Lagrangian Measurement of Small Scale Atmospheric Turbulence by Floating Fine Particles*

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I. Introduction

The turbulent motion in fluid was considered at first as the unstable phenomenon, so the stability theories have developed. On the other hand, statistical theories which started from the analogy to the kinetic theory of gases and, considering turbulent fluctuations as the random variables, found the probability distribution function, have developed remarkably up to the present. As turbulent motion is regarded naturally as a quite irregular phenomenon, it is appropriate to treat it statistically as it has been studied theoretically and experimentally. At the same time, however, it is also necessary to observe the phenomenon more essentially. Namely it seems to be necessary to observe what mechanism the turbulent field is moving practically with. Therefore, we intended to examine the practical mechanism of the variation of the turbulent field by tracing actual movement of air particles and to carry out Lagrangian measurements and analysis which has little been studied. The object of our observations is the turbulence in the natural wind near the ground. Differing from the artificial turbulent fields in wind tunnels, this field is completely turbulent and eddies appear and disappear freely in the field. The observations of the atmospheric turbulence hitherto have been carried only in the cases of very large scales, so, in order to make clear elementary process of turbulent motion, we intended to examine the motion in smaller scales as far as possible. J. G. Edinger tried the original experiment in which he released bubbles $2\frac{1}{2}$ inches in diameter at 1000 ft above the ground[1]. Differing from his case, we made experiments near the ground, using suspended matter far lighter than the bubbles, and measured the motion three dimensionally.

II. Experimental Method

a. Suspended matter and its suspending method

Our observational method is to suspend small particles in the natural wind and to take photographs of their trajectories. So these particles

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ought to have the following characteristics: (a) Their falling terminal velocities should be small; (b) They should follow faithfully small air motions; (c) They should be taken photographs well of; (d) When they are released, they should not disturb the air movements. As the suspended matter, at first we chose small bubbles of resin-soap. Though the diameters of the bubbles were very small (1～3 mm), their terminal falling velocities were much larger than those which had been expected (about 10 cm/sec). Fiberous clusters formed by sublimation of metaldehyde were used. A nichrome wire 0.2 mm in diameter was wound to form a downward cone whose diameter and height were 1 mm each, and only a very small quantity of needle crystals of metaldehyde was put in the cone. The nichrome wire was instantaneously heated to slightly red by a.c. sourse with several tens volts, in order to make the effects of heating as small as possible. The value of voltage was determined according to the wind speed in the observation. Then several particles of the sublimated metaldehyde flew away from the nichrome wire and moved leeward with the wind. A photograph of these particles is shown in Fig. 1. These particles have various shapes and are about 2～3 to 5 mm in size. Their average terminal velocity is about 1～2 cm/sec.

b. Photographying method

The trajectories of those particles were photographed stereographically in the open air at night. On the films, continuous trajectories were obtained by continuous lights and discrete points were marked on these lines by xenon strobo-flashes which were operated with a given frequency adequate to the wind speed (generally 20 cycles per second).

In order to photograph for a long distance leeward, and to keep the accuracy of measurement, two pairs of cameras, each base length of the pair was 75 cm, were used. The field as far as about 2 m leeward from the source was photographed by a pair, and overlapping several tens centimeters, following range of 2 m leeward was photographed by another pair. The source was set about one meter high above the ground. Three markes with the same height were placed in the immediate neighbourhood of the source, in the overlapping field of two sets of cameras and in the end of the lee-field, for it was necessary in the later procedure to know the elevation angle and the levelness of the cameras. In addition to the periodic illumination described above, another periodic illumination with lower frequency (about 4～5 cycles per sec.) was superposed, in order to know the corresponding images of particles in overlapping field of each set. Four 35 mm size cameras ($1/f=1:3.5$) were set on a plate and were placed on the ground with an angle of elevation, at a distance about 3.5 meters from the expected mean trajectories of the particles.

As the continuous light source, three illumination lamps (500 watts)

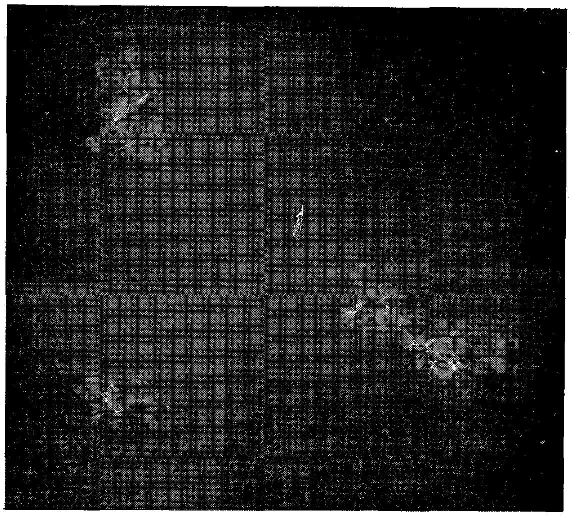


Fig. 1. Particles of sublimated metaldehyde. (3×)

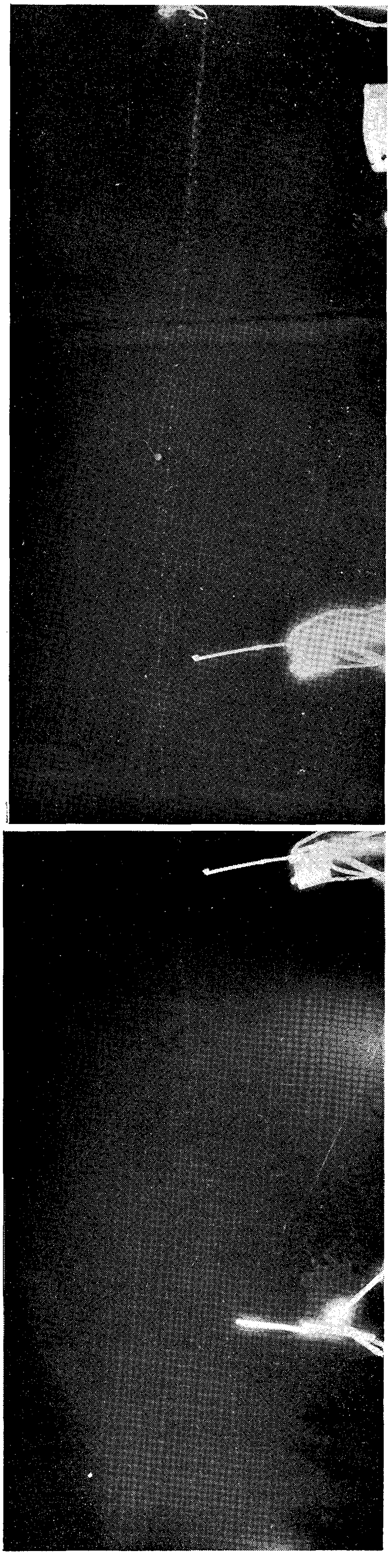


Fig. 3. A pair of photographs of trajectory.

put on the ground were used. Two lamp houses of the strobo-flashes were set at nearly the same height as the source of particles and were put on turn-tables to make the panning illumination of the particles. For the strobo-flash, two xenon discharge tubes* were used. The arrangement of measuring apparatus described above is illustrated in Fig. 2. A photograph obtained by the method is shown in Fig. 3.

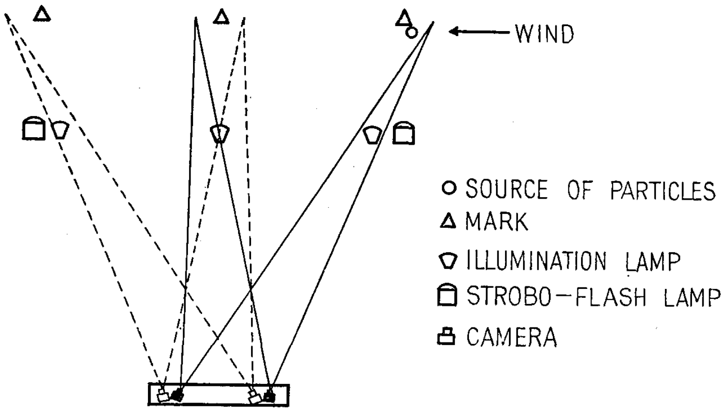


Fig. 2. Arrangement of measuring apparatus (plan).

The heating of nichrom wire and the operation of the strobo-flashes were synchronized with the shutter of a camera. When the synchronous circuit of a camera was closed, one relay was operated, then the trigger circuits of strobo-discharge tubes and the heating circuit were closed. The nichrome wire was heated only for a moment by the actions of a relay-discharge tube and another relay. This circuit diagram is shown in Fig. 4.

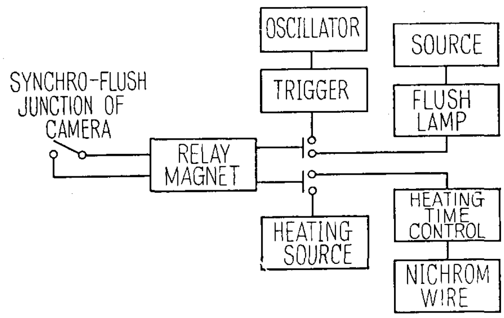


Fig. 4. Block diagram of apparatus system.

c. Conditions of observation

The observations were made in the natural wind near the ground. The area of the observation was an unoccupied ground, about 500 m square, near the University, and in its surroundings there were some small houses here and there. The observations were made at some bleezing nights during May to August 1957. This reason is that when the wind speed is higher, the passing times of the particles become shorter, and it becomes more difficult to put the crystals of metaldehyde in the heat source and the sublimated particles becomes too fine to be photographed. The particle-source was always set at the height of about 1 m. Though the air temperatures at the hights of 0.5, 1 and 1.5 m were measured by an Assman thermometer, the effects of the meteorological factors were not taken into consideration especially. In order to

* These tubes are connected in series and are operated by discharging 4 μF condensor whose voltage is 3.6 kV, using 20 μF condensor as a reservoir. The wattage of a tube is about 4 watt-seconds per pulse and the duration is several tens micro-seconds.

check the mean wind speed, a hot-wire probe was set near the source and the indicating meter was placed to be in the photographic field.

III. Analysis

a. Determination of three dimensional positions of particles

From a set of four photographs obtained by the above method, three dimensional positions of every image of particles were determined, using the method which one of the authors has resorted to in former experiment.[2] Namely the four photographic plates* prepared by printing from the negatives were set in the cameras just in the same condition as they were photographed, and each of them were lighted from behind. When the images of a particle on a movable screen projected by the cameras coincided perfectly, the position of the screen relative to the cameras was exactly the same to the particle position when it was photographed. A movable small vertical screen on which a vertical scale was affixed, was moved on a sheet of paper spread out on the horizontal table to find the position where a pair of images of a particle coincided, then the position was marked on the paper and at the same time its height read from the scale was recorded. In such a way positions of the particles in every moment when the strobo-flashes were operated were determined three dimensionally, one after the other.

b. Turbulent velocity and intensity of turbulence

From the trajectory thus obtained the mean wind direction was determined, and choosing x-axis along this direction, z-axis vertically, y-axis perpendicularly to them and an arbitrary point as the origin, coordinates of every points were measured. The measured positions for each direction against the time are plotted in Fig. 5. Since time intervals between succssive points are known, the fluctuating velocities u', v' and w' at each interval can be obtained. These u', v' and w' against the time are shown in Fig. 6. Using these data the turbulent intensities were calculated.

c. Lagrangian correlation curve

From the data above shown the Lagrangean correlation coefficients between $u'(t)$ at a time t and $u'(t+\xi)$ at a later time $t+\xi$,

$$R(\xi)=\frac{\overline{u'(t)u'(t+\xi)}}{\overline{u'^2}} \qquad (1)$$

were calculated. These results are shown in Fig. 7.

* For the film streches by the heat of light from the back.

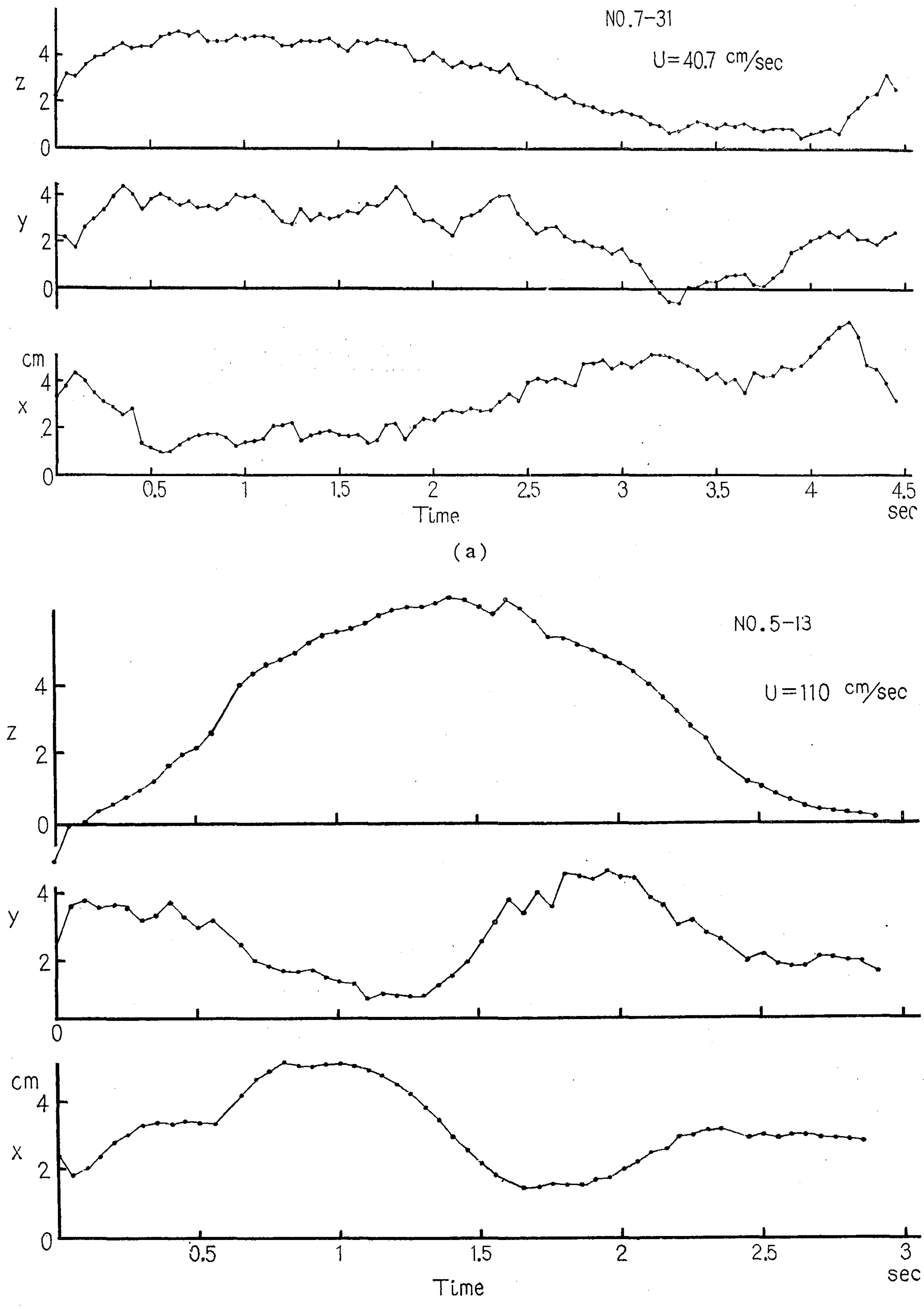
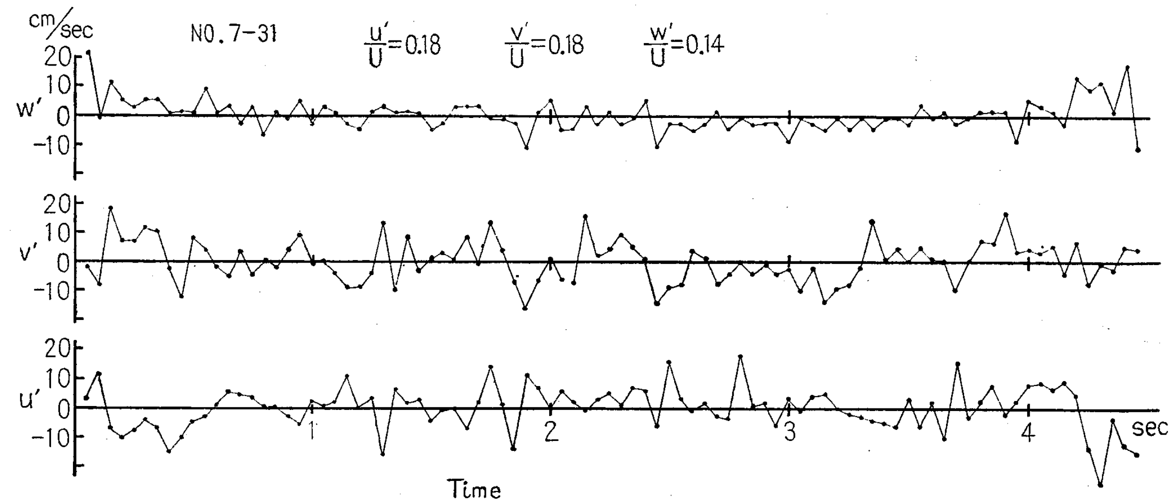
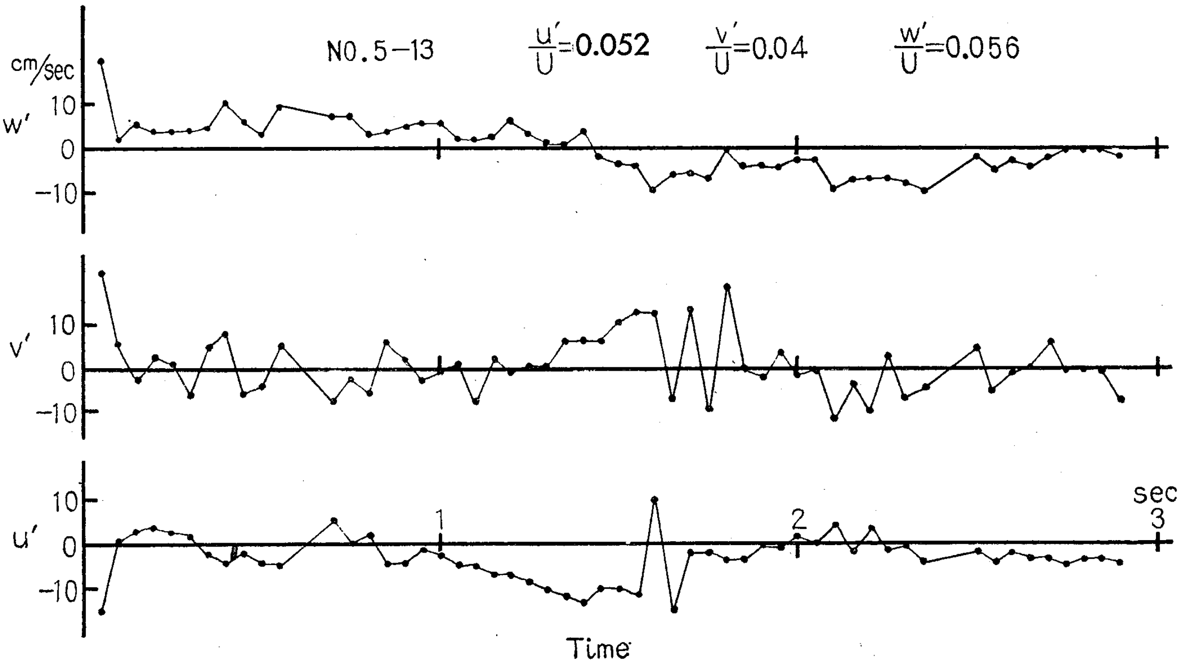


Fig. 5. Three dimensional variation of particle position from the displacement by mean motion.



(a)



(b)

Fig. 6. Variation of fluctuating velocity.

d. Standard deviation

$\overline{y}^2$ which indicates the transversal standard deviation of the positions of the particles is given by

$$\overline{y}^2 = 2\overline{v}'^2 \int_0^T \int_0^t R(\xi)\, d\xi\, dt \,. \qquad (2)$$

Then using correlation curves above obtained, $\overline{y}^2$ ($\overline{x}^2$ and $\overline{z}^2$ also similarly) can be calculated. While in our experiments co-ordinates of particles are known at a constant time interval, so $\overline{y}^2$ is obtained from $\overline{(y_{t+\xi}-y_t)}^2$

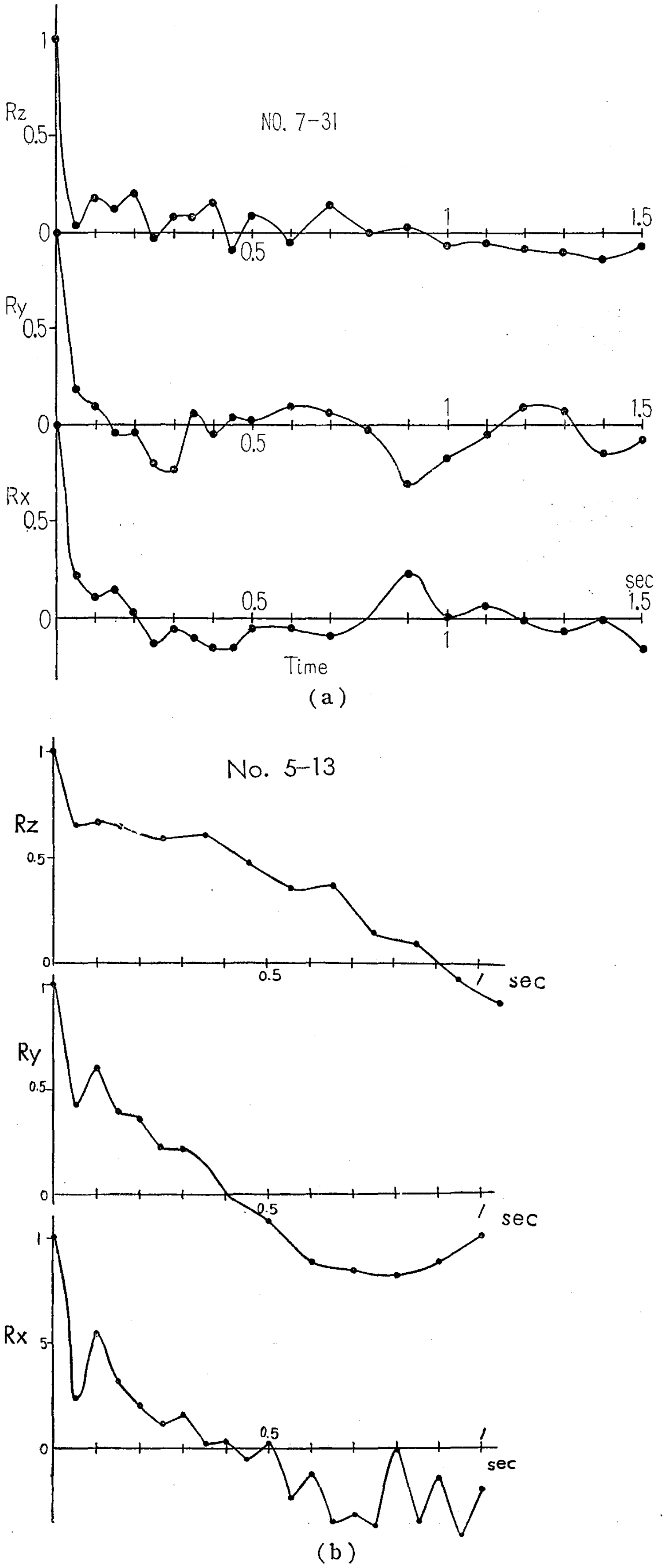
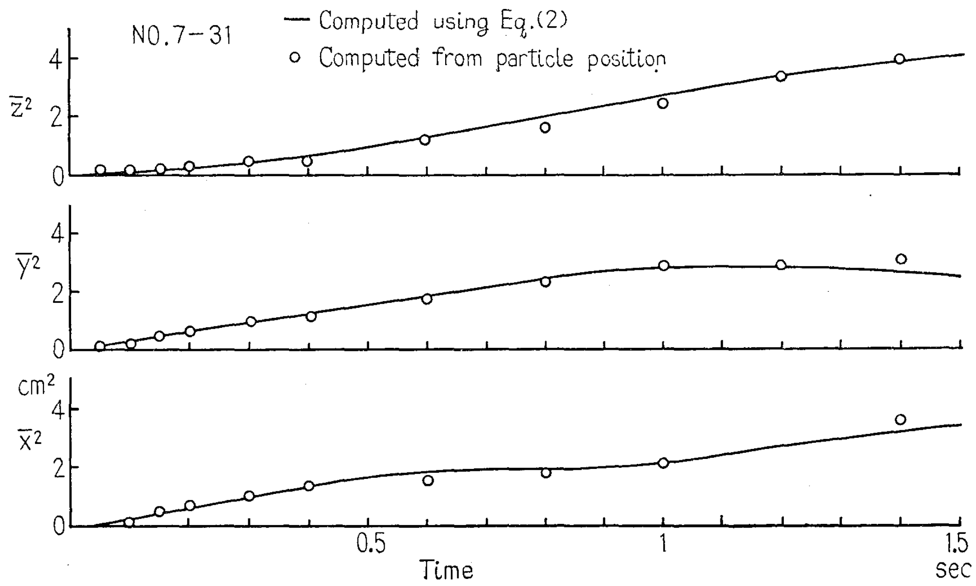
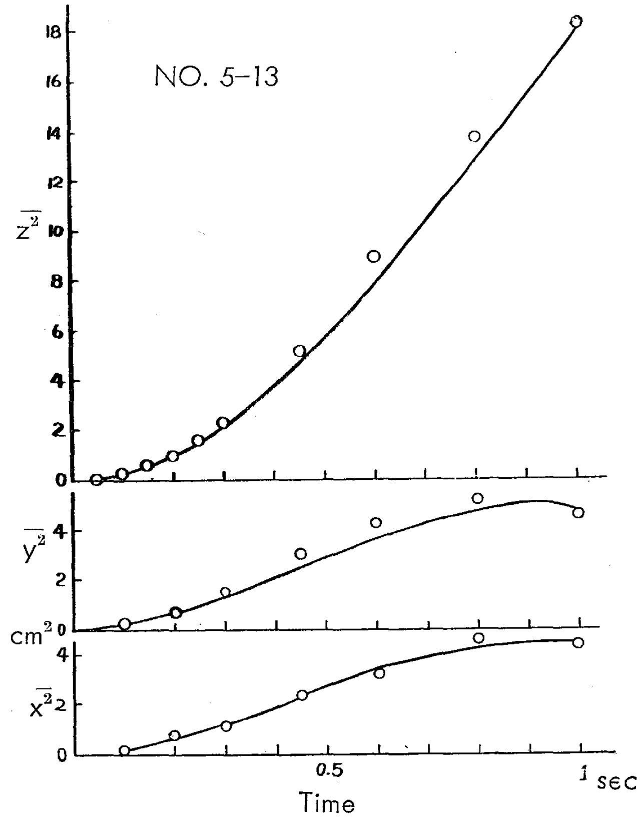


Fig. 7. Lagrangian correlation of fluctuating velocity.

directly. $\bar{x}^2$, $\bar{y}^2$ and $\bar{z}^2$ which are obtained by the above two methods against time are compared with in Fig. 8.

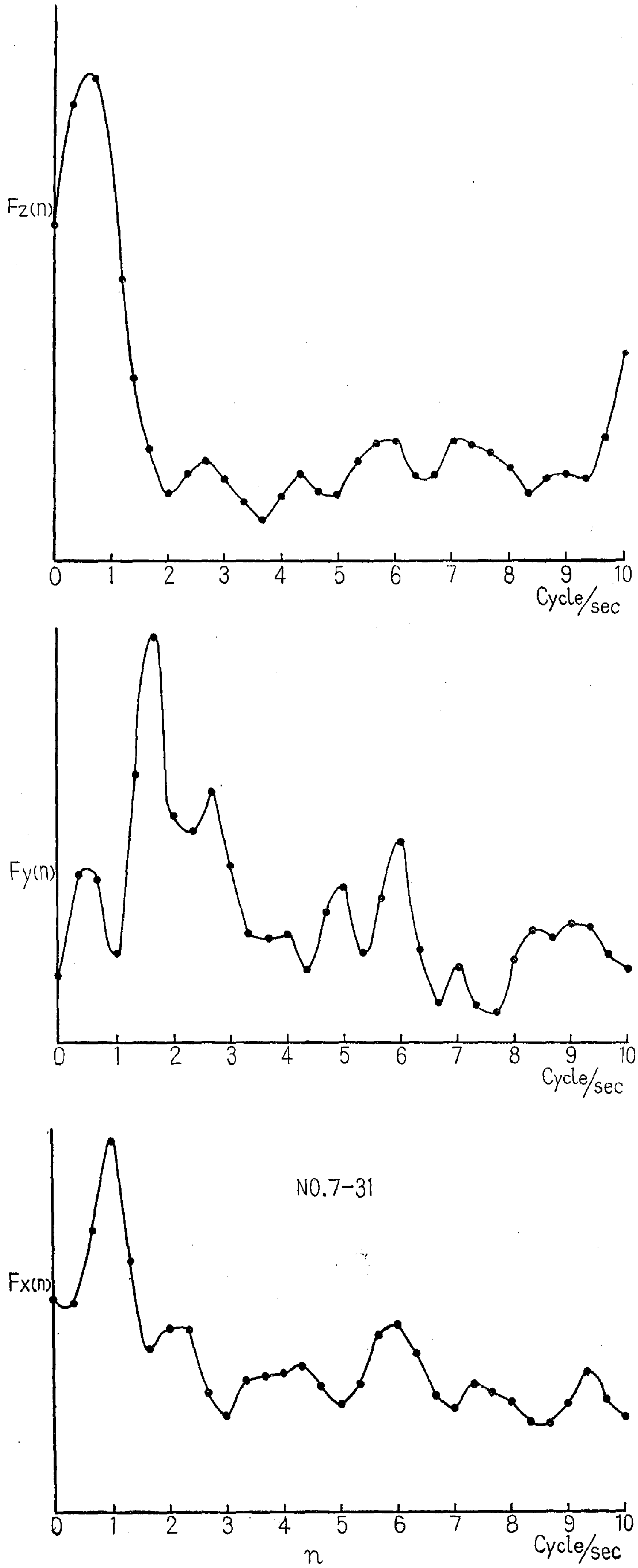


(a)



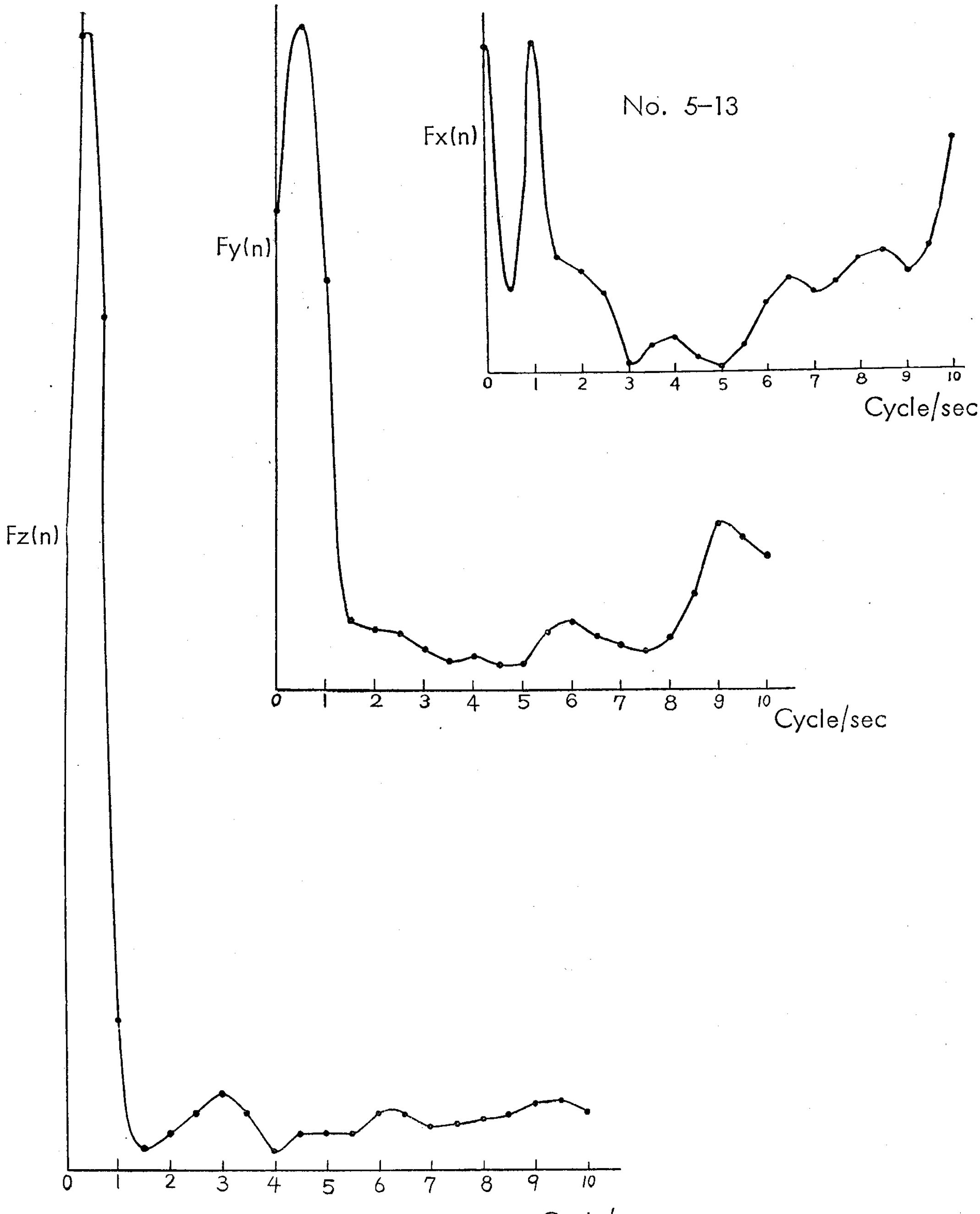
(b)

Fig. 8. Comparison between values of standard deviations of particle positions computed by using Eq. (2) and those computed directly from particle positions.



(a)

Fig. 9. Energy spectrum.



(b)

Fig. 9. Energy spectrum.

e. Energy spectrum

The energy spectrum $F(n)$ for frequency n is given by

$$F(n)=4\int_0^\infty R(n)\cos 2\pi n\xi \, d\xi \, . \qquad (3)$$

Therefore, the energy spectra are estimated from the computed correlation coefficients. In our cases, since the curve for $R(\xi)$ has been computed only in a finite period, Tukey and Hamming's method, which makes it possible to obtain spectrum within a certain range of n, was used to compute $F(n)$[3]. Some results are given in Fig. 9.

IV. Result and Discussion

In our observations, we intended to make clear the mechanism of the turbulent motion as finely and as concretely as possible. Namely we

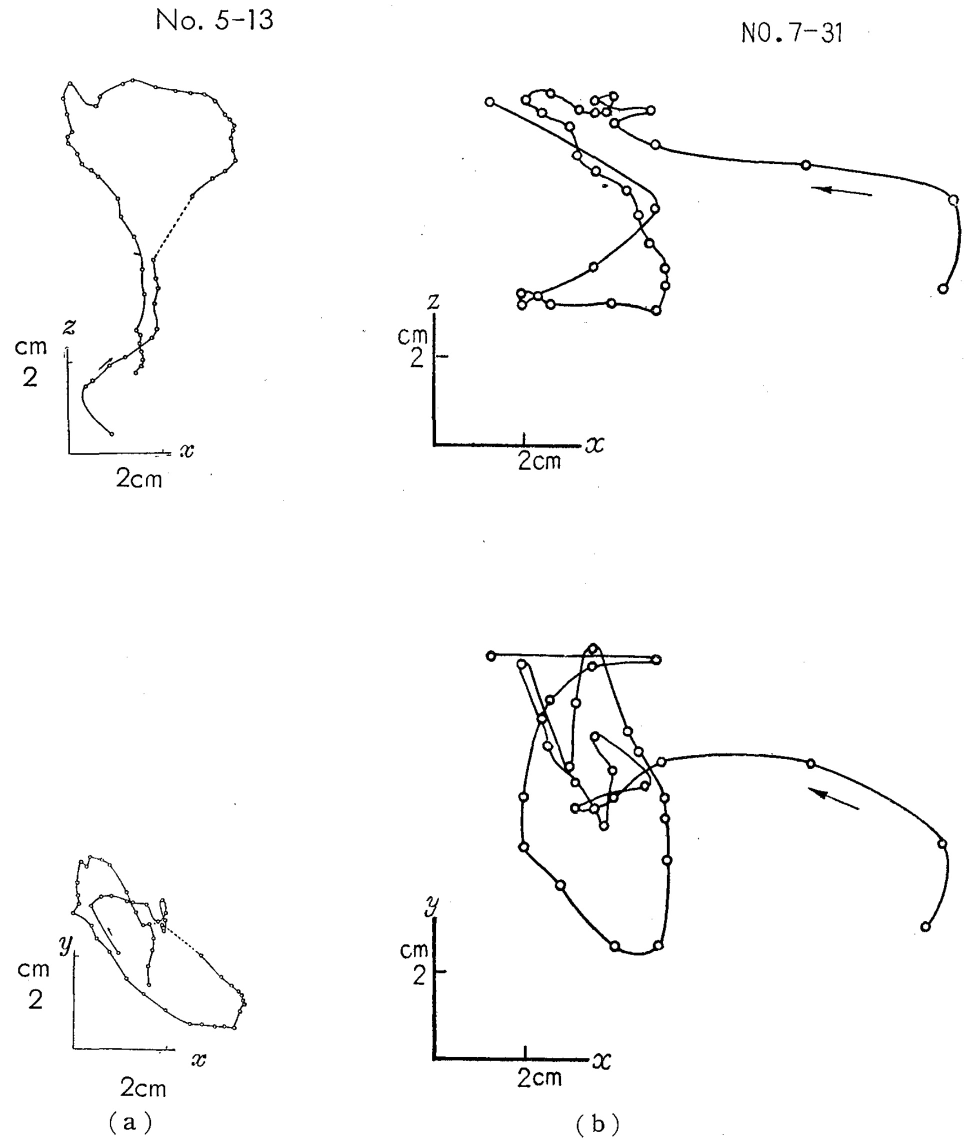


Fig. 10. Track of particle relative to steady mean displacement.

asked, for example, in what procedure each minute portion in a spreading smoke plume practically moves and the total shape of the plume is formed. So we pursuited the motions of the particles which follow faithfully the micro-movements of air parcels, and the deviations of position of the particles from the steady translation with the mean velocity are given in Fig. 5. We shall see that the particles drifting with the mean velocity move around three dimensionally in a domain about 5 cm in scale. These data are plotted in pairs in Fig. 10, by which we might recognize that an eddy moves the particle for a time and then the other eddy moves it for a time and so on, and that each eddy has the axis with different direction. The periods of these fluctuations of the position shall be deduced from the figure. Further we treated these periods quantitatively. The position of peaks in the energy spectrum should be the same to those in the spectrum of the variation of particle position, so we calculated the energy spectra. (Fig. 9) For example, in the graph y in Fig. 5 (a), we can see typical period: one of which is 2.1 sec between the instants 2.1 sec and 4.2 sec and the other is 0.6 sec between the instants 1.8 sec and 2.4 sec. These correspond to the peaks for 0.5 and 1.6 cycle/sec in the graph $F_y(n)$ in Fig. 9 (a).

In our case, the standard deviations of the position of the particles can be obtained directly by computing $\overline{(y_{t+\xi}-y_t)^2}$ and at the same time we can also compute them from Taylor's equation (2) which involves the correlation function. So we can prove the equation (2) experimentally. As is shown in Fig. 8, both results agree fairly well.

Previously we had measured heat wakes within 5 cm leeward from the source to investigate the diffusion phenomena in a small scale[4]. In the cases in which the wind speed was smaller in the former experiment, the range of the measurement corresponds to that in the present experiment, so we can compare both standard deviations of the spread. Both results agree fairly.

As it has not been frequent that the conditions of the observation were satisfied, we have obtained only some data, so we are intending to continue the observations and to find relationship between the mechanism and the meteorological factors.

Literatures

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