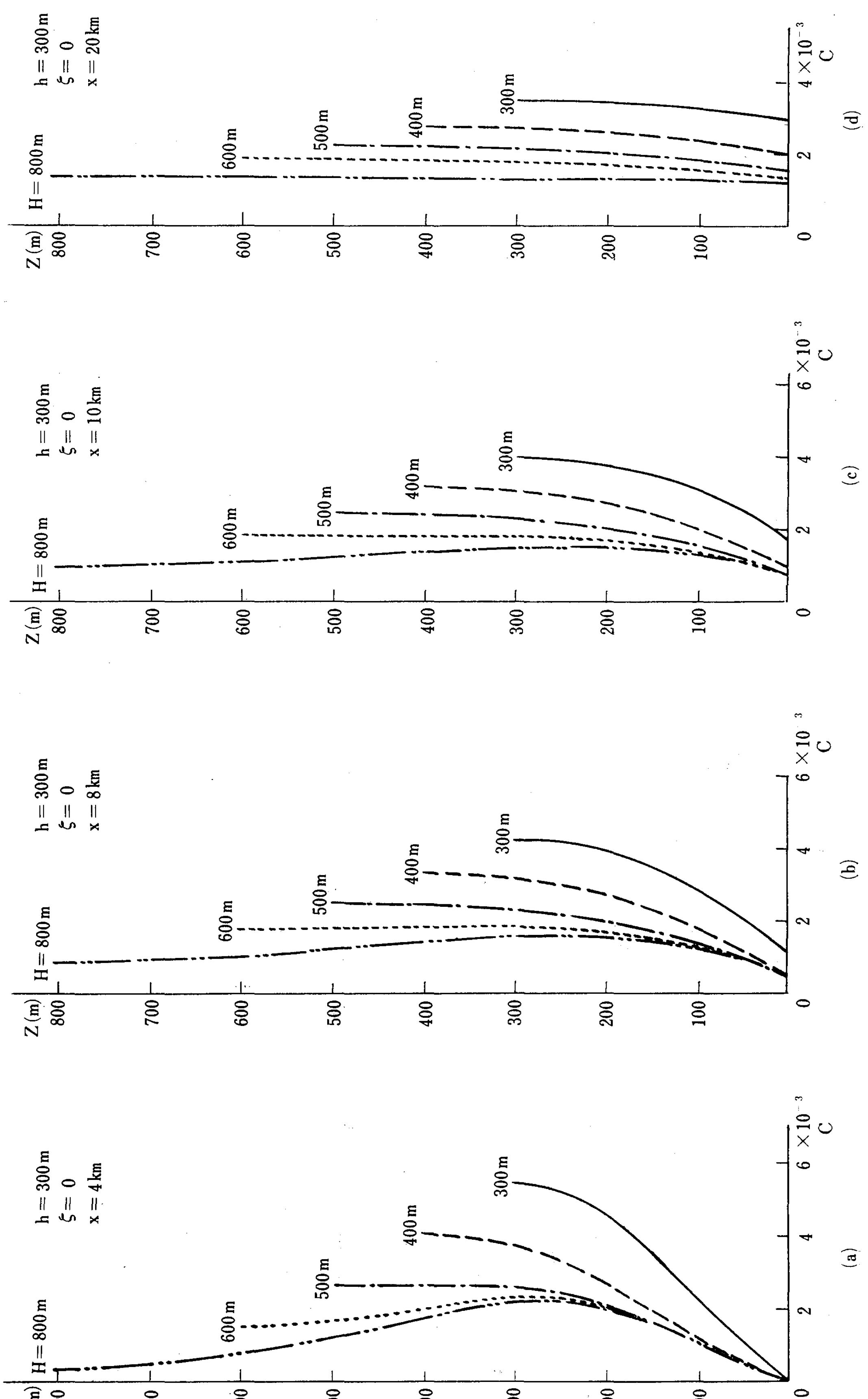

Fig. 1E. Concentration profiles: $\zeta=0$, $h=300$ m,

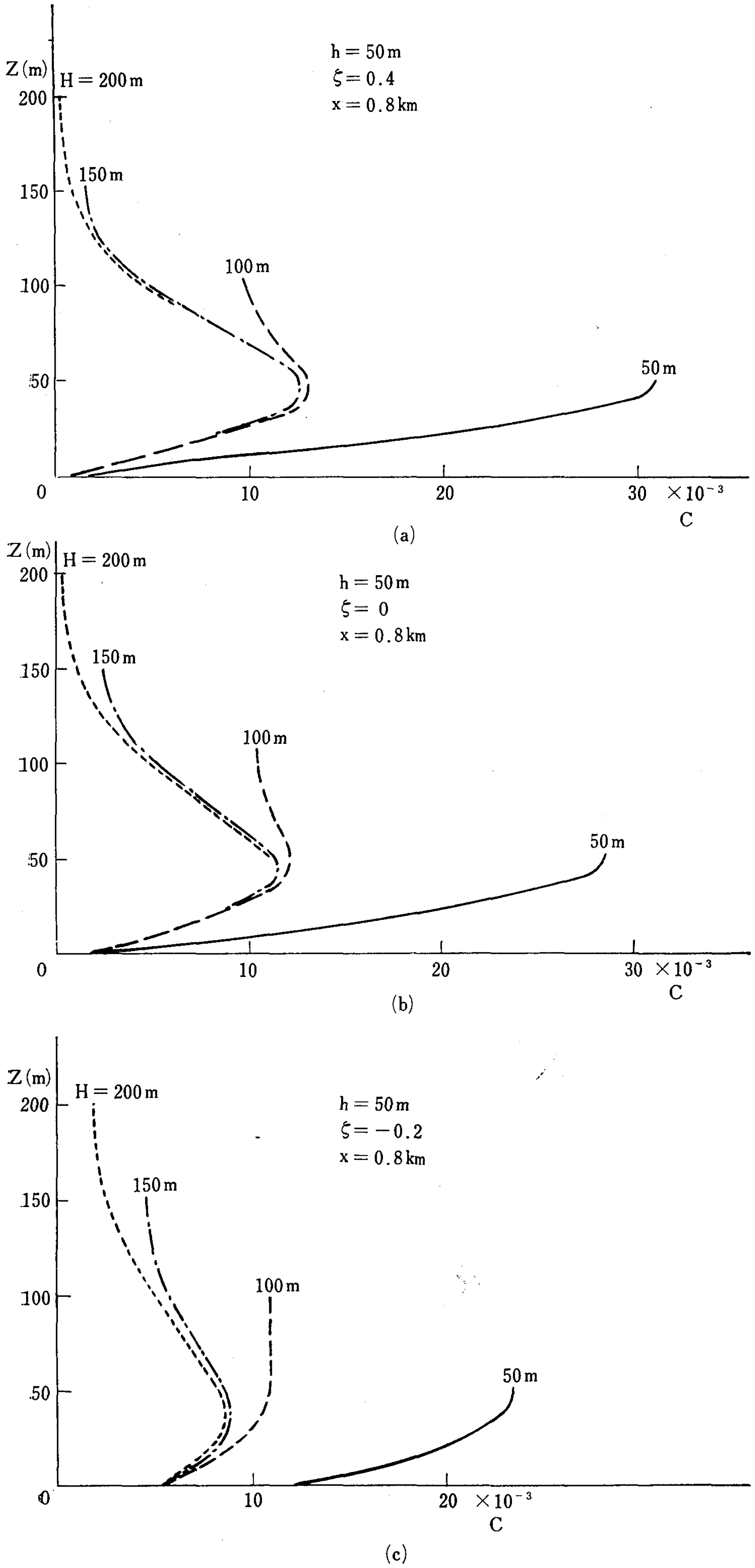

Fig. 2A. Concentration profiles: $h=50$ m, $x=0.8$ km.

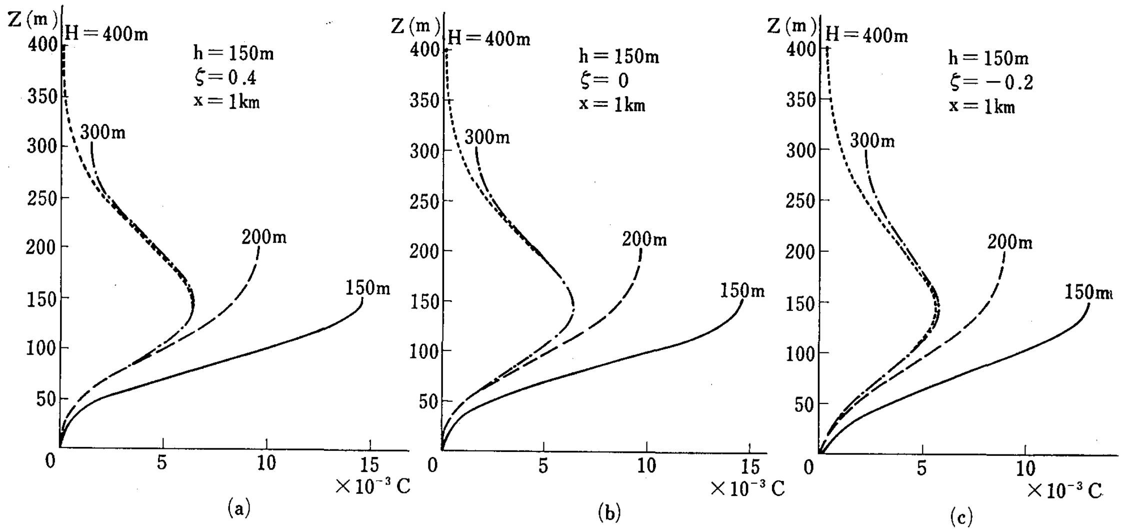

Fig. 2B. Concentration profiles: h=150 m, x=1 km.

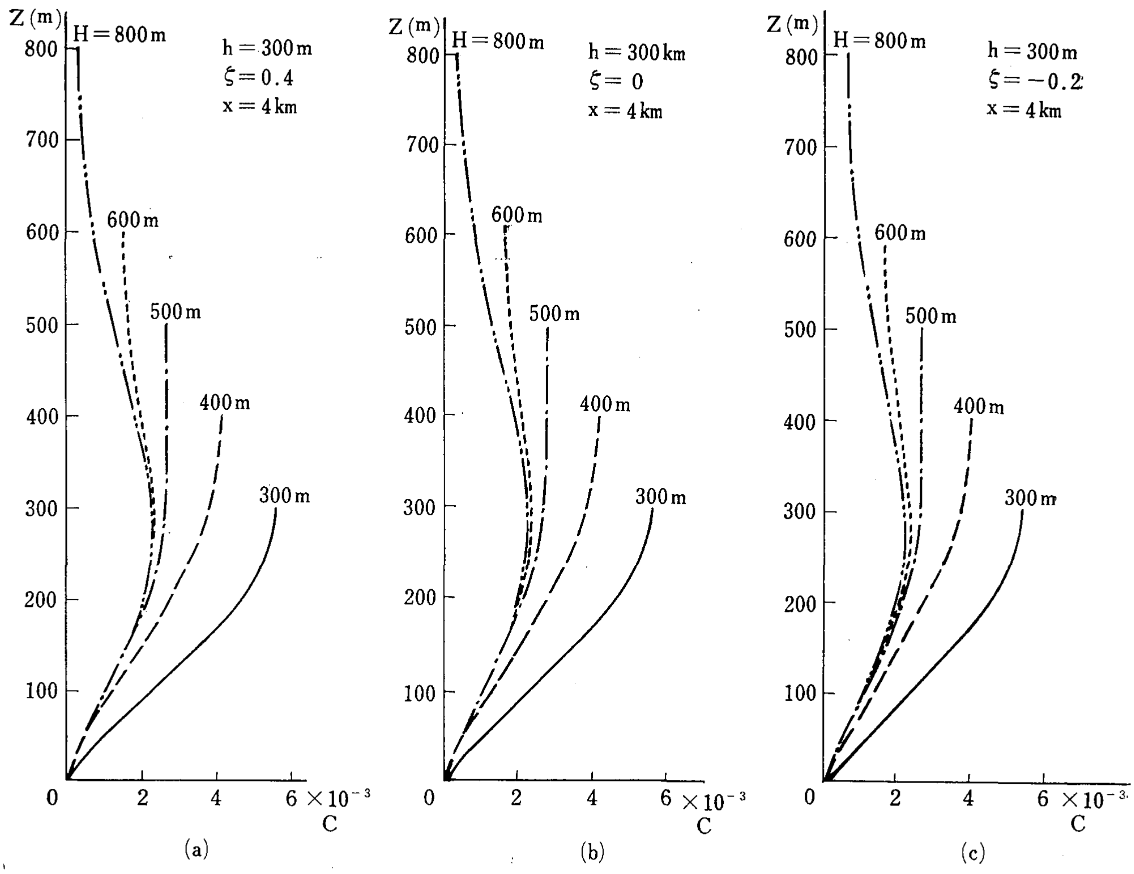

Fig. 2C. Concentration profiles: h=300 m, x=4 km.

Table 2. Relations between H, h and $C_{xm}/C_{H\infty}$. In this Table, e. g. $5.702^{(-3)}$ means 5.702×10^{-3}.

ζ			0.4				0				−0.2			
h (m)	H (m)	H/h	x_m(km)	$C_{H\infty}$	C_{xm}	$C_{xm}/C_{H\infty}$	x_m(km)	$C_{H\infty}$	C_{xm}	$C_{xm}/C_{H\infty}$	x_m(km)	$C_{H\infty}$	C_{xm}	$C_{xm}/C_{H\infty}$
50	50	1	2.0	$5.702^{(-3)}$	$1.300^{(-2)}$	2.26	2.0	$6.658^{(-3)}$	$1.559^{(-2)}$	2.34	1.0	$6.420^{(-3)}$	$1.487^{(-2)}$	2.47
	100	2			$5.805^{(-3)}$	1.03			$6.953^{(-3)}$	1.04			$6.741^{(-3)}$	1.12
	150	3			$5.701^{(-3)}$	1.00			$6.633^{(-3)}$	1.00			$6.167^{(-3)}$	1.02
	200	4			$5.700^{(-3)}$	1.00			$6.625^{(-3)}$	1.00			$6.110^{(-3)}$	1.00
100	100	1	4.0	$2.800^{(-3)}$	$6.349^{(-3)}$	2.27	4.0	$3.058^{(-3)}$	$7.028^{(-3)}$	2.30	2.0	$2.193^{(-3)}$	$5.400^{(-3)}$	2.46
	150	1.5			$3.266^{(-3)}$	1.17			$3.706^{(-3)}$	1.21			$2.864^{(-3)}$	1.31
	200	2			$2.846^{(-3)}$	1.02			$3.139^{(-3)}$	1.03			$2.440^{(-3)}$	1.11
	300	3			$2.800^{(-3)}$	1.00			$3.053^{(-3)}$	1.00			$2.341^{(-3)}$	1.07
150	150	1	8.0	$2.271^{(-3)}$	$5.362^{(-3)}$	2.34	8.0	$2.308^{(-3)}$	$5.491^{(-3)}$	2.34	4.0	$1.587^{(-3)}$	$3.837^{(-3)}$	2.31
	200	1.33			$3.416^{(-3)}$	1.48			$3.537^{(-3)}$	1.48			$2.358^{(-3)}$	1.45
	300	2			$2.388^{(-3)}$	1.04			$2.449^{(-3)}$	1.03			$1.733^{(-3)}$	1.06
	400	2.67			$2.279^{(-3)}$	1.00			$2.319^{(-3)}$	1.00			$1.660^{(-3)}$	1.01
200	200	1	10.0	$1.645^{(-3)}$	$3.843^{(-3)}$	2.34	10.0	$1.686^{(-3)}$	$3.968^{(-3)}$	2.35	8.0	$1.539^{(-3)}$	$3.571^{(-3)}$	2.32
	300	1.5			$2.073^{(-3)}$	1.26			$2.167^{(-3)}$	1.29			$1.985^{(-3)}$	1.29
	400	2			$1.711^{(-3)}$	1.04			$1.768^{(-3)}$	1.05			$1.615^{(-3)}$	1.05
	500	2.5			$1.654^{(-3)}$	1.00			$1.697^{(-3)}$	1.01			$1.522^{(-3)}$	1.00
300	300	1	20.0	$1.210^{(-3)}$	$2.969^{(-3)}$	2.45	15.0	$1.216^{(-3)}$	$3.004^{(-3)}$	2.47	10.0	$8.395^{(-4)}$	$1.995^{(-3)}$	2.38
	400	1.33			$1.986^{(-3)}$	1.65			$2.024^{(-3)}$	1.66			$1.227^{(-3)}$	1.46
	500	1.67			$1.528^{(-3)}$	1.26			$1.556^{(-3)}$	1.28			$9.81^{(-4)}$	1.17
	600	2.00			$1.332^{(-3)}$	1.10			$1.351^{(-3)}$	1.11			$9.00^{(-4)}$	1.07
									$1.234^{(-3)}$	1.01			$8.62^{(-4)}$	1.03

Table 2, where C_{x_m} is the ground level concentration at the position where it becomes maximum, and C_{H_∞} is that when $H=\infty$.

Generally, when $H/h=1$, the values of C_{x_m}/C_{H_∞} are about 2.3～2.5, when H/h is 1.3, C_{x_m}/C_{H_∞} is about 1.5, and when H/h is 1.6, C_{x_m}/C_{H_∞} is about 1.2. So the effects of the flux-zero level are not so severe, except when H is very near to h.

B) Sutton's formula

B-1) *Fundamental formula.* Sutton's formula for the concentrations of a continuous, infinite line source which lies at $x=0$, $y=-L$ to $y=L$ and $z=h$, is given by

$$C=\frac{q}{uC_z x^{1-\frac{n}{2}}\sqrt{\pi}}\left\{\exp\left(-\frac{(z+h)^2}{C_z^2 x^{2-n}}\right)+\exp\left(-\frac{(z-h)^2}{C_z^2 x^{2-n}}\right)\right\}$$

$$=\frac{q}{u}\frac{1}{\sqrt{B_1\pi}}\left\{\exp\left(-\frac{(z+h)^2}{B_1}\right)+\exp\left(-\frac{(z-h)^2}{B_1}\right)\right\} \qquad 8\text{-}1),$$

where $B=C_z^2x^{2-n}$, and C_z and n are Sutton's parameters. In the case when there exists the flux-zero level at $z=H$, we can obtain the formula of concentration by considering the images of the source referring to the levels at $z=0$ and $z=H$.

$$C_2=\frac{q}{u\sqrt{B_1\pi}}\sum_{n=-\infty}^{\infty}\left[\exp\left\{-\frac{(z+h+n\cdot 2H)^2}{B_1}\right\}\right.$$

$$\left.+\exp\left\{-\frac{(z-h+n\cdot 2H)^2}{B_1}\right\}\right] \qquad 8\text{-}2).$$

As

$$\sum_{n=-\infty}^{\infty}\delta(\xi-n)=\sum_{n=-\infty}^{\infty}e^{i2\pi n\xi} \qquad 8\text{-}3),$$

so

$$\sum_{n=-\infty}^{\infty}\exp\left\{-\frac{(z\pm h+2Hn)^2}{B_1}\right\}$$

$$=\int_{-\infty}^{\infty}\exp\left\{-\frac{(z\pm h+2H\xi)^2}{B_1}\right\}\sum_{n=-\infty}^{\infty}\delta(n-\xi)d\xi$$

$$=\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\exp\left\{-\frac{(z\pm h+2H\xi)^2}{B_1}+i2\pi n\xi\right\}d\xi \qquad 8\text{-}4).$$

Putting

$$2H\xi\pm h+z=\eta \qquad 8\text{-}5),$$

we have

$$\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\exp\left\{-\frac{\eta^2}{B_1}+i\frac{2\pi n}{2H}(\eta\mp h-z)\right\}\frac{d\eta}{2H}$$

$$=\frac{1}{2H}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\exp\left\{-\left(\frac{\eta}{\sqrt{B_1}}-\frac{i\pi n\sqrt{B_1}}{2H}\right)^2\right.$$

$$\left.+\left(\frac{\pi n\sqrt{B_1}}{2H}\right)^2-\frac{i\pi n}{H}(\pm h-z)\right\}d\eta$$

$$=\frac{1}{2H}\sum_{n=-\infty}^{\infty}\sqrt{\pi B_1}\exp\left\{-\frac{B_1}{4}\left(\frac{\pi n}{H}\right)^2-\frac{i\pi}{H}(\pm h+z)n\right\} \qquad 8\text{-}6).$$

Therefore, we have

$$C_2=\frac{1}{\sqrt{\pi B_1}}\frac{\sqrt{\pi B_1}}{2H}\sum_{n=-\infty}^{\infty}\exp\left\{-\frac{B_1}{4}\left(\frac{\pi n}{H}\right)^2-\frac{i\pi}{H}zn\right\}2\cos\left(\frac{\pi}{H}hn\right)$$

$$=\frac{1}{H}\sum_{n=-\infty}^{\infty}\exp\left\{-\frac{B_1}{4}\left(\frac{\pi n}{H}\right)^2-\frac{i\pi z}{H}n\right\}\cos\left(\frac{\pi}{H}hn\right)$$

$$=\frac{1}{H}\sum_{n=-\infty}^{\infty}\exp\left\{-\frac{B_1}{4}\left(\frac{\pi n}{H}\right)^2\right\}\cos\left(\frac{n\pi z}{H}\right)\cos\left(\frac{n\pi h}{H}\right) \qquad 8\text{-}7).$$

If $\frac{B_1\pi^2}{4H^2}\gg 1$, namely when $\sqrt{B_1}\gg\frac{2H}{\pi}$,

$$C_z\sim 1/H,$$

which is independent of z.

B-2) *Results of the calculation.* Some of the results of the calculations are shown in Fig. 3, in which $C_z=0.4$ and $n=0.25$.

In Sutton's formula, n, C_z and x are combined together in the form $C_z^2x^{2-n}=B_1$ $(=2\sigma_z^2)$, so we calculated by using B_1 and several values of h and H. The ratio C_{x_m}/C_{H_∞} and H/h are shown in Table 3, where C_{x_m} is the ground level concentration at the position where that concentration becomes maximum and C_{H_∞} is the concentration at the same place when $H=\infty$.

The general features of the results are almost the same to those obtained by Sakagami's formula, because these formulae differ mainly in the region near the ground and when the source height is high, such as more than 100 m, the difference becomes very small.

C) Consideration on the condition of the occurrence of the "flux-zero" level

The boundary conditions at the boundary, $z=H$, of two different domains are

$$C(H)=C'(H) \qquad \text{at } z=H, \qquad 9\text{-}1),$$

$$K_z\frac{\partial C}{\partial z}K_z'\frac{\partial C'}{\partial z}$$

where C' is the concentration and K_z' is the diffusion coefficient in the

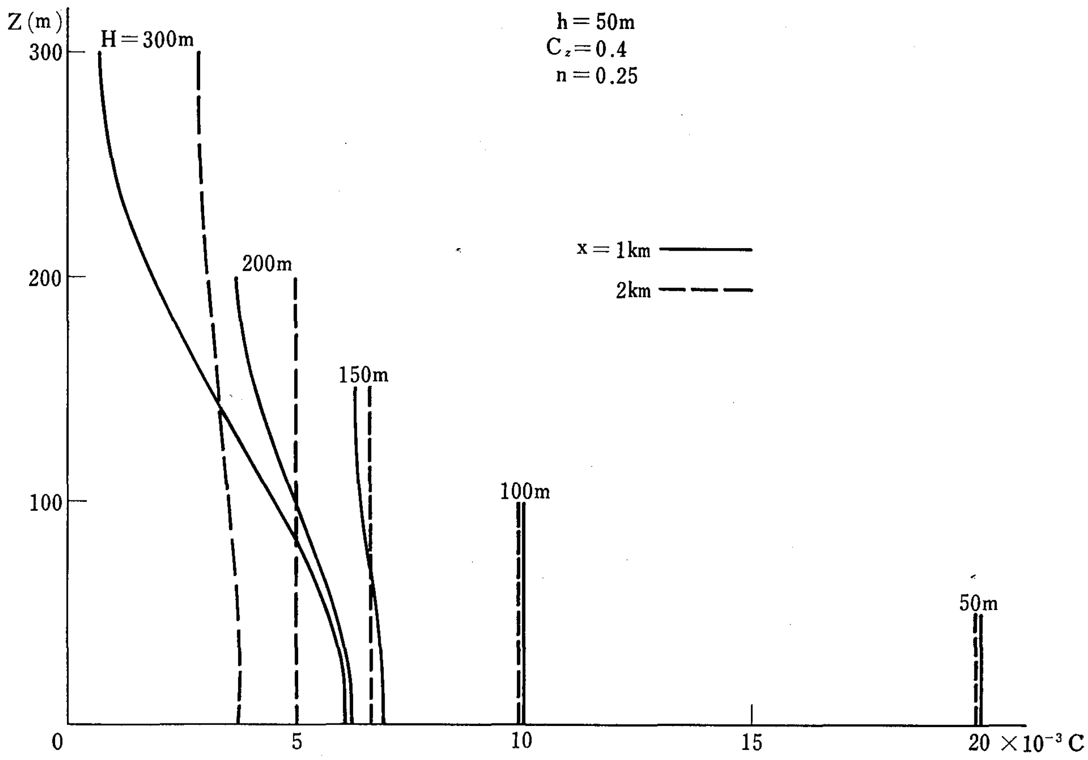

Fig. 3A. Concentration profiles: $h=50$ m, $x=1$ and 2 km.

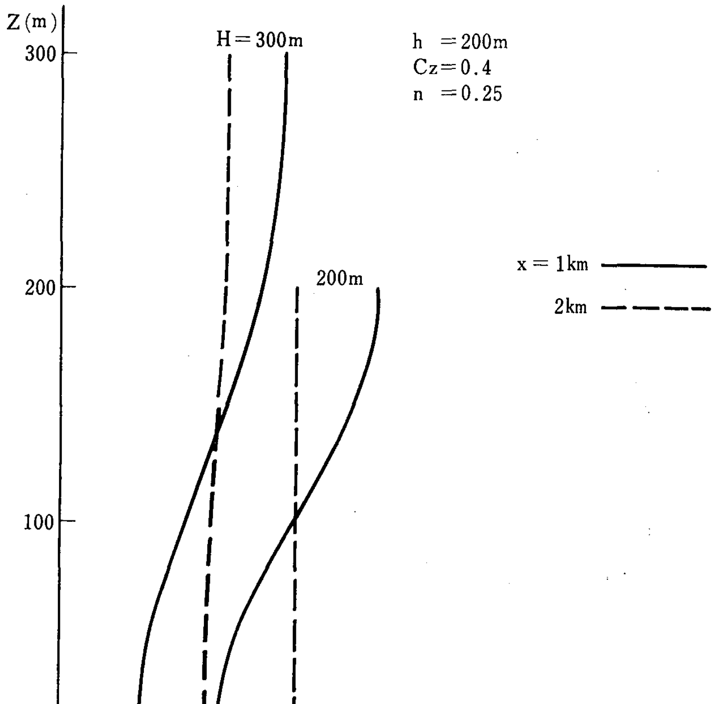

Fig. 3B. Concentration profiles: $h=200$ m, $x=1$ and 2km.

Table 3. Relations between H, h and $C_{xm}/C_{H\infty}$. In this Table, e. g. $3.51^{(-3)}$ means 3.51×10^{-3}.

		h	H	50	100	150	200	300	400	500	600
σ_z	B_1		H/h	1	2	3	4	6	8	10	12
20	0.80$^{(3)}$	50	C	3.51$^{(-3)}$	1.75$^{(-3)}$	1.75$^{(-3)}$	1.75$^{(-3)}$	1.75$^{(-3)}$	1.75$^{(-3)}$	1.75$^{(-3)}$	1.75$^{(-3)}$
			$C/C_{H\infty}$	2.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		100	C		2.99$^{(-7)}$	1.49$^{(-7)}$	1.49$^{(-7)}$	1.49$^{(-7)}$	1.49$^{(-7)}$	1.49$^{(-7)}$	1.49$^{(-7)}$
			$C/C_{H\infty}$		1.79	1.00	1.00	1.00	1.00	1.00	1.00
		150	C			9.14$^{(-13)}$	2.44$^{(-13)}$	2.44$^{(-13)}$	2.44$^{(-13)}$	2.44$^{(-13)}$	2.44$^{(-13)}$
			$C/C_{H\infty}$			3.75	1.00	1.00	1.00	1.00	1.00
		200	C				1.54$^{(-21)}$	7.70$^{(-24)}$	7.70$^{(-24)}$	7.70$^{(-24)}$	7.70$^{(-24)}$
			$C/C_{H\infty}$				2.00	1.00	1.00	1.00	1.00
		300	C					0.00	0.00	0.00	0.00
			$C/C_{H\infty}$					0.00	0.00	0.00	0.00
40	0.32$^{(4)}$	50	C	1.83$^{(-2)}$	9.14$^{(-3)}$	9.14$^{(-3)}$	9.14$^{(-3)}$	9.14$^{(-3)}$	9.14$^{(-3)}$	9.14$^{(-3)}$	9.14$^{(-3)}$
			$C/C_{H\infty}$	2.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		100	C		1.75$^{(-3)}$	8.77$^{(-4)}$	8.77$^{(-4)}$	8.77$^{(-4)}$	8.77$^{(-4)}$	8.77$^{(-4)}$	8.77$^{(-4)}$
			$C/C_{H\infty}$		2.00	1.00	1.00	1.00	1.00	1.00	1.00
		150	C			3.53$^{(-5)}$	1.76$^{(-5)}$	1.76$^{(-5)}$	1.76$^{(-5)}$	1.76$^{(-5)}$	1.76$^{(-5)}$
			$C/C_{H\infty}$			2.01	1.00	1.00	1.00	1.00	1.00
		200	C				1.49$^{(-7)}$	1.44$^{(-8)}$	7.44$^{(-8)}$	7.44$^{(-3)}$	7.44$^{(-8)}$
			$C/C_{H\infty}$				2.00	1.00	1.00	1.00	1.00
		300	C					2.44$^{(-14)}$	1.22$^{(-14)}$	1.22$^{(-14)}$	1.22$^{(-14)}$
			$C/C_{H\infty}$					2.00	1.00	1.00	1.00
60	0.72$^{(4)}$	50	C	2.00$^{(-2)}$	9.99$^{(-3)}$	9.40$^{(-3)}$	9.40$^{(-3)}$	9.40$^{(-3)}$	9.40$^{(-3)}$	9.40$^{(-3)}$	9.40$^{(-3)}$
			$C/C_{H\infty}$	2.13	1.06	1.00	1.00	1.00	1.00	1.00	1.00
		100	C		6.63$^{(-3)}$	3.37$^{(-3)}$	3.32$^{(-3)}$	3.32$^{(-3)}$	3.32$^{(-3)}$	3.32$^{(-3)}$	3.32$^{(-3)}$
			$C/C_{H\infty}$		2.00	1.02	1.00	1.00	1.00	1.00	1.00
		150	C			1.17$^{(-3)}$	5.87$^{(-4)}$	5.84$^{(-4)}$	5.84$^{(-4)}$	5.84$^{(-4)}$	5.84$^{(-4)}$
			$C/C_{H\infty}$			2.00	1.00	1.00	1.00	1.00	1.00
		200	C				1.03$^{(-4)}$	5.14$^{(-5)}$	5.14$^{(-5)}$	5.14$^{(-5)}$	5.14$^{(-5)}$
			$C/C_{H\infty}$				2.00	1.00	1.00	1.00	1.C0
		300	C					9.91$^{(-8)}$	4.96$^{(-8)}$	4.96$^{(-8)}$	4.96$^{(-8)}$
			$C/C_{H\infty}$					2.00	1.00	1.00	1.00
80	0.13$^{(5)}$	50	C	2.00$^{(-2)}$	1.00$^{(-2)}$	8.28$^{(-3)}$	8.21$^{(-3)}$	8.21$^{(-3)}$	8.21$^{(-3)}$	8.21$^{(-3)}$	8.21$^{(-3)}$
			$C/C_{H\infty}$	2.44	1.22	1.01	1.00	1.00	1.00	1.00	1.00
		100	C		9.15$^{(-3)}$	5.01$^{(-3)}$	4.58$^{(-3)}$	4.58$^{(-3)}$	4.58$^{(-3)}$	4.58$^{(-3)}$	4.58$^{(-3)}$
			$C/C_{H\infty}$		2.00	1.09	1.00	1.00	1.00	1.00	1.00
		150	C			3.44$^{(-3)}$	1.80$^{(-3)}$	1.72$^{(-3)}$	1.72$^{(-3)}$	1.72$^{(-3)}$	1.72$^{(-3)}$
			$C/C_{H\infty}$			2.00	1.05	1.00	1.00	1.00	1.00
		200	C				8.77$^{(-4)}$	4.38$^{(-4)}$	4.38$^{(-4)}$	4.38$^{(-4)}$	4.38$^{(-4)}$
			$C/C_{H\infty}$				2.00	1.00	1.00	1.00	1.00
		300	C					1.76$^{(-5)}$	8.82$^{(-6)}$	8.82$^{(-6)}$	8.82$^{(-6)}$
			$C/C_{H\infty}$					2.00	1.00	1.00	1.00
100	0.20$^{(5)}$	50	C	2.00$^{(-2)}$	1.00$^{(-2)}$	7.41$^{(-3)}$	7.06$^{(-3)}$	7.04$^{(-3)}$	7.04$^{(-3)}$	7.04$^{(-3)}$	7.04$^{(-3)}$
			$C/C_{H\infty}$	2.84	1.42	1.05	1.00	1.00	1.00	1.00	1.00
		100	C		9.86$^{(-3)}$	5.92$^{(-3)}$	4.93$^{(-3)}$	4.84$^{(-3)}$	4.84$^{(-3)}$	4.84$^{(-3)}$	4.84$^{(-3)}$
			$C/C_{H\infty}$		2.04	1.22	1.02	1.00	1.00	1.00	1.00
		150	C			5.18$^{(-3)}$	2.94$^{(-3)}$	2.59$^{(-3)}$	2.59$^{(-3)}$	2.59$^{(-3)}$	2.59$^{(-3)}$
			$C/C_{H\infty}$			2.00	1.14	1.00	1.00	1.00	1.00
		200	C				2.16$^{(-3)}$	1.08$^{(-3)}$	1.08$^{(-3)}$	1.08$^{(-3)}$	1.08$^{(-3)}$
			$C/C_{H\infty}$				2.00	1.00	1.00	1.00	1.00
		300	C					1.77$^{(-4)}$	8.87$^{(-5)}$	8.87$^{(-5)}$	8.87$^{(-5)}$
			$C/C_{H\infty}$					2.00	1.00	1.00	1.00
200	0.80$^{(6)}$	50	C	2.00$^{(-2)}$	1.00$^{(-2)}$	6.67$^{(-3)}$	5.05$^{(-3)}$	3.98$^{(-3)}$	3.87$^{(-3)}$	3.87$^{(-3)}$	3.87$^{(-3)}$
			$C/C_{H\infty}$	5.17	2.58	1.72	1.30	1.03	1.00	1.00	1.00
		100	C		1.00$^{(-2)}$	6.67$^{(-3)}$	5.00$^{(-3)}$	3.71$^{(-3)}$	3.53$^{(-3)}$	3.52$^{(-3)}$	3.52$^{(-3)}$
			$C/C_{H\infty}$		2.84	1.87	1.42	1.05	1.00	1.00	1.00
		150	C			6.67$^{(-3)}$	4.95$^{(-3)}$	3.33$^{(-3)}$	3.03$^{(-3)}$	3.01$^{(-3)}$	3.01$^{(-3)}$
			$C/C_{H\infty}$			2.22	1.64	1.11	1.01	1.00	1.00
		200	C				4.93$^{(-3)}$	2.96$^{(-3)}$	2.47$^{(-3)}$	2.42$^{(-3)}$	2.42$^{(-3)}$
			$C/C_{H\infty}$				2.04	1.22	1.02	1.00	1.00
		300	C					2.59$^{(-3)}$	1.47$^{(-3)}$	1.30$^{(-3)}$	1.30$^{(-3)}$
			$C/C_{H\infty}$					1.99	1.13	1.00	1.00

domain above H.

If the height H is considerably high, the condition of the flux-zero can be expressed approximately by the next equation:

$$\left(K_z\frac{\partial C}{\partial z}\right)_{z=H}=\frac{1}{\sqrt{\pi}}f_1(H)\frac{1}{2K_z\sqrt{t}}\left[-1+\frac{K_z\sqrt{K_z'}-K_z'\sqrt{K_z}}{K_z\sqrt{K_z'}+K_z'\sqrt{K_z}}\right]=0 \quad 9\text{-}2),$$

which is obtained by using the results of Kodaira's paper,[3] where $f_1(z)$ is the initial concentration distribution in the domain lower than H. Namely, we have

$$K_z\sqrt{K_z'}+K_z'\sqrt{K_z}=K_z\sqrt{K_z'}-K_z'\sqrt{K_z}$$

$$K_z'\sqrt{K_z}=0 \quad 9\text{-}3).$$

As K_z is not equal to zero, so K_z' must be zero. So we can conclude that the flux-zero level occurs only when the upper domain is so extremly stable that merely laminar flow exists.

The vertical diffusion coefficient does not generally vanish even in the inversion state, except in extremely stable state. So the flux-zero level does not so frequently occur even when the inversion layer is above the neutral or lapse layer.

Literatures

1) Sakagami, J.: On the Turbulent Diffusion in the Atmosphere near the Ground, 1954, Natural Science Report, Ochanomizu Univ., **5** (1), pp. 79–91.

2) Sakagami, J.: On the Relations between the Diffusion Parameters and Meteorological Conditions, 1960, ditto, **10** (1), pp. 19–30.

3) Kodaira, Y.: Conduction of Heat in an Infinite Solid made of two Different Parts, 1950, Geophysical Magazine, **21** (3), pp. 217–220.