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Heat Diffusion Close to a Solid Wall in Turbulent Boundary Layer

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Introduction. In order to investigate quantitatively the problems of the air pollution, it is quite necessary to research in detail the diffusion phenomena in the atmosphere. Furthermore, it is not sufficient only to obtain the knowledge of variation of standard deviation of diffusive quantities, we must have informations about the distribution of those quantities themselves. For this purpose, we should determine the differential equation which governs the phenomena at first and then solve that equation.

Field experiments of the atmospheric diffusion are not easy to carry out frequently and the results are very sensitive to various circumstances under which the experiments are carried out. So they are not adequate to investigate in detail the mechanism of the phenomena. Therefore, as the fundamental research on the turbulent diffusion close to the ground, we made a series of diffusion experiments close to a solid surface in the turbulent boundary layer in a wind tunnel, and examined two types of solution, which of them was more adequate to describe the phenomena.

Instrumentation. The experiments were made in a small wind tunnel in Hermann-Föttinger Institut, Technische Universität Berlin, where the author stayed about half a year in 1963 by lucky opportunity. The wind tunnel is of blow down type and the test section has lateral side walls 18 cm in height and has a flat plate 10 cm in breadth and 50 cm in length at the bottom whose temperature can be adjusted by circulating cold or hot water. At the leading edge of the plate, a circular rod 8 mm in diameter was set in order to make the boundary layer over the plate turbulent.

As the tracer of the diffusion, we used heat. We set a nichrome ribbon, 0,03 mm in thickness, 1 mm in breadth, parallel to the plate and perpendicular to the wind (y-direction) at the position of 10 cm leeward from the leading edge, then the ribbon was heated electrically.

The temperature distributions in the direction vertical to the plate (z-direction) in the vertical center plane of the wind tunnel, were measured by a copper-constantan thermo-junction about 0,07 mm in diameter.

The leeward lengths (x) of the measuring positions from the heat source were 5, 10, 20 and 30 cm.

The wind velocity was measured by a hot-wire anemometer calibrated by a Pitot-tube, and the anemometer was of linearized constant-temperature type. Mean velocities (U) and velocity fluctuations in the x-direction ($\sqrt{\overline{u'^2}}$) were measured by the anemometer with a single platinum-plated tungsten wire, 5 μ in diameter, and the fluctuations in z-direction ($\sqrt{\overline{w'^2}}$) were measured by an X-meter, the length of the side of the probe was 2,5 mm.

Mean wind velocities and velocity fluctuations. We made measurements in three conditions: (W) The temperature of the plate (T_w) is higher about 5°C than the air temperature (T_a), (N) both temperatures are almost the same and (C) the temperature of the plate is lower about 5°C than the air temperature. The results are shown in Fig. 1, 2 and 3. Any remarkable differences in the mean velocities

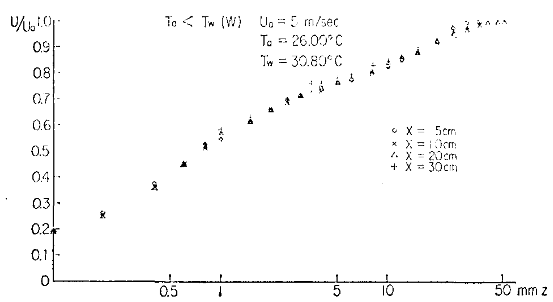

Fig. 1. Profiles of mean wind velocity: (W).

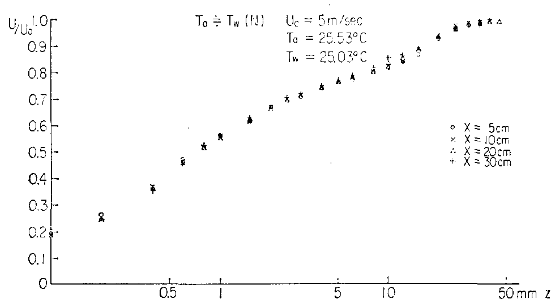

Fig. 2. Profiles of mean wind velocity: (N)

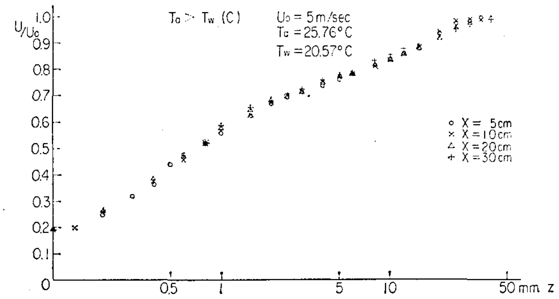

Fig. 3. Profiles of mean wind velocity: (C).

in these conditions were not recognized. The thickness of the laminar sublayer was about 1 mm. The results of the velocity fluctuations $\sqrt{\overline{u'^2}}$ and $\sqrt{\overline{w'^2}}$ are shown in Fig. 4, 5, 6 and Fig. 7, 8, 9 respectively. From these figures, we can clearly recognize that both regions lower

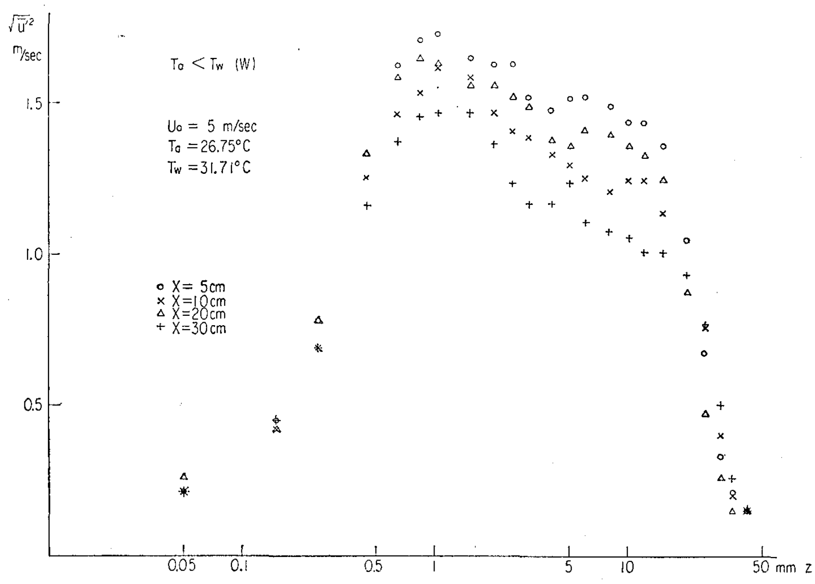

Fig. 4. Profiles of $\sqrt{\overline{u'^2}}$: (W).

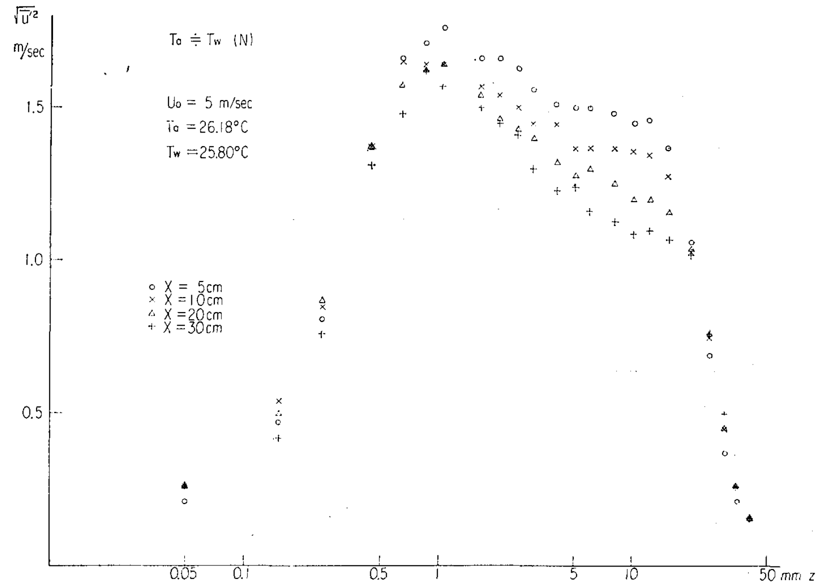

Fig. 5. Profiles of $\sqrt{\overline{u'^2}}$: (N).

than 0,5 mm and higher than 20 mm are different from the intermediate region as the turbulent field, and the diffusion in that region should be investigated. In that intermediate region, the values of $\sqrt{\overline{w'^2}}$ are nearly constant in z-direction, but those of $\sqrt{\overline{u'^2}}$ decrease with the height. The quantity which contributes the vertical diffusion is $\sqrt{\overline{w'^2}}/\sqrt{\overline{u'^2}}$, so it should be considered that the vertical diffusion coefficient increases with the height.

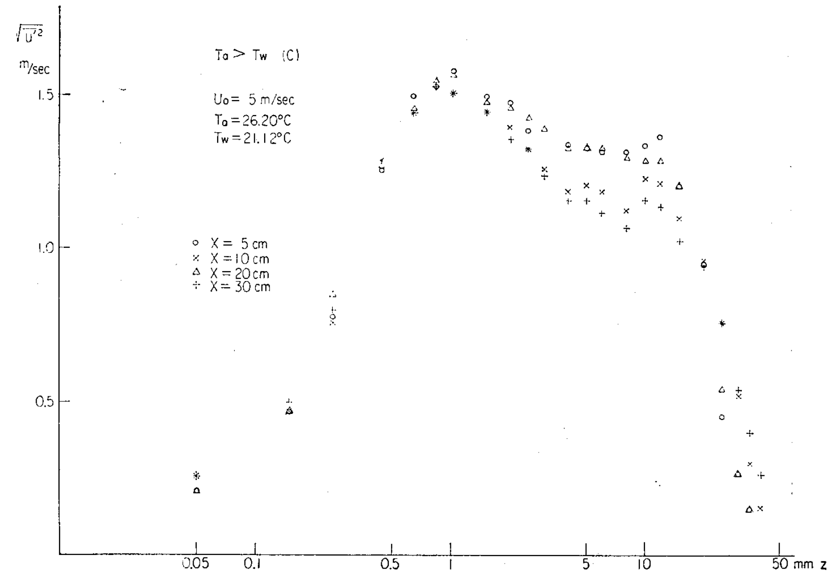

Fig. 6. Profiles of $\sqrt{\overline{u'^2}}$: (C).

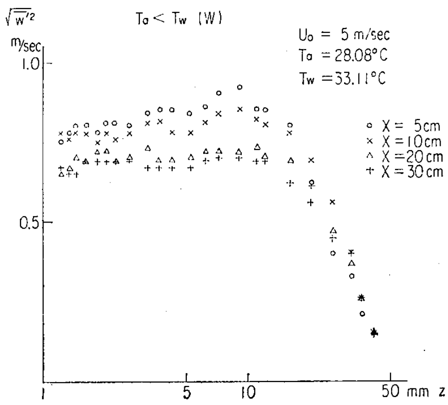

Fig. 7. Profiles of $\sqrt{\overline{w'^2}}$: (W).

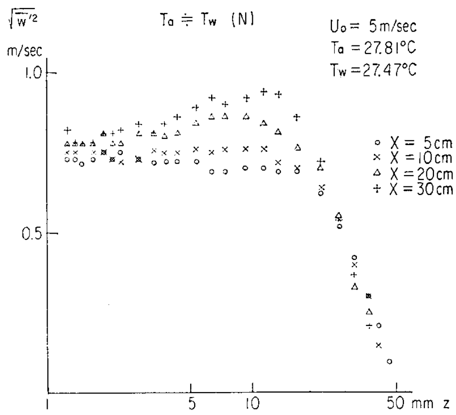

Fig. 8. Profiles of $\sqrt{\overline{w'^2}}$: (N).

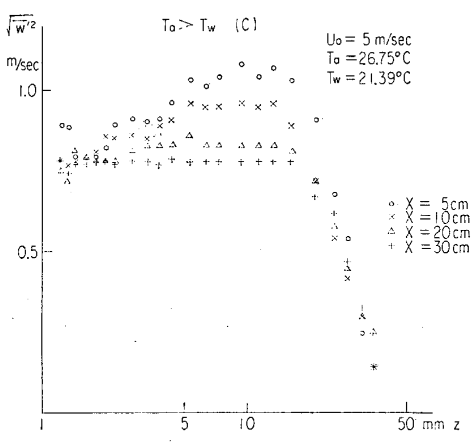

Fig. 9. Profiles of $\sqrt{\overline{w'^2}}$: (C).

Temperature distribution. We measured the temperature profiles (T_a) in z-direction at each measuring position without the heat source and then measured those (T) with the source and calculated the differences (ΔT) between them.

In case of the source height h was 1,4 mm, two profiles of the temperature differences for which the free stream velocity (U_0) is 2 and 5 m/sec respectively, are shown in Fig. 10. Any remarkable difference cannot be noticed in both cases, so subsequent experiments were made for $U_0=5$ m/sec only.

The profiles of the air temperature T_a and the temperature differences ΔT are shown in Fig. 11, 12 and 13, (A), (B) for $h=2$ mm, and those of ΔT in Fig. 14, 15 and 16 for $h=5$ mm.

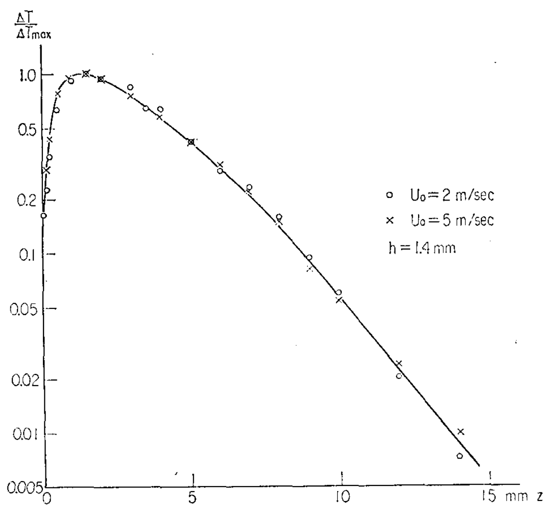

Fig. 10. Profiles of ΔT at $U_0=2$ m/sec and 5 m/sec.

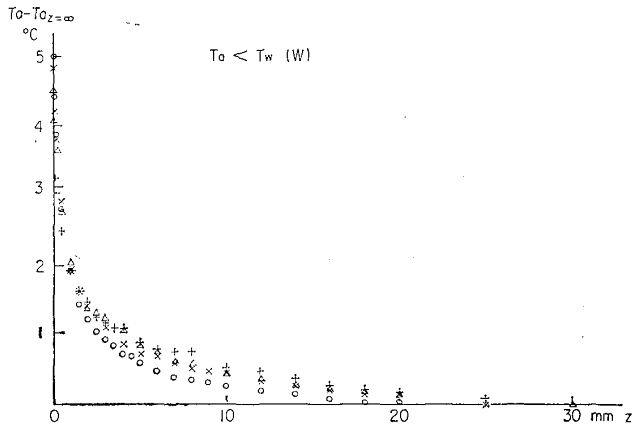

Fig. 11 A. Profiles of T_a: (W).

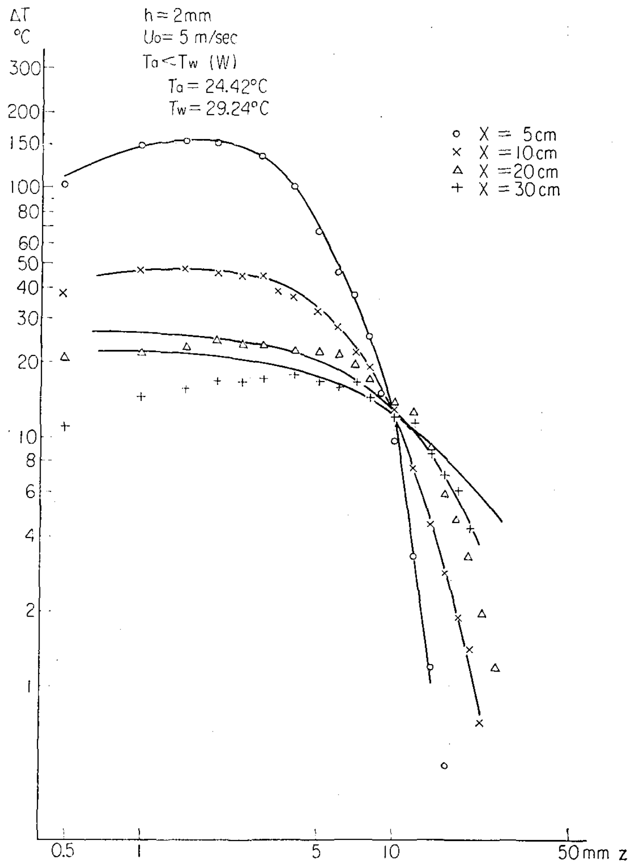

Fig. 11 B. Profiles of ΔT: $h=2$ mm (W).

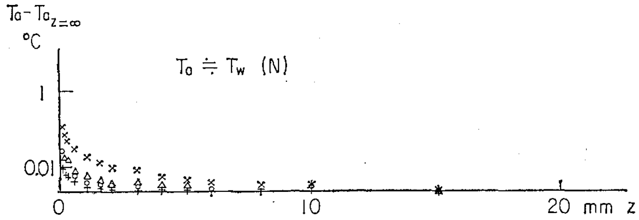

Fig. 12 A. Profiles of T_a: (N).

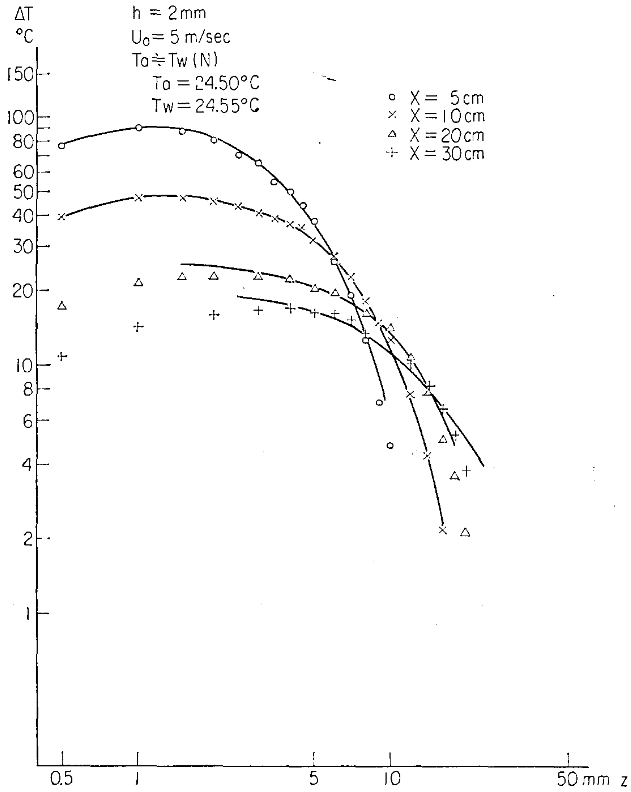

Fig. 12 B. Profiles of ΔT: h=2 mm, (N).

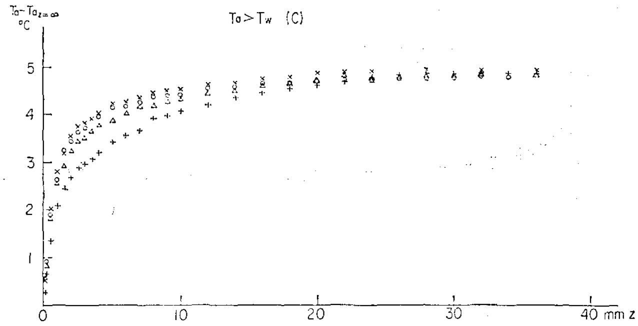

Fig. 13 A. Profiles of T_a: (C).

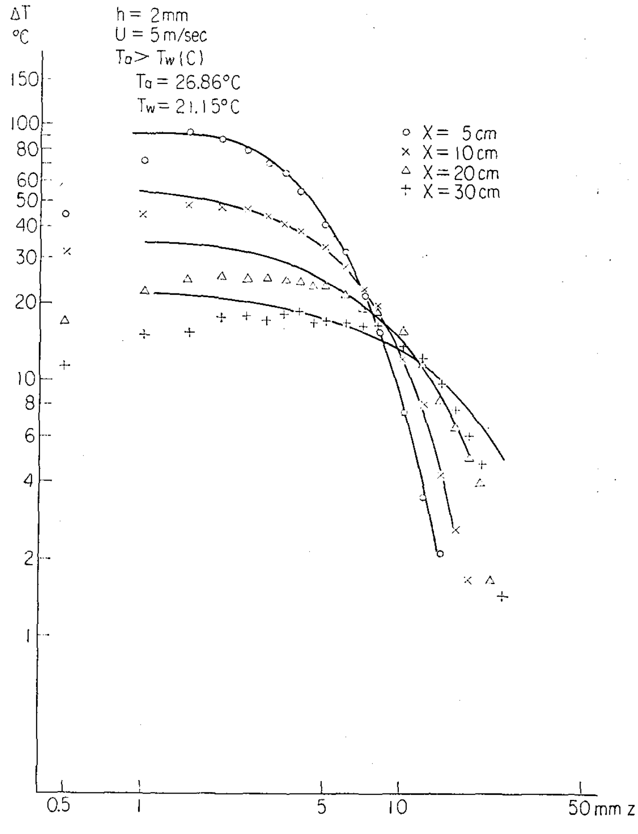

Fig. 13 B. Profiles of ΔT: h=2 mm, (C).

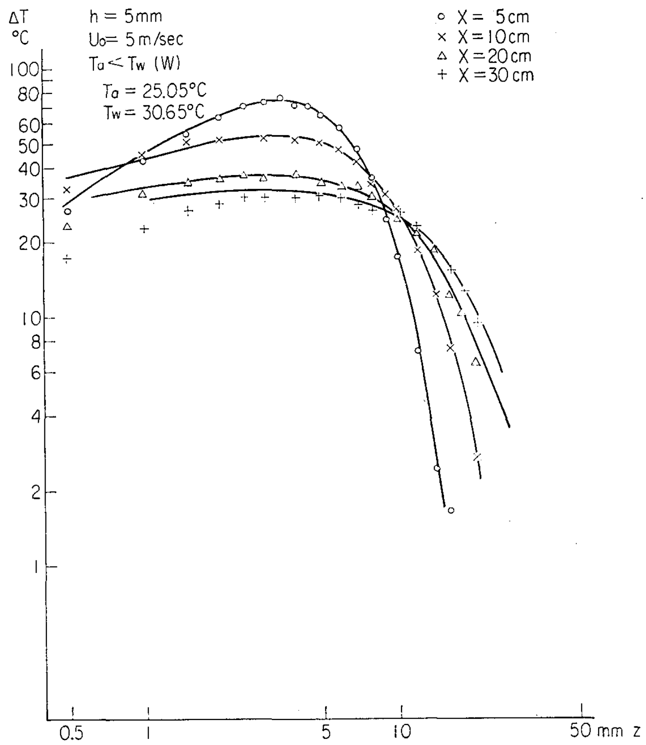

Fig. 14. Profiles of ΔT: h=5 mm, (W).

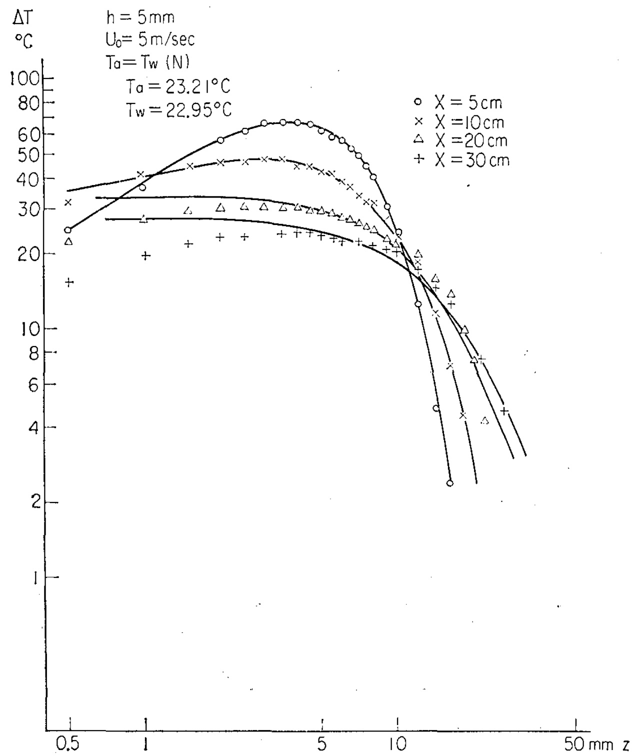

Fig. 15. Profiles of ΔT: h=5 mm, (N).

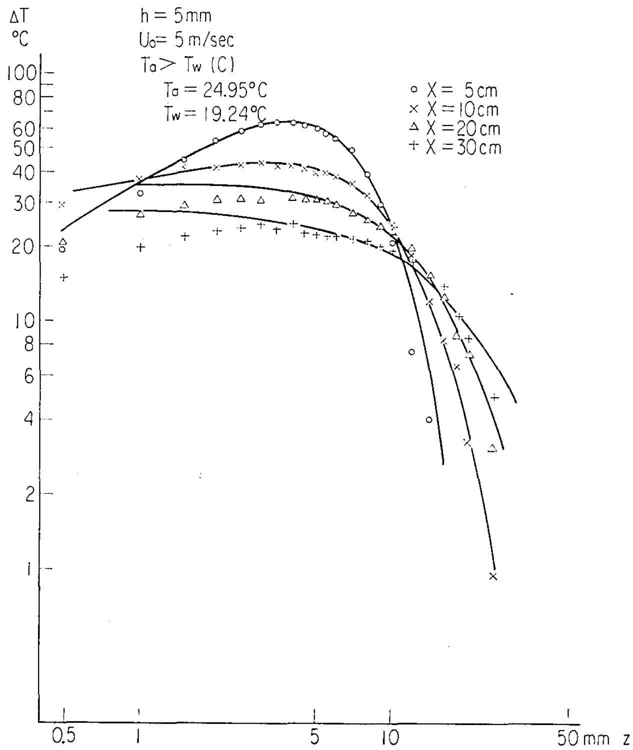

Fig. 16. Profiles of ΔT: $h=5$ mm, (C).

Analysis In order to analyse these results, we used two types of formulae, namely one is the solution of the Fick's differential equation with constant diffusion coefficient. As the solution of this equation for a continuous line source, we have

$$\Delta T = \frac{q}{u} \frac{e^{-\frac{(h+z)^2}{B}} + e^{-\frac{(h-z)^2}{B}}}{\sqrt{B\pi}}, \tag{1}$$

where q is the source intensity, u is the mean velocity at the height of the source and B is a function of x. Almost of all formulae hitherto used for the atmospheric diffusion were of this type. Judging from a number of results of the field experiments for the atmospheric diffusion, the author has noticed the incompleteness of this type, and he assumed in 1941 that the vertical diffusion coefficient is proportional to the vertical height z, and adopted the next differential equation:

$$\frac{\partial T}{\partial t} = K_x \frac{\partial^2 T}{\partial x^2} + K_y \frac{\partial^2 T}{\partial y^2} + \frac{\partial}{\partial z}\left(K_z \frac{\partial T}{\partial z}\right) \tag{2}$$

$$K_x = K_y = a,\ K_z = bz, \tag{3}$$

and obtained the solution for a continuous line source:

$$\Delta T = \frac{q}{u} \frac{e^{-\frac{h+z}{B}}}{B} J_0\left(i\frac{2\sqrt{hz}}{B}\right). \qquad (4)^{1,2,3)}$$

The height of the heat source h was difficult to set exactly, and though the heating current was reduced as possible as it was permissible, the center of the wake came up gradually by buoyancy, so the actual height was not determined exactly. It is convenient, therefore, for the analysis to transform the equations (1) and (4).

If we put

$$z/h=\gamma \tag{5}$$

$$h/\sqrt{B}=\eta\,, \tag{6}$$

$$h/B=\lambda\,, \tag{7}$$

the eqation (1) becomes

$$\Delta T_1=\frac{q}{u}\frac{e^{-\eta^2(1+\gamma)^2}+e^{-\eta^2(1-\gamma)^2}}{\sqrt{\pi}\,h}\eta\,, \tag{8}$$

and the equation (4) becomes

$$\Delta T_2=\frac{q}{u}\frac{1}{B}e^{-\lambda(1-\sqrt{\gamma})^2}e^{-2\lambda\sqrt{\gamma}}J_0(i2\lambda\sqrt{\gamma})\,. \tag{9}$$

From the equations (8) and (9), we get

$$\log \Delta T_1=\log\,[(e^{-\eta^2(1+\gamma)^2}+e^{-\eta^2(1-\gamma)^2})\eta]+C_1\,, \tag{10}$$

and

$$\log \Delta T_2=\log\,[e^{-\lambda(1-\sqrt{\gamma})^2}e^{-2\lambda\sqrt{\gamma}}J_0(i2\lambda\sqrt{\gamma})/B]+C_2\,. \tag{11}$$

The curves of ΔT_1 and ΔT_2 against γ calculated from (10) and (11) for various values of η and λ are shown in Fig. 17 and 18. It is natural

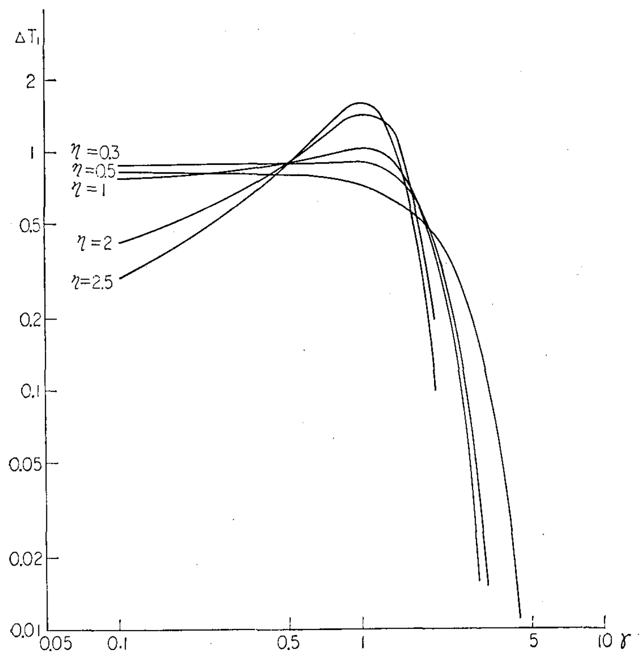

Fig. 17. ΔT_1 calculated from eq. (10).

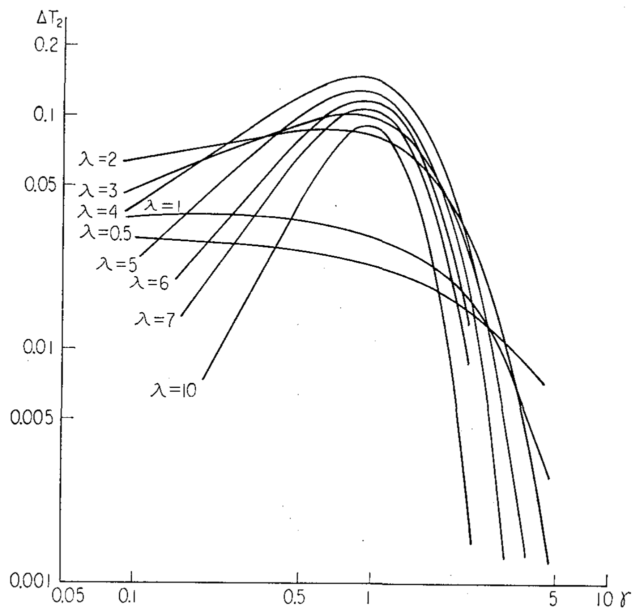

Fig. 18. ΔT_2 calculated from eq. (11).

that both curves resembles each other when r is large, but they clearly differ from each other when r is small. The results already shown in Fig. 11–16 are plotted on logarithmic papers. By superposing these curves on those in Fig. 17 or Fig. 18, and translating in the direction of abscissa, we can compare both curves corresponding to the actual height H at that leeward position. Then by translating in the direction of ordinate, we can examine the shape of the curves, unconcerned with the values of the source intensity q which are difficult to determine directly in the case of heat diffusion.

For the analysis of the measured results, it should be taken into consideration that in the region higher than 20 mm, the turbulent velocity components become very small as it can be noticed from Fig. 4–9, so the grade of diffusion diminishes remarkably. For this reason, the temperature profiles become steeper in that region. In the region lower than 1 mm, the laminar sublayer existed. So the profiles for z between 0,5 and 20 mm should be treated as the results of the turbulent diffusion.

On the other hand, because of that we used heat as the tracer, the boundary condition under which the equations (1) and (4) were obtained as the solutions of the differential equations, namely that there was no flux through the boundary, could not be satisfied exactly. So in the results other than those at $x=5$ and 10 cm, there appeared the effect of existence of the flux through the boundary.

The results at $x=5$ and 10 cm, the equation (1) do not fit at all, but the equation (4) agrees well. In Fig. 11–16, the marks show the measured values and the curves show the calculated ones with the equation (4).

Consequently, it can be concluded that the diffusion phenomena close to a wall is described more adequately by the differential equation with the diffusion coefficient proportional to the height, rather than that with the constant coefficient. This conclusion agrees well to the results of analyses of the atmospheric diffusion which have been made by the author.[4,5,6,7,8]

Numerical results of the analysis are shown in Table 1. Fig. 19 shows the relationship between B and x. The coefficient of the turbulent diffusion is considered to be proportional to the wind velocity, so when we write b in equation (3) $b=\beta U$,

Table 1. Measured values of actual height H and B.

h=2 mm				h=5 mm			
Condition	x (cm)	H (mm)	B (mm)	Condition	x (cm)	H (mm)	B (mm)
(W)	5	2, 3	1, 1	(W)	5	3, 8	1, 1
	10	2, 5	2, 5		10	4, 9	2, 1
	20	4, 0	5, 0		20	5, 4	4, 0
	30	4, 5	7, 5		30	6, 0	6, 0
(N)	5	2, 0	1, 4	(N)	5	1, 4	1, 3
	10	2, 3	2, 3		10	4, 9	2, 1
	20	4, 3	4, 3		20	5, 0	4, 6
	30	5, 0	6, 2		30	5, 2	6, 5
(C)	5	2, 5	1, 3	(C)	5	4, 5	1, 3
	10	2, 6	2, 6		10	5, 2	2, 5
	20	3, 2	5, 3		20	5, 0	4, 8
	30	5, 0	8, 3		30	6, 0	7, 5

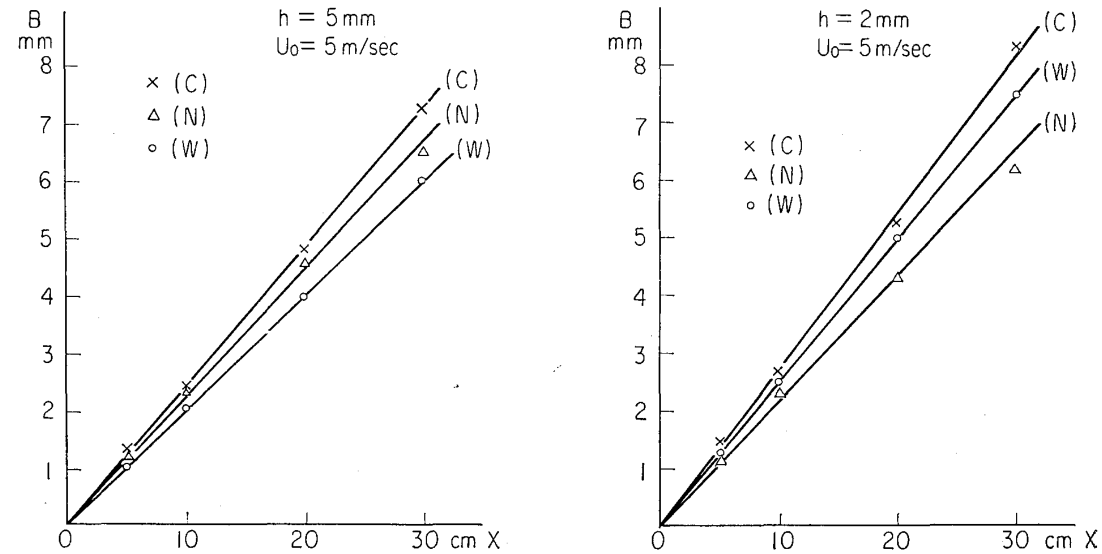

Fig. 19. Relations between B and x.

$$B \doteqdot bx/U = \beta x\,, \tag{13}$$

and the values of β is about 1,3. The temperature difference between the air and the plate in these experiments did not show any remarkable difference for the diffusion.

Comparison with other experiments. During the author was preparing this paper, he received two reprints concerning the experiments of turbulent diffusion from Professor J. E. Cermak, Colorado State University. His works are very elaborate, but the aim of the research differs somewhat from that of this paper. So some analyses on the line of this paper were made by using his results of the diffusion from and elevated point source.[9] Some concentration profiles are shown in Fig. 20, 21 and 22, in which the marks represent the

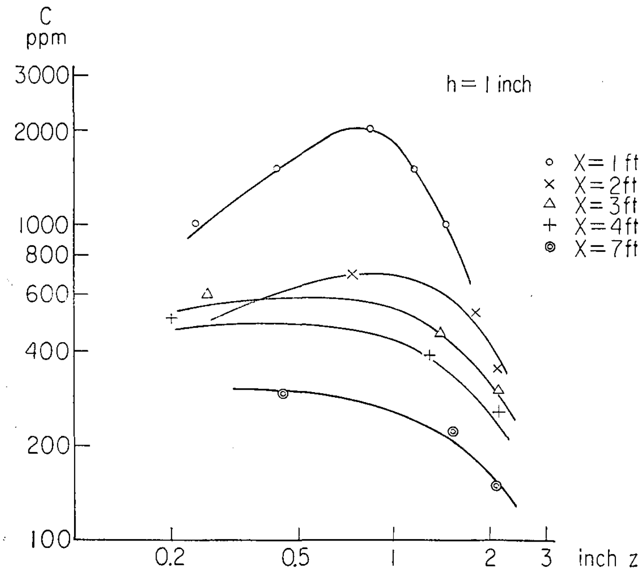

Fig. 20. Concentration profiles: $h=1$ inch.

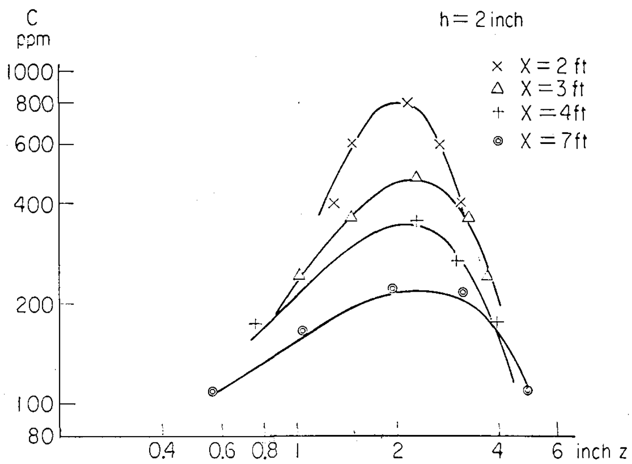

Fig. 21. Concentration profiles: $h=2$ inch.

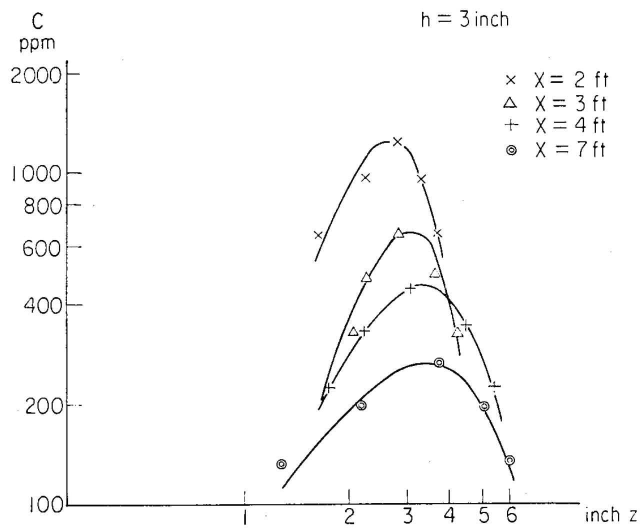

Fig. 22. Concentration profiles: h=3 inch.

Table 2. Measured values of actual height H and B.

h (inch)	x (ft)	H (inch)	B (inch)
1	1	0, 85	0, 17
	2	0, 9	0, 33
	3	0, 9	0, 45
	4	0, 9	0, 56
	7	0, 9	1, 0
2	2	1, 2	0, 11
	3	1, 2	0, 15
	4	1, 2	0, 20
	7	1, 4	0, 35
3	2	0, 9	0, 045
	3	1, 0	0, 07
	4	1, 1	0, 11
	7	1, 2	0, 17

measured values and the curves are the calculated ones from the equation (4). As well as the present experiments, the results can be described well by the equation (4). The numerical results are shown in Table 2 and the relation between B and x is shown in Fig. 23.

Future planning. In these experiments, we used heat as the tracer for the reason of convenience. However, for the investigations of the diffusion in the air layer with temperature gradient, heat is not adequate tracer. Furthermore, the obtainable temperature difference between the air and the plate was not so large and the size of the wind tunnel was not sufficient, so we are now going to make

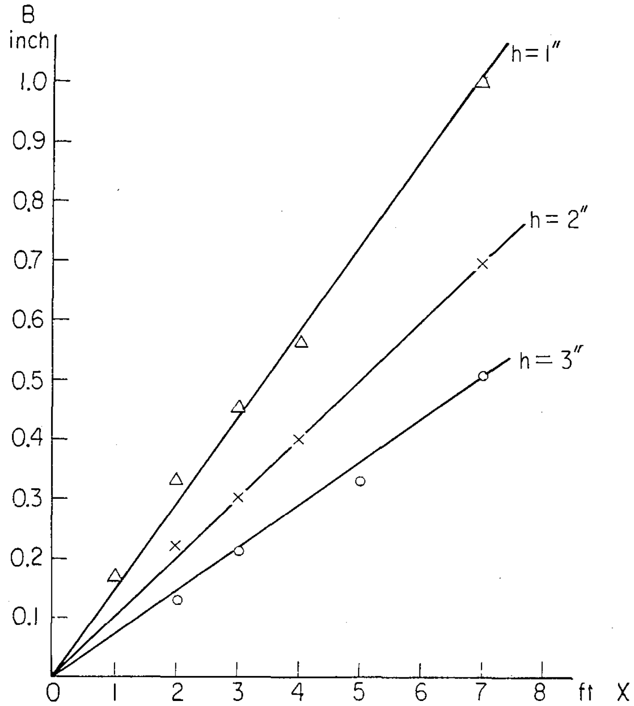

Fig. 23. Relations between B and x.

experiments in the wind tunnel in our University by using fluorescent particles or gas as the tracer.

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