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On the Vertical Atmospheric Diffusion Close to the Ground—An Analysis of the Data of the Dispersion of Conidio-Spores

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Introduction

In diffusion problems arising in our circumstances, influence of the earth's surface is most essential. In almost all observations, vertical concentration profiles have not been observed or observed only at a few positions, so there have been scarcely any results suitable to examine theoretical results, in what extent they can express that influence.

Results of observations, concerning vertical profiles of sedimentated numbers of conidio spores dispersed from paddy rices infected by rice blast in a paddy field, have been reported.[1] These data are adequate to the detailed examinations of the theoretical conclusions of the vertical diffusion. They are the results of captured number of spores on samplers, so they are not the results of concentration themselves. However, the captured number can be regarded proportional to the concentration at least in the first approximation, so in this paper, the vertical diffusion is considered by using these data, even though the capturing efficiency of the samplers is unknown.

Some theoretical considerations were made in that report, but the theory was inadequate to apply to small scale experiments as such ones, and comparison with the observed data was not throughgoing. So in this paper, the data are analysed more accurately as the results of the diffusion from a two-dimensional source.

General description of experiments

According to that report, the samplers were set at the heights of 10, 40, 70, 100, 130, 160, 250 and 490 cm from the ground at the center of a paddy field whose area was 18 m $\times$ 25.3 m. Among the data, those obtained by horizontal slide glasses, on which solution of glyceline and gelatine mixture was rubbed, are analysed. Sampling time was 24 hours, from 0900 till 0900 in the next morning.

As for meteorological observations, wind speed at the height of 2 m and air temperature and humidity at the heights of 1.0 and 4.5 m were measured.

Total number of runs was 35, but in some runs the sedimentated number of the spores was insufficient for quantitative treatment, so 16 runs among them were adopted for this analysis. The results are shown in Table 1, in which thick line shows the height of grass level. Assuming that the height of the grass level is $z=0$, the numbers (N) of spores for each run are plotted against the height (z) from the grass level in Fig. 1.

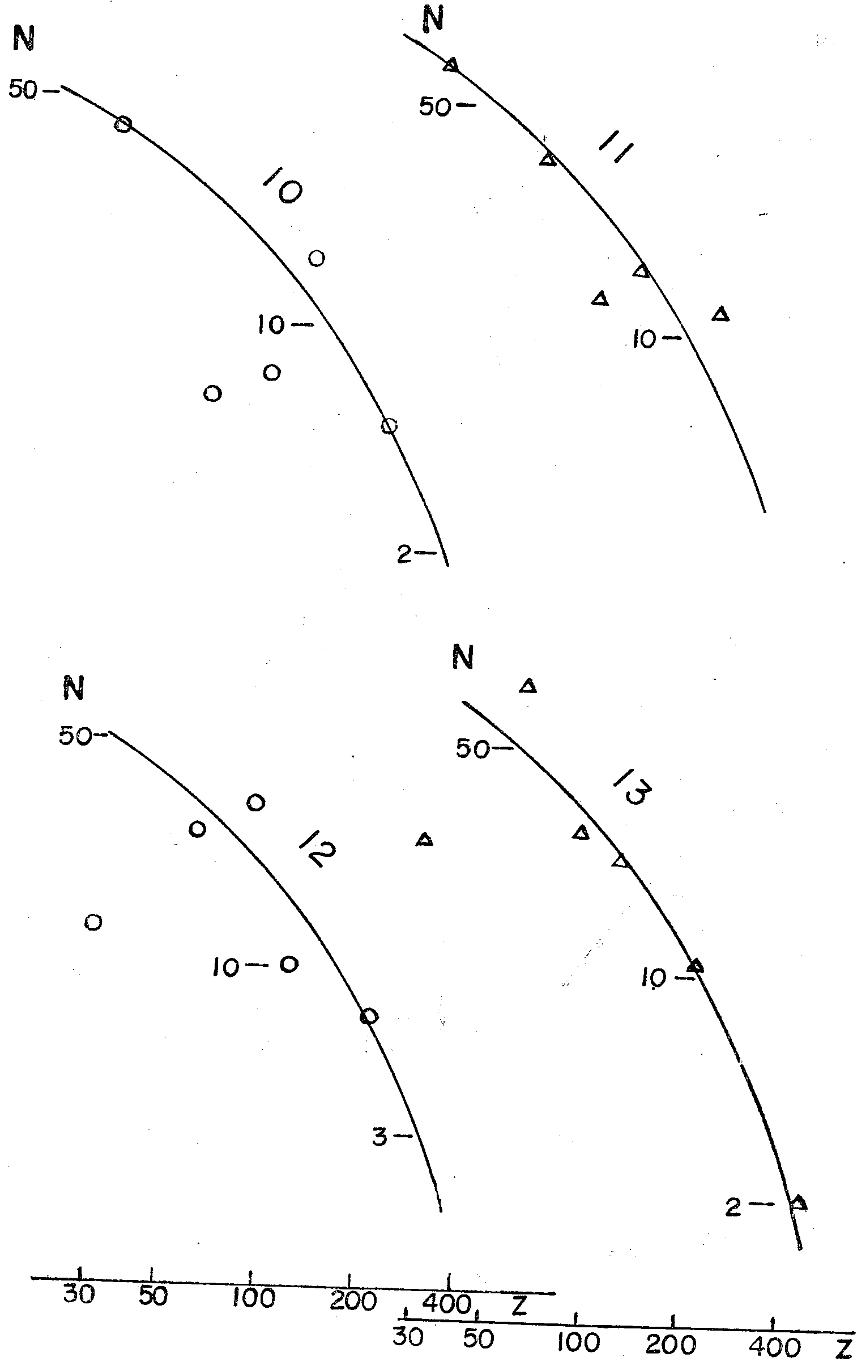
Table 1. Number of sedimentated spores.

Height of measuring post (cm)		10	40	70	100	130	160	250	490
Height from top-level of grass (cm)				30	60	90	120	210	450
Date	10	88	68	40	6	7	16	5	0
	11	18	130	77	35	13	16	12	0
	12	152	111	13	26	32	10	7	1
	13	43	44	36	79	28	23	11	2
	15	324	252	172	76	52	34	11	7
	16	759	446	105	53	58	27	14	5
	17	594	312	296	146	78	67	21	19
	18	169	457	248	95	63	38	15	5
	19	574	393	185	74	39	23	12	4
	21	805	464	210	118	42	36	39	13
	22	1325	599	360	269	45	85	22	15
	23	1361	695	305	228	235	89	31	19
Height from top-level of grass (cm)					30	60	90	180	420
Date	27	526	331	108	45	27	28	20	0
	28	567	474	203	141	38	30	17	4
	29	587	225	86	65	35	18	11	2
	30	193	541	307	131	100	72	22	15

Theoretical consideration

We take the origin at the center of the rectangular source, x-axis leeward, z-axis vertically upward and y-axis perpendicularly to them (Fig. 2).

The number of spores dC' at the position $(x, 0, 0)$ scattered from the band shaped continuous source, which is in parallel with y-axis


N
50
10
10
2
N
50
11
10
N
50
12
10
3
N
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30
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100
200
400
Z
30
50
100
200
400
Z

Fig. 1 a.

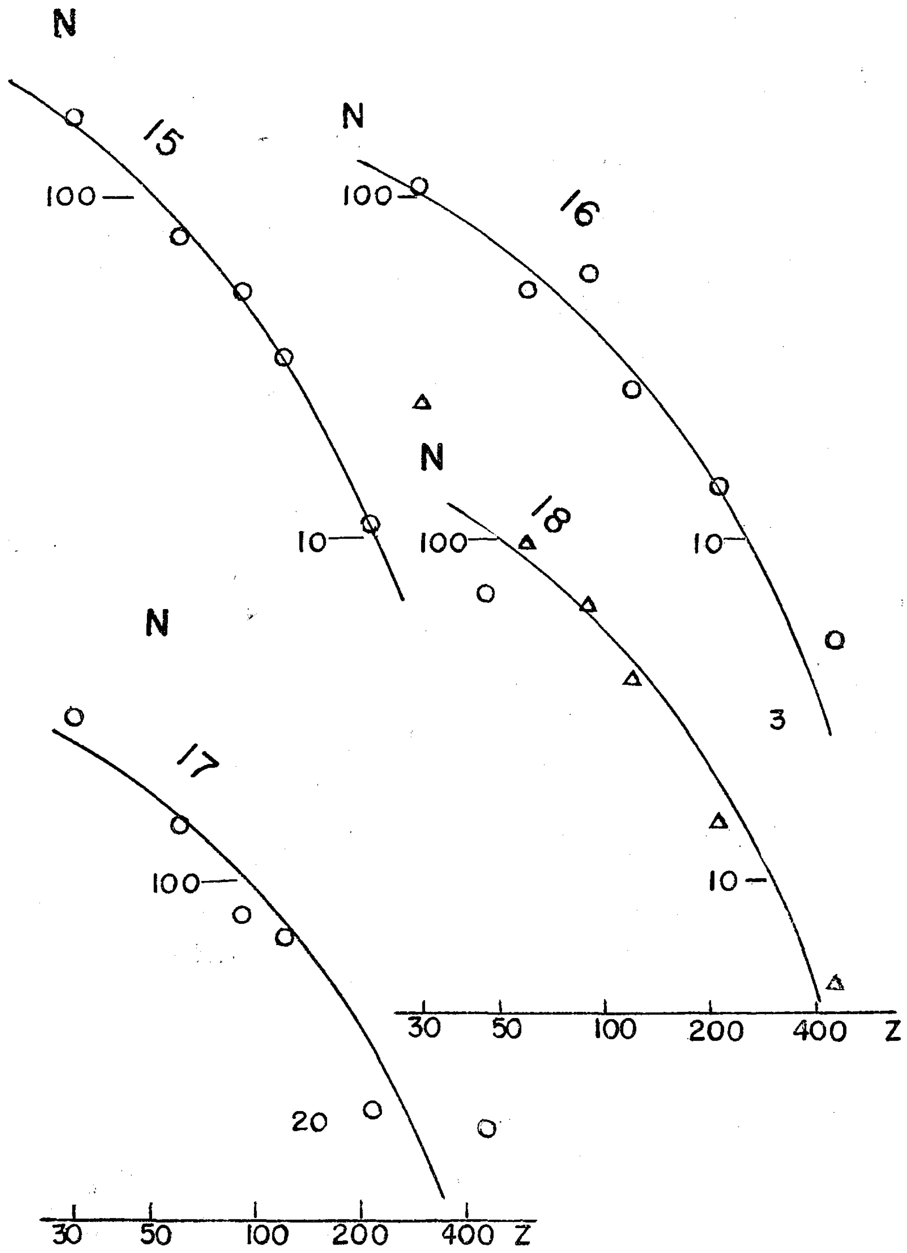
N
15
100
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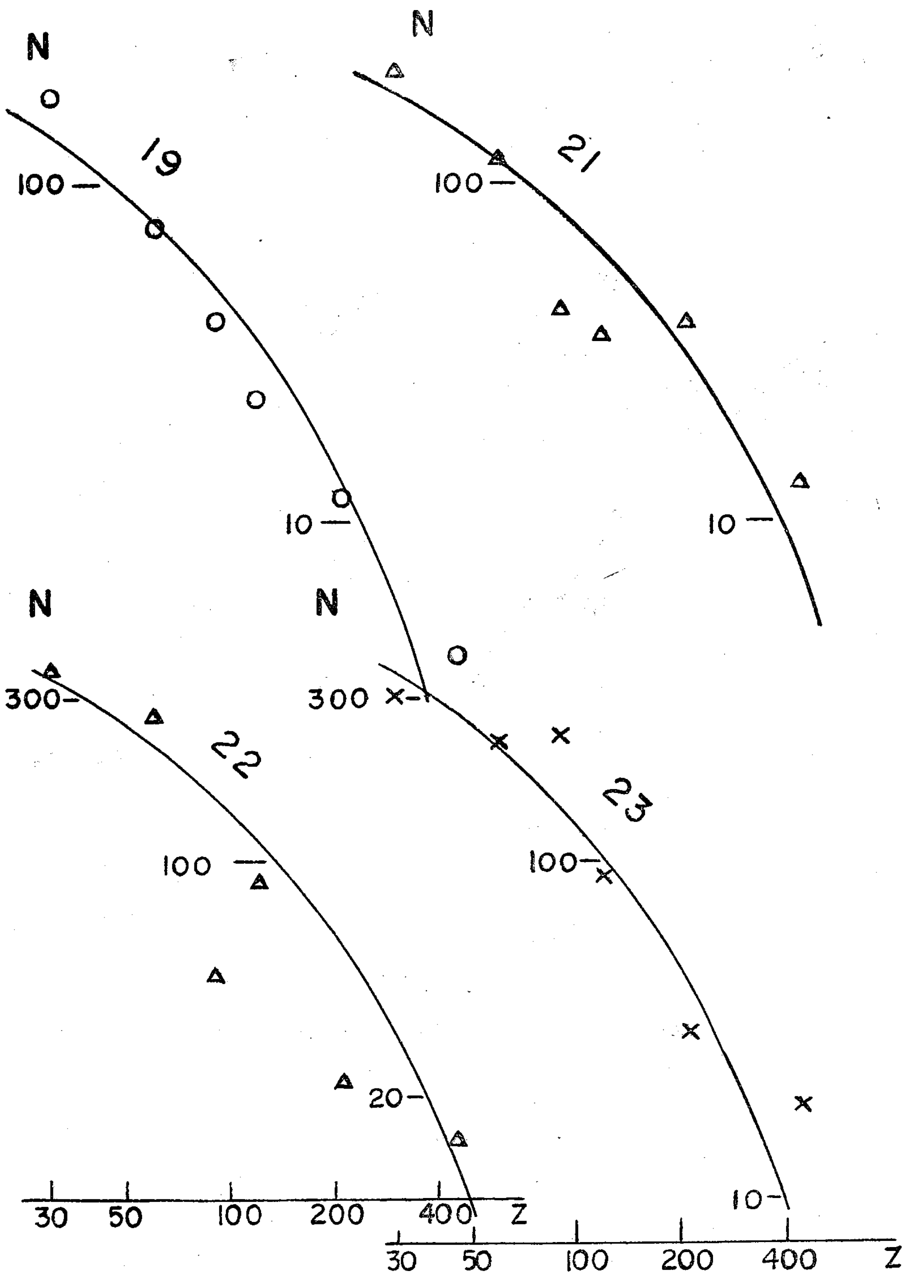
Fig. 1 b.


N
19
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21
100
10
N
300
22
100
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23
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Fig. 1 c.

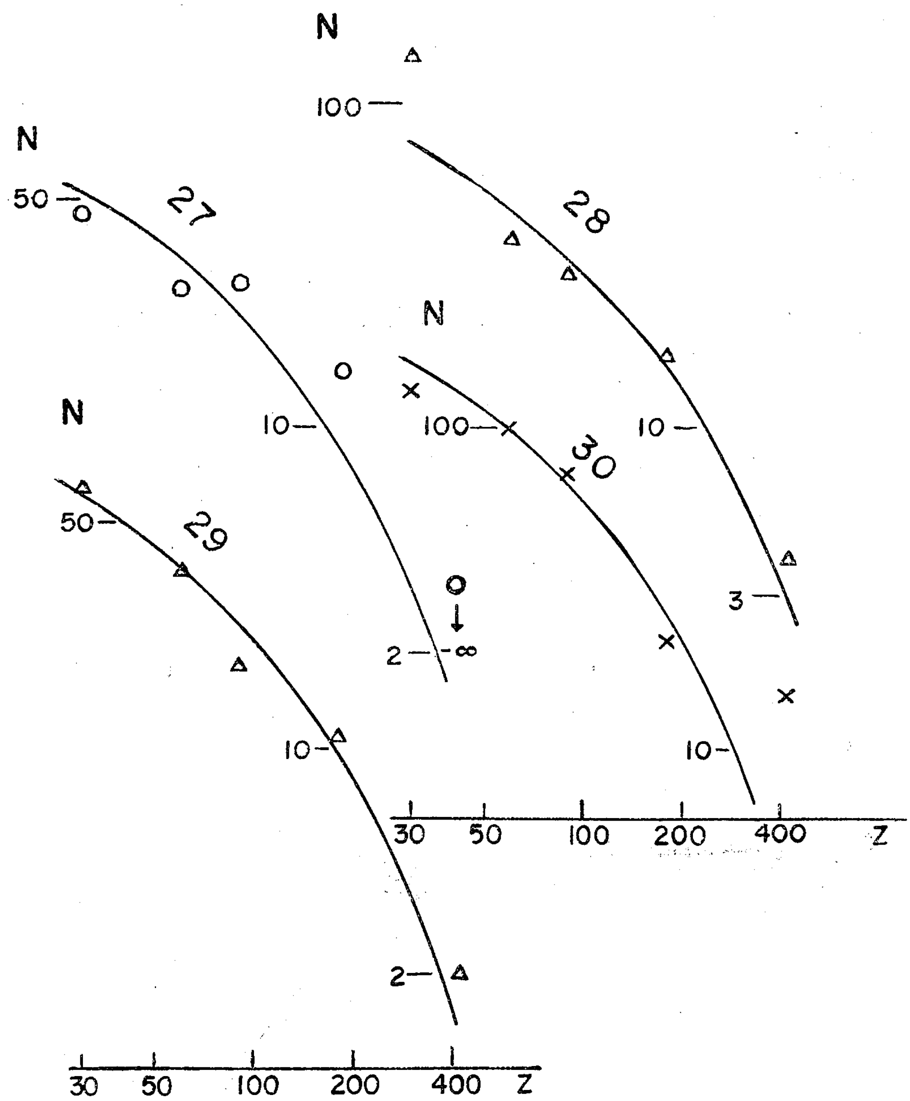

Fig. 1 d.

Fig. 1. Number of spores versus height from the top level of grass.

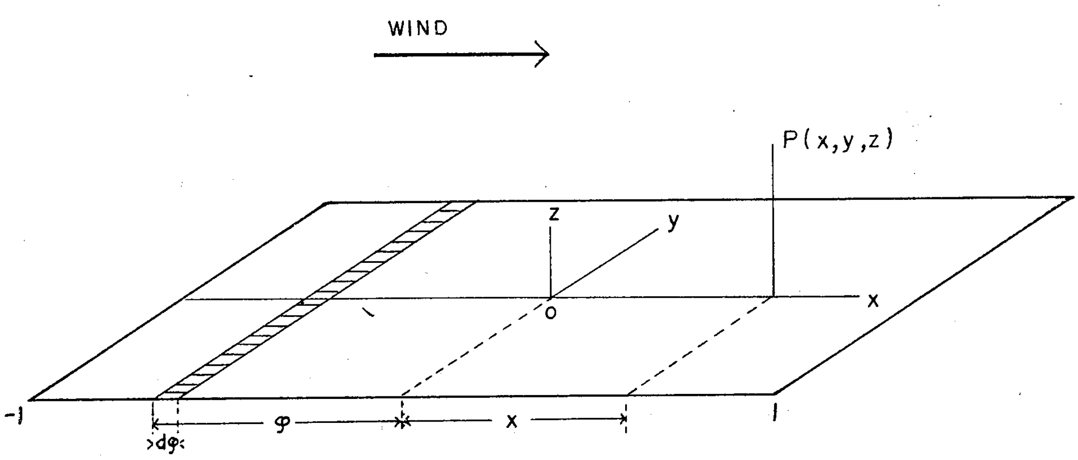

Fig. 2. Scheme of source for theoretical consideration.

and at the position of $\varphi \sim \varphi + d\varphi$, is derived from the author's formula :[2,3]

$$dC' = \frac{\omega Q}{u} \frac{e^{-\frac{z}{B_0(x-\varphi)}}}{B_0(x-\varphi)} d\varphi \qquad (1),$$

where

$$B_0(x-\varphi) = b\left\{\sqrt{\left(\frac{x-\varphi}{u}\right)^2 + \left(\frac{3}{4}\frac{4a}{u}\right)^2} - \frac{3}{4}\frac{4a}{u}\right\} = \beta\{\sqrt{(x-\varphi)^2 + (3\alpha)^2} - 3\alpha\} \quad (2),$$

$$a = \alpha u, \qquad b = \beta u \qquad (3),$$

Q is source strength (number/m^2/day), u is mean wind speed, a and b are diffusion constants for horizontal and vertical direction respectively, and ω is the capturing efficiency of the sampler, which is left unknown.

The total concentration from the source is given by

$$C = \int_{-l}^{x} dC' = \frac{\omega Q}{u} \int_{-l}^{x} \frac{e^{-\frac{z}{B_0(x-\varphi)}}}{B_0(x-\varphi)} d\varphi \qquad (4).$$

Putting $x - \varphi = \lambda$, eq. 4 becomes

$$C = \frac{\omega Q}{u} \int_0^{x+l} \frac{e^{-\frac{z}{B_0(\lambda)}}}{B_0(\lambda)} d\lambda \qquad (5)$$

and

$$B_0(\lambda) = \beta\{\sqrt{\lambda^2 + (3\alpha)^2} - 3\alpha\} \qquad (6).$$

As the result of physical consideration, λ is generally regarded as large compared with 3α, except that z is close to zero, so

$$B_0(\lambda) \approx \beta\lambda \qquad (7).$$

Therefore,

$$C = \frac{\omega Q}{u} \int_0^{x+l} \frac{e^{-\frac{z}{\beta\lambda}}}{\beta\lambda} d\lambda \qquad (8).$$

Putting $\phi = \dfrac{\beta\lambda}{z}$, we obtain

$$C = \frac{\omega Q}{u} \int_{\frac{z}{\beta_0(x+l)}}^{\infty} \frac{e^{-\phi}}{\phi} d\phi$$

$$= \frac{\omega Q}{u}\left[-E_i\left(-\frac{z}{\beta(x+l)}\right)\right] = \frac{\omega Q}{u\beta}[-E_i(-\mu z)] \qquad (9),$$

where

$$\mu = 1/(x+l),$$

and

$$\int_{\xi}^{\infty} \frac{e^{-\psi}}{\psi} d\psi = -E_i(-\xi) \tag{10}$$

is the exponential integral.

In the present observation $x=0$, so

$$\mu = \frac{1}{\beta l} \tag{11}.$$

Determination of parameters from observed data

The results given in Fig. 1 show z-dependence of the number of spores at a given leeward position. From eq. 9, we obtain

$$\log_{10} C = \log_{10}[-E_i(-\mu z)] + \log_{10} \frac{\omega Q}{u\beta}.$$

Putting $p=-E_i(-\xi)$, and plotting a curve $\log_{10} p$ against $\log_{10}\xi$, we obtain Fig. 3. We put this curve on the graph given in Fig. 1 and translate it in parallel with both axes to fit the marks of observed data in Fig. 1. Then, if the value of C corresponding to $p=1$ be C_1, we obtain

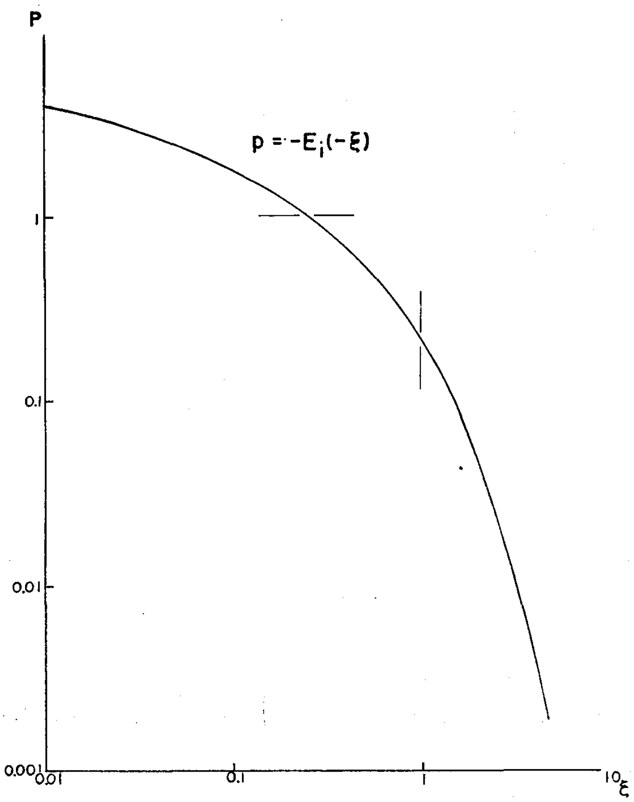

Fig. 3. Graph of p versus ξ.

$$C_1=\frac{Q}{u\beta} \tag{12},$$

and if the value of z corresponding to $\xi=1$ be z_0, we obtain

$$\mu z_0=1\,, \qquad \therefore\ \mu=\frac{1}{\beta l}=\frac{1}{z_0} \tag{13}.$$

In this case $2l$ is 21.6 m which is the mean of the lengths of both sides of the field, so we obtain $\beta=z_0/10.8$ and $\omega Q=C_1uz_0/10.8$.

Comparison with observation

The results of fitting of the p-curve are shown in full lines in Fig. 1. As these curves agree fairly well to the arrangement of the marks, the above theoretical results may be regarded adequate. The determined values of β and ωQ are shown in Table 2, together with the wind speed and air temperature averaged by the day.

The author has adopted a quantity

Table 2. Values of β, ωQ and ζ.

Data	u	T_{450}	$T_{450}-T_{100}$	ζ	β	ωQ
10	0.9	26.6	+0.1	0.189	0.17	3.8
11	0.7	24.0	+0.2	0.624	0.15	5.3
12	1.1	——	——	——	0.18	7.5
13	0.9	——	——	——	0.18	9.1
15	0.9	25.6	+0.5	0.941	0.11	15.6
16	1.5	27.1	+0.1	0.068	0.19	21.1
17	1.3	24.7	0	0	0.19	45.4
18	0.9	24.8	−0.1	−0.189	0.19	16.1
19	1.1	25.4	+0.3	0.379	0.15	18.5
21	0.9	27.8	0	0	0.23	22.9
22	1.1	26.4	−1.4	−0.177	0.22	50.8
23	1.1	27.6	+0.2	0.252	0.16	48.3
27	1.1	29.1	+0.3	0.379	0.16	7.4
28	1.1	28.4	+0.3	0.379	0.21	10.9
29	1.3	29.3	+0.1	0.091	0.17	9.7
30	0.9	28.5	+0.1	0.189	0.17	18.4

$$\zeta=\frac{\partial T}{\partial \log_{10}z}\Big/\left(\frac{V_*}{\kappa}\log_{10}z_0\right)^2$$

as a stability parameter, where T is the air temperature, V_* is the shearing velocity, κ is the Karman's constant and z_0 is the roughness parameter. In the present observations, the wind speed was observed only at the height of 2 m and air temperature was observed only at two

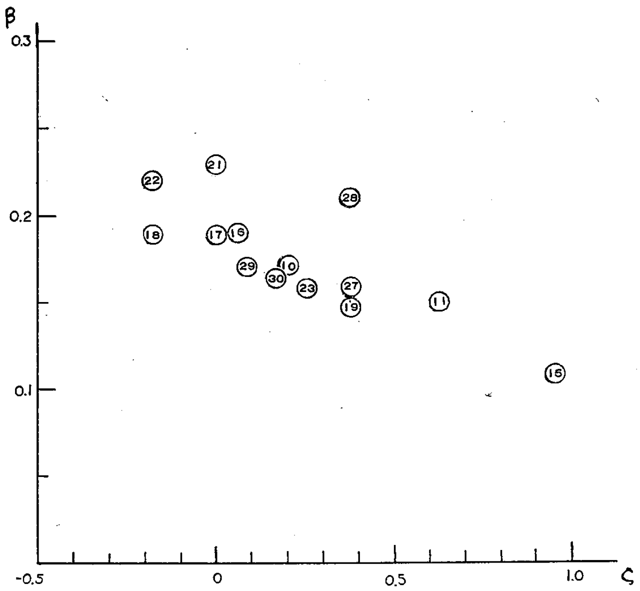

Fig. 4. Curve of β versus ζ.

heights, so accurate values of ζ were not calculated. Therefore, the values of ζ in Table 2 are approximate ones. The curve of β versus ζ is shown in Fig, 4, and this shows that β has a definite relation with ζ.

Comparison with another formula

The author's formula has been derived in order to be adapted to observed vertical profiles of concentration. This formula turns to be the same to that which is usually used, for example, Sutton's formula, when the source and the posts of observation are sufficiently high above the ground. However, when the source and the posts of observation are close to the ground, especially when the source is on the ground, the vertical concentration profile becomes proportional to $\exp(-z^2/B)$ according to the Sutton's formula,[4] but it becomes proportional to $\exp(-z/B)$ according to the author's formula. In several papers, comparisons of these two kinds of formulae with observed data have been reported and it has been verified that the author's formula is more adequate than the another,[5),6),7)] but the present data are the most adequate ones to verify immediately the theoretical conclusions.

The formula of the concentration derived from the Sutton's formula is given by

$$C=\frac{2\omega Q}{u\pi c_y c_z}\int_{-l}^{x}\frac{e^{-\frac{z^2}{c_z{}^2(x-\xi)^{2-n}}}}{(x-\xi)^{2-n}}d\xi=\frac{2\omega Q}{u\pi c_y c_z}\int_{0}^{x+l}\frac{e^{-\frac{z^2}{c_z{}^2\eta^{2-n}}}}{\eta^{2-n}}d\eta \tag{14}$$

Putting $a=z^2/c_z^2$ and $\rho=a/\eta^{2-n}$, the above formula becomes

$$C=\frac{2\omega Q}{u\pi c_y c_z}a^{\frac{1}{2-n}-1}\left(\frac{1}{2-n}\right)\int_{\frac{a}{(x+l)^{2-n}}}^{\infty} e^{-\rho}\rho^{-\frac{1}{2-n}}d\rho \tag{15}$$

The integral in eq. 15 can be expressed by the incomplete Γ-function of the 2nd kind:

$$\Gamma(\nu, \lambda)=\int_{\lambda}^{\infty} e^{-\rho}\rho^{\nu-1}d\rho \tag{16}$$

where $\nu<0$. The correspondence between ν and n is shown in Table 3.

Table 3. Correspondence between n and ν.

n	0	0.5	1	1.5
ν	0.5	0.333	0	−1

Therefore, when n lies between $0\leqq n<1$, i. e. ν lies between $0.5\geqq\nu>0$, eq. 15 can be expressed explicitly by

$$\begin{aligned} C&=\frac{2\omega Q}{u\pi c_y c_z}\frac{1}{2-n}a^{\frac{1}{2-n}-1}\Gamma\left(1-\frac{1}{2-n}, \frac{a}{(x+l)^{2-n}}\right) \\ &=\frac{2\omega Q}{u\pi c_y c_z}\frac{1}{2-n}a^{\frac{1}{2-n}-1}\Gamma\left(1-\frac{1}{2-n}, \frac{z^2}{\sigma}\right) \end{aligned} \tag{19}$$

where $\sigma=c_z^2(x+l)^{2-n}$.

When $n=1$, ν becomes 0, so eq. 15 can be expressed by

$$\begin{aligned} C&=\frac{2\omega Q}{u\pi c_y c_z}\left[-E_i\left(-\frac{a}{(x+l)}\right)\right] \\ &=\frac{2\omega Q}{u\pi c_y c_z}\left[-E_i\left(-\frac{z^2}{\sigma_1}\right)\right] \end{aligned} \tag{18}$$

where E_i is the exponential integral which has been introduced in eq. 10, and $\sigma_1=c_z^2(x+l)$.

When $n>1$ $(2>n)$, ν becomes negative, the integral (eq. 15) converges only when $a/(x+l)^{2-n}$ is not zero.

If we plot $\log_{10}\left(\int_{\lambda^2}^{\infty} e^{-\rho}\rho^{-\frac{1}{2-n}}d\rho\right)=\log_{10}P$ against $\log_{10}\lambda$, we obtain Fig. 5, and it may be called P-curve for convenience. Just as described above for the p-curve, this P-curve is put on marks in Fig. 1 in order to be examined the degree of agreement with the theoretical results.

Among the P-curves, those for the values $n=0.333$ or $n=0.5$ which are within the range of values usually adopted, have considerably different shape compared with the observed curve, and that for $n=1$ which corresponds rather rare case as the Sutton's parameter, shows some resemblance, even so, the degree of resemblance may not be

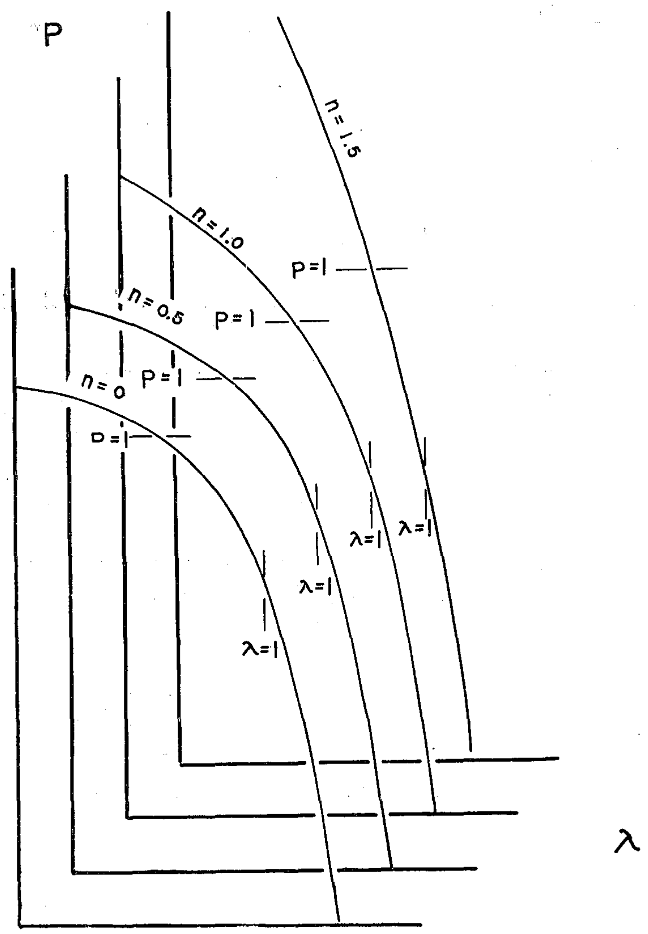

Fig. 5. Graph of p versus λ.

so good ; those for $n>1$, the shape differs more considerably.

So the Sutton's formula is not adequate for the vertical profile when the influence of the earth's surface is effective.

Consideration about source intensity

Estimated values of ωQ are already shown in Table 2. These values have not any relations to wind speed or air temperature, but if we plot these values against the date of the run, we obtain Fig. 6. This curve increases exponentially at first, after showing final steep increase, it suddenly decreases, then it repeats the same procedure. This characteristic might be explained pathologically, but in this paper, it should be left untouched. However, as these values which are calculated indirectly show clear regularities, the present theoretical consideration may be regarded as appropriate.

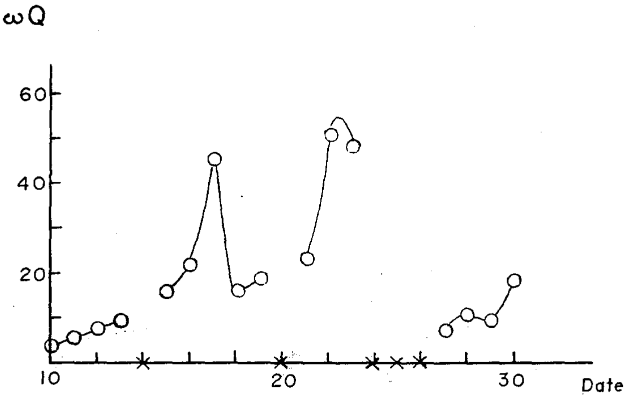

Fig. 6. Curve of ωQ versus date of observation.

Conclusion

Using the data which are suitable to the examinations of the theoretical results for vertical diffusion, it is verified that the author's formula is more adequate than the Sutton's one, namely that the vertical profile varies as z^1 rather than z^2.

The experiments were rather preliminary ones and successive experiments are also going to be carried out this year. The data which will be obtained in those experiments will contribute much more informations for the vertical diffusion.

The author wishes to express his cordial thanks to the members who have carried out the experiments for their good will allowing the author to treat the data, and especially to Dr. E. Inoue, National Institute of Agricultural Sciences, for his suggestions to analyse the data.

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