

On the Atmospheric Diffusion of Gas or Aerosol Near the Ground

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INTRODUCTION

Since 1935 till the End of the World War II, the author had engaged in measuring and analysing the diffusion of gas or aerosol from various kinds of sources in spaces which were ranging from several meters to several kilometers in horizontal direction and within 5 meters high from the ground.²⁾ On the day of the End of the War, all reports and data were completely burnt during the author's absence, so there is not any numerical datum which can be used at present. Nevertheless it may have some meanings to report the results which remain in his memorandum, because the results are also useful for the problems of agricultural meteorology, industrial sanitary and disaster prevention, so we dare to report some of these results.

I THEORY

I-1 Differential equation of diffusion

It may be reasonable to assume that the vorticity transport theory holds true in a free stream and the momentum transport theory holds true in the region near boundary.⁽¹⁾ So when we treat the atmospheric diffusion in spaces within 1.5 km. in horizontal direction and 5 m. in vertical direction, we make next assumptions :

1) We consider a coordinate system which moves along with mean wind whose velocity is u (m/sec.), and take x -axis along the mean wind direction leeward, z -axis vertically from the ground and y -axis perpendicularly to them. The coordinates of a point source at the time $t=0$ be $(0, 0, h)$ and concentration of the diffusive matters be C (mg/m³).

2) The horizontal diffusion occurs as in the vorticity transport theory, and the vertical diffusion occurs as in the momentum transport theory.

3) The vertical diffusion coefficient is assumed to be a linear function of height from the ground ($K_z=bz$),⁽²⁾ and the two horizontal diffusion coefficients are assumed to be equal and constant ($K_x=K_y=a$).

Then we obtain next differential equation :

1) This report was written at the end of 1946 and at that time the author was a member of the Meteorological Research Institute.

2) These measurements were carried on in the Army Science Research Institute.

$$\frac{\partial C}{\partial t} = a \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) + \frac{\partial}{\partial z} \left(bz \frac{\partial C}{\partial z} \right) \quad 1-1-1.$$

The procedure to obtain the solution appropriate to a instantaneous point source has been reported by the author,⁽³⁾ so we shall describe only the final result.

$$C = \frac{M}{4ab\pi t^2} \exp \left(-\frac{x^2+y^2}{4at} - \frac{h+z}{bt} \right) J_0 \left(i \frac{2\sqrt{hz}}{bt} \right) \quad 1-1-2,$$

where M is the total amount of the diffusive matter.

I-2 Formula referred to a coordinate system fixed to the ground

Now we refer to a coordinate system fixed to the ground. The point on the ground directly under the source at $t=0$ is taken as the origin. Eq. (1-1-2) becomes

$$C = \frac{M}{4ab\pi t^2} \exp \left(-\frac{(x-ut)^2+y^2}{4at} - \frac{h+z}{bt} \right) J_0 \left(i \frac{2\sqrt{hz}}{bt} \right) \quad 1-2-1.$$

We adopt a quantity Ct , which is defined by

$$Ct = \frac{1}{60} \int_0^\infty C dt \quad (\text{mg. min./m}^3.) \quad 3) \quad 1-2-2.$$

So we get

$$Ct = \frac{M}{60 \times 4ab\pi} \int_0^\infty \frac{e^{-\frac{(x-ut)^2+y^2}{4at} - \frac{h+z}{bt}}}{t^2} J_0 \left(i \frac{2\sqrt{hz}}{bt} \right) dt \quad 1-2-3.$$

As this formula is not tractable for calculation, we make an approximate formula. The term $\exp \left(-\frac{(x-ut)^2}{4at} \right) / t^2$ varies more rapidly with time t than any other term in the integrand, so the value of t which makes the term maximum

$$t_0 = \sqrt{\frac{x^2}{u^2} + \left(\frac{4a}{u^2} \right)^2} - \frac{4a}{u^2} \quad 1-2-4.$$

is substituted for all t except that in the term $(x-ut)$.

$$Ct = \frac{M}{60 \times 4ab\pi t_0} e^{-\frac{y^2}{4at_0} - \frac{h+z}{bt_0}} J_0 \left(i \frac{2\sqrt{hz}}{bt_0} \right) \int_0^\infty e^{-\frac{(x-ut)^2}{4at_0}} dt.$$

When we put $4at_0 = A_0$ and $bt_0 = B_0$,

$$= \frac{M}{60A_0B_0\pi} e^{-\frac{y^2}{A_0} - \frac{h+z}{B_0}} J_0 \left(i \frac{2\sqrt{hz}}{B_0} \right) \int_0^\infty e^{-\frac{(x-ut)^2}{A_0}} dt.$$

Using the error integral

$$\Phi(\rho) = \frac{2}{\sqrt{\pi}} \int_0^\rho e^{-\eta^2} d\eta \quad 1-2-5,$$

$$Ct = \frac{M}{60\sqrt{A_0}\pi B_0} e^{-\frac{y^2}{A_0} - \frac{h+z}{B_0}} J_0 \left(i \frac{2\sqrt{hz}}{B_0} \right) \frac{1}{u} \frac{1 + \Phi \left(\frac{x}{\sqrt{A_0}} \right)}{2} \quad 1-2-6.$$

3) If we take physiological effects into consideration, it is not sufficient to determine the values of Ct , but the values of C as function of t must be determined.⁽⁴⁾

I-3 Continuous point source

The instant when the concentration is measured be t , the time of duration of the source be λ (sec.) and the emitting amount of matter per unit time be q (mg/sec.). The concentration is given by

$$C = \frac{1}{4ab\pi} \int_0^t q(\varsigma) \frac{e^{-\frac{(x-u(t-\varsigma))^2+y^2}{4a(t-\varsigma)} - \frac{h+z}{b(t-\varsigma)}}}{(t-\varsigma)^2} J_0\left(i \frac{2\sqrt{hz}}{b(t-\varsigma)}\right) d\varsigma \quad 1-3-1.$$

Putting $t-\varsigma=\xi$,

$$= \frac{1}{4ab\pi} \int_0^t q(t-\xi) \frac{e^{-\frac{(x-u\xi)^2+y^2}{4a\xi} - \frac{h+z}{b\xi}}}{\xi^2} J_0\left(i \frac{2\sqrt{hz}}{b\xi}\right) d\xi \quad t \leq \lambda \quad 1-3-2,$$

$$= \frac{1}{4ab\pi} \int_{t-\lambda}^t q(t-\xi) \frac{e^{-\frac{(x-u\xi)^2+y^2}{4a\xi} - \frac{h+z}{b\xi}}}{\xi^2} J_0\left(i \frac{2\sqrt{hz}}{b\xi}\right) d\xi \quad t \geq \lambda \quad 1-3-3.$$

Eqs. (1-3-2) and (1-3-3) are not tractable, so we make approximate formulae. In this case q is assumed to be independent of time. Similar to the procedure in which we have obtained (1-2-6), using

$$\xi_0 = \sqrt{\frac{x^2}{u^2} + \left(\frac{4a}{u^2}\right)^2} - \frac{4a}{u^2} \quad 1-3-4,$$

we obtain

$$\begin{aligned} C &= q \int_0^t \frac{e^{-\frac{(x-u\xi)^2+y^2}{4a\xi_0} - \frac{h+z}{b\xi_0}}}{4ab\pi\xi_0^2} J_0\left(i \frac{2\sqrt{hz}}{b\xi_0}\right) d\xi \\ &= q \frac{e^{-\frac{y^2}{4a\xi_0} - \frac{h+z}{b\xi_0}}}{4ab\pi\xi_0^2} J_0\left(i \frac{2\sqrt{hz}}{b\xi_0}\right) \int_0^t e^{-\frac{(x-u\xi)^2}{4a\xi_0}} d\xi. \end{aligned}$$

Putting $4a\xi_0=A_0$ and $b\xi_0=B_0$,

$$\begin{aligned} C &= q \frac{e^{-\frac{y^2}{A_0} - \frac{h+z}{B_0}}}{\pi A_0 B_0} J_0\left(i \frac{2\sqrt{hz}}{B_0}\right) \int_0^t e^{-\frac{(x-u\xi)^2}{A_0}} d\xi \\ &= q \frac{e^{-\frac{y^2}{A_0} - \frac{h+z}{B_0}}}{\sqrt{A_0}\pi B_0} J_0\left(i \frac{2\sqrt{hz}}{B_0}\right) \frac{1}{u} \frac{\phi\left(\frac{x}{\sqrt{A_0}}\right) - \phi\left(\frac{x-ut}{\sqrt{A_0}}\right)}{2} \quad t \leq \lambda \quad 1-3-5, \end{aligned}$$

$$= q \frac{e^{-\frac{y^2}{A_0} - \frac{h+z}{B_0}}}{\sqrt{A_0}\pi B_0} J_0\left(i \frac{2\sqrt{hz}}{B_0}\right) \frac{1}{u} \frac{\phi\left(\frac{x-ut-u\lambda}{\sqrt{A_0}}\right) - \phi\left(\frac{x-ut}{\sqrt{A_0}}\right)}{2} \quad t \geq \lambda \quad 1-3-6.$$

$$Ct = \frac{1}{60} \int_0^\infty \frac{dt}{4ab\pi} \int_0^t q(\varsigma) \frac{e^{-\frac{(x-u(t-\varsigma))^2+y^2}{4a(t-\varsigma)}}}{(t-\varsigma)^2} J_0\left(i \frac{2\sqrt{hz}}{b(t-\varsigma)}\right) d\varsigma.$$

Changing the order of the integration (cf. Fig. 1-1), we get

$$= \frac{1}{60} \int_0^\infty q(\varsigma) d\varsigma \int_\varsigma^\infty ((t-\varsigma)) dt.$$

Putting $t-\varsigma=\xi$,

$$\begin{aligned} &= \frac{1}{60} \int_0^\infty q(\varsigma) d\varsigma \int_0^\infty ((\xi)) d\xi \\ &= \frac{M}{60} \int_0^\infty ((\xi)) d\xi \end{aligned}$$

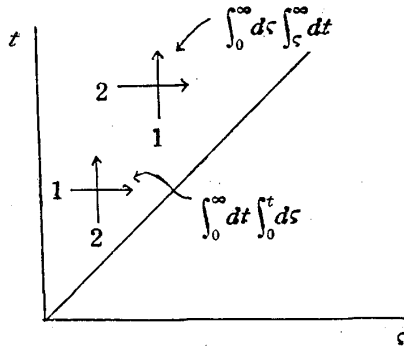


Fig. 1-1

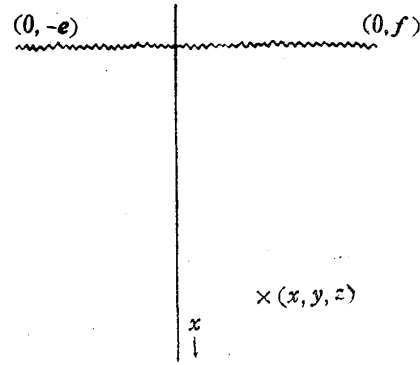


Fig. 1-2

where $M = \int_0^\infty q(\varsigma) d\varsigma$, so the final result becomes

$$Ct \div \frac{M}{60} \int_0^\infty \frac{e^{-\frac{(x-ut)^2+y^2}{4at} - \frac{h+z}{bt}}}{4ab\pi\xi^2} J_0\left(\frac{2\sqrt{hz}}{b\xi}\right) d\xi \quad 1-3-7.$$

Its approximate formula is given by

$$Ct \div \frac{M}{60} \frac{e^{-\frac{y^2}{A_0} - \frac{h+z}{B_0}}}{\sqrt{A_0}\pi} \frac{J_0\left(i\frac{2\sqrt{hz}}{B_0}\right)}{B_0} \frac{1}{u} \frac{1+\phi\left(\frac{x}{\sqrt{A_0}}\right)}{2} \quad 1-3-8.$$

I-4 Line source

For this source, we shall consider only the case when the wind direction is normal to the line of source.

I-4-1 Instantaneous line source

The current coordinate along the line be η , (Fig. 1-2), so the concentration is given by

$$C = \frac{1}{4ab\pi t^2} e^{-\frac{(x-ut)^2}{4at} - \frac{h+z}{bt}} J_0\left(i\frac{2\sqrt{hz}}{bt}\right) \int_{-e}^f q(\eta) e^{-\frac{(y-\eta)^2}{4at}} d\eta.$$

Assuming that $q(\eta) = \text{constant} = M_1$,

$$\begin{aligned} C &= \frac{M_1}{AB\pi} e^{-\frac{(x-ut)^2}{A} - \frac{h+z}{B}} J_0\left(i\frac{2\sqrt{hz}}{B}\right) \int_{-e}^f e^{-\frac{(y-\eta)^2}{A}} d\eta \\ &= \frac{M_1}{AB\pi} e^{-\frac{(x-ut)^2}{A} - \frac{h+z}{B}} J_0\left(i\frac{2\sqrt{hz}}{B}\right) \frac{\phi\left(\frac{y+e}{\sqrt{A}}\right) - \phi\left(\frac{y-f}{\sqrt{A}}\right)}{2} \end{aligned} \quad 1-4-1,$$

where $4at = A$ and $bt = B$.

$$\begin{aligned} Ct &= \frac{1}{60 \times 4ab\pi} \int_0^\infty \frac{e^{-\frac{(x-ut)^2}{A} - \frac{h+z}{B}}}{t^2} J_0\left(i\frac{2\sqrt{hz}}{B}\right) dt \int_{-e}^f q(\eta) e^{-\frac{(y-\eta)^2}{A}} d\eta \\ &= \frac{M_1}{60\pi} \int_0^\infty \frac{e^{-\frac{(x-ut)^2}{A} - \frac{h+z}{B}}}{AB} J_0\left(i\frac{2\sqrt{hz}}{B}\right) dt \int_{-e}^f e^{-\frac{(y-\eta)^2}{A}} d\eta. \end{aligned}$$

$$\text{Putting } t_0 = \sqrt{\left(\frac{x}{u}\right)^2 + \left(\frac{3}{4} \frac{4a}{u^2}\right)^2} - \frac{3}{4} \frac{4a}{u^2}, \quad A_0 = 4at_0 \text{ and } B_0 = bt_0 \quad 1-4-2,$$

we get

$$\begin{aligned}
Ct &= \frac{M_1 e^{-\frac{h+z}{B_0}}}{60\sqrt{A_0}\pi B_0} J_0\left(i\frac{2\sqrt{hz}}{B_0}\right) \frac{\phi\left(\frac{y+e}{\sqrt{A_0}}\right) - \phi\left(\frac{y-f}{\sqrt{A_0}}\right)}{2} \int_0^\infty e^{-\frac{(x-ut)}{A_0}} dt \\
&= \frac{M_1}{60} \frac{e^{-\frac{h+z}{B_0}}}{B_0} J_0\left(i\frac{2\sqrt{hz}}{B_0}\right) \frac{\phi\left(\frac{y+e}{\sqrt{A_0}}\right) - \phi\left(\frac{y-f}{\sqrt{A_0}}\right)}{2} \frac{1}{u} \frac{1+\phi\left(\frac{x}{\sqrt{A_0}}\right)}{2}
\end{aligned}
\quad 1-4-3.$$

I-4-2 Continuous line source

Putting the emitting amount per unit length and per unit time be $q(\eta, \varsigma)$ (mg/m. sec.), we get for the concentration next formula :

$$C = \frac{1}{4ab\pi} \int_0^t q(\varsigma, \eta) \frac{e^{-\frac{(x-u(t-\varsigma))^2}{4a(t-\varsigma)} - \frac{h+z}{b(t-\varsigma)}}}{(t-\varsigma)^2} J_0\left(i\frac{2\sqrt{hz}}{b(t-\varsigma)}\right) d\varsigma \int_{-e}^f e^{-\frac{(y-\eta)^2}{4a(t-\varsigma)}} d\eta.$$

Using $t-\varsigma=\xi$ and considering the case where q is independent of η ,

$$C = \int_0^t \frac{q(t-\xi)}{\sqrt{A\pi B}} e^{-\frac{(x-u\xi)^2}{A} - \frac{h+z}{B}} J_0\left(i\frac{2\sqrt{hz}}{B}\right) \frac{\phi\left(\frac{y+e}{\sqrt{A}}\right) - \phi\left(\frac{y-f}{\sqrt{A}}\right)}{2} d\xi \quad t \leq \lambda \quad 1-4-4,$$

$$= \int_{t-\lambda}^t \frac{q(t-\xi)}{\sqrt{A\pi B}} e^{-\frac{(x-u\xi)^2}{A} - \frac{h+z}{B}} J_0\left(i\frac{2\sqrt{hz}}{B}\right) \frac{\phi\left(\frac{y+e}{\sqrt{A}}\right) - \phi\left(\frac{y-f}{\sqrt{A}}\right)}{2} d\xi \quad t \geq \lambda \quad 1-4-5.$$

Using ξ_0 which makes the term $\exp\left(-\frac{(x-u\xi)^2}{4a\xi}\right)/\xi^{3/2}$ maximum, i. e.

$$\xi_0 = \sqrt{\left(\frac{x}{u}\right)^2 + \left(\frac{3}{4} \frac{4a}{u^2}\right)^2} - \frac{3}{4} \frac{4a}{u^2} \quad 1-4-6,$$

we get

$$C = \frac{q}{B_0} e^{-\frac{h+z}{B_0}} J_0\left(i\frac{2\sqrt{hz}}{B_0}\right) \frac{\phi\left(\frac{y+e}{\sqrt{A_0}}\right) - \phi\left(\frac{y-f}{\sqrt{A_0}}\right)}{2} \frac{1}{u} \frac{\phi\left(\frac{x}{\sqrt{A_0}}\right) - \phi\left(\frac{x-ut}{\sqrt{A_0}}\right)}{2} \quad t \leq \lambda \quad 1-4-7,$$

$$= \frac{q}{B_0} e^{-\frac{h+z}{B_0}} J_0\left(i\frac{2\sqrt{hz}}{B_0}\right) \frac{\phi\left(\frac{y+e}{\sqrt{A_0}}\right) - \phi\left(\frac{y-f}{\sqrt{A_0}}\right)}{2} \frac{1}{u} \frac{\phi\left(\frac{x-ut-u\lambda}{\sqrt{A_0}}\right) - \phi\left(\frac{x-ut}{\sqrt{A_0}}\right)}{2} \quad t \geq \lambda \quad 1-4-8.$$

$$Ct = \frac{1}{60} \int_0^\infty dt \int_0^t \frac{e^{-\frac{(x-u(t-\varsigma))^2}{4a(t-\varsigma)} - \frac{h+z}{b(t-\varsigma)}}}{4ab(t-\varsigma)^2\pi} J_0\left(i\frac{2\sqrt{hz}}{b(t-\varsigma)}\right) d\varsigma \int_{-e}^f q(\varsigma, \eta) e^{-\frac{(y-\eta)^2}{4a(t-\varsigma)}} d\eta.$$

Changing the order of integration and putting $t-\varsigma=\xi$,

$$= \frac{1}{60} \int_0^\infty q(\varsigma) d\varsigma \int_\varsigma^\infty ((t-\varsigma)) dt = \frac{1}{60} \int_0^\infty q(\varsigma) d\varsigma \int_0^\infty ((\xi)) d\xi = M \int_0^\infty ((\xi)) d\xi,$$

where $M = \int_0^\infty q(\varsigma) d\varsigma$, so we get

$$Ct = \frac{M}{60} \int_0^\infty e^{-\frac{(x-u\xi)^2}{A} - \frac{h+z}{B}} J_0\left(i\frac{2\sqrt{hz}}{B}\right) \frac{\phi\left(\frac{y+e}{\sqrt{A}}\right) - \phi\left(\frac{y+f}{\sqrt{A}}\right)}{2} d\xi \quad 1-4-9.$$

As its approximate formula, we obtain

$$Ct \doteq \frac{M}{60} \frac{e^{-\frac{h+z}{B_0}}}{B_0} J_0\left(i \frac{2\sqrt{hz}}{B_0}\right) \frac{\phi\left(\frac{y+e}{\sqrt{A_0}}\right) - \phi\left(\frac{y-f}{\sqrt{A_0}}\right)}{2} \frac{1}{u} \frac{1 + \phi\left(\frac{x}{\sqrt{A_0}}\right)}{2} \quad 1-4-10.$$

Especially when $h=0$, i. e. when the source is on the ground,

$$Ct \doteq \frac{M}{60} \frac{e^{-\frac{z}{B_0}}}{B_0} \frac{\phi\left(\frac{y+e}{\sqrt{A_0}}\right) - \phi\left(\frac{y-f}{\sqrt{A_0}}\right)}{2} \frac{1}{u} \frac{1 + \phi\left(\frac{x}{\sqrt{A_0}}\right)}{2} \quad 1-4-11.$$

I-4-3 Minimum necessary amount

We assume that we intend to cover with diffusive matter over a linear domain ($x=x, y=-s \sim +s$), and that we expect a given value of $Ct (=Ct_0)$ even at both ends of the domain ($x=x, y=\pm s$), so we must determine the emitting amount per unit length M and the length of the source $2r$. In this case the total emitting amount $N=2r \cdot M$ has a minimum value. From (1-4-10) or (1-4-11), we obtain

$$Ct_0 = M \cdot K \cdot \frac{\phi\left(\frac{s+r}{\sqrt{A_0}}\right) - \phi\left(\frac{s-r}{\sqrt{A_0}}\right)}{2},$$

where K is a function of x and u . So,

$$N = 2r \cdot M = \frac{2r \cdot 2 \cdot Ct_0}{K \left\{ \phi\left(\frac{s+r}{\sqrt{A_0}}\right) - \phi\left(\frac{s-r}{\sqrt{A_0}}\right) \right\}}.$$

Putting $s/\sqrt{A_0} = \sigma$, $r/\sqrt{A_0} = \rho$, we get

$$N = \frac{2\rho\sqrt{A_0} \cdot 2Ct_0}{K \{ \phi(\sigma+\rho) - \phi(\sigma-\rho) \}} \quad 1-4-12.$$

To obtain its minimum value, we make $dN/d\rho=0$, i. e.

$$\phi(\sigma+\rho) - \phi(\sigma-\rho) - \rho \{ \phi'(\sigma+\rho) + \phi'(\sigma-\rho) \} = 0 \quad 1-4-13.$$

Therefore, when s is given, together with A_0 , σ can be calculated, and then ρ is calculated from this equation and finally the length r is determined. The relation between σ and ρ is shown in Table 1.

Table 1

σ	0.707	0.8	0.9	1.0	1.5	2.0	3.0
ρ	0	0.648	1.094	1.328	2.145	2.772	3.952
ρ/σ	0	0.810	1.215	1.328	1.430	1.386	1.317
$2/\{\phi(\sigma+\rho) - \phi(\sigma-\rho)\}$	∞	2.538	1.648	1.474	1.220	1.159	1.097
σ	4.0	5.0	6.0	7.0	8.0	9.0	10
ρ	5.063	6.142	7.218	8.254	9.305	10.34	11.37
ρ/σ	1.266	1.228	1.203	1.179	1.163	1.148	1.137
$2/\{\phi(\sigma+\rho) - \phi(\sigma-\rho)\}$	1.071	1.054	1.045	1.039	1.033	1.029	1.028
σ	20	50	100	200	500	1000	
ρ	21.59	51.75	102.0	202.2	502.4	1002.5	
ρ/σ	1.079	1.035	1.020	1.011	1.004	1.003	
$2/\{\phi(\sigma+\rho) - \phi(\sigma-\rho)\}$	1.011	1.006	1.001	1.000	1.000	1.000	

When r is sufficiently large ($\rho=\infty$), the emitting amount M' which results $Ct=Ct_0$ at $x=x, y=0$, is given by

$$Ct_0 = \frac{M'}{60u} \frac{e^{-\frac{h+z}{B_0}}}{B_0} J_0\left(i \frac{2\sqrt{hz}}{B_0}\right) \frac{1 + \phi\left(\frac{x}{\sqrt{A_0}}\right)}{2} \quad (h \neq 0)$$

$$= \frac{M'}{60u} \frac{e^{-\frac{h+z}{B_0}}}{B_0} \frac{1 + \Phi\left(\frac{x}{\sqrt{A_0}}\right)}{2} \quad (h=0).$$

The relation between M' and M is given by

$$M = 2M' / \{\Phi(\sigma + \rho) - \Phi(\sigma - \rho)\} \quad 1-4-14.$$

Factor $2 / \{\Phi(\sigma + \rho) - \Phi(\sigma - \rho)\}$ is also shown in Table 1.

I-5 2-dimensional source

2-dimensional sources are generally situated on the ground. The current coordinates in the domain of the source (Σ) be $(\varphi, \psi, 0)$ (Fig. 1-3). Here we want only to treat the rectangular domain of the source $(-l, -m)$, $(-l, m)$, $(l, -m)$ and (l, m) .

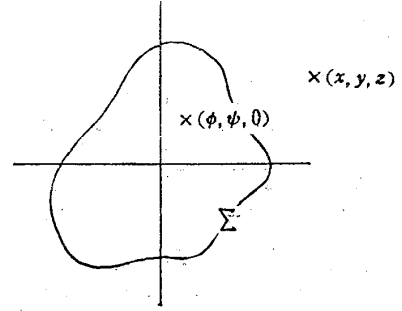


Fig. 1-3

I-5-1 Instantaneous 2-dimensional source

The emitting amount of the source per unit area be q . The concentration is given by

$$C = \int_{-l}^l d\varphi \int_{-m}^m \frac{q(\varphi, \psi)}{4ab\pi t^2} e^{-\frac{(x-ut-\varphi)^2 + (y-\psi)^2}{4at} - \frac{z}{bt}} d\psi.$$

Now we suppose that $q(\varphi, \psi)$ is independent of φ and ψ . So we get

$$C = qe^{-\frac{z}{B}} \frac{1}{B} \frac{\Phi\left(\frac{x-ut+l}{\sqrt{A}}\right) - \Phi\left(\frac{x-ut-l}{\sqrt{A}}\right)}{2} \frac{\Phi\left(\frac{y+m}{\sqrt{A}}\right) - \Phi\left(\frac{y-m}{\sqrt{A}}\right)}{2} \quad 1-5-1.$$

Using $t_0 = \sqrt{\left(\frac{x-\varphi}{u}\right)^2 + \left(\frac{4a}{u^2}\right)^2} - \frac{4a}{u^2}$,

$$Ct = \int_{-l}^l \frac{q}{B_0} e^{-\frac{z}{B_0}} \frac{1}{u} \frac{1 + \Phi\left(\frac{x-\varphi}{\sqrt{A_0}}\right)}{2} \frac{\Phi\left(\frac{y+m}{\sqrt{A_0}}\right) - \Phi\left(\frac{y-m}{\sqrt{A_0}}\right)}{2} d\varphi. \quad 1-5-2.$$

If we put $4a = 1.5u$, as we shall discuss latter (II-6-3), the value of $|\Phi((x-\varphi)/\sqrt{A_0})|$ only varies between 0.95 and 1.00 owing to the variation of φ , so we may assume that this value is approximately equal to 1. Then we get

$$x \geq l \quad Ct = \int_{-l}^l \frac{q}{B_0} e^{-\frac{z}{B_0}} \frac{1}{u} \frac{\Phi\left(\frac{y+m}{\sqrt{A_0}}\right) - \Phi\left(\frac{y-m}{\sqrt{A_0}}\right)}{2} d\varphi \quad 1-5-3,$$

$$l \geq x \geq -l \quad Ct = \int_{-l}^x \frac{q}{B_0} e^{-\frac{z}{B_0}} \frac{1}{u} \frac{\Phi\left(\frac{y+m}{\sqrt{A_0}}\right) - \Phi\left(\frac{y-m}{\sqrt{A_0}}\right)}{2} d\varphi. \quad 1-5-4.$$

I-5-2 Continuous 2-dimensional source

If we assume that the emitting amount per unit area and per unit time be $q(\zeta, \varphi, \psi)$, we get next formula for the concentration

$$C = \frac{1}{4ab\pi} \int_{-l}^l d\varphi \int_{-m}^m d\psi \int_0^t q(\zeta, \varphi, \psi) e^{-\frac{(x-\varphi-u(t-\zeta))^2 + (y-\psi)^2}{4a(t-\zeta)} - \frac{z}{b(t-\zeta)}} d\zeta.$$

Assuming that $q(\zeta, \varphi, \psi)$ is independent of φ and ψ , ($=q(\zeta)$) and putting $t-\zeta = \xi$, we get

$$C = \int_0^t q(t-\xi) \frac{e^{-\frac{z}{b\xi}}}{b\xi} \frac{\phi\left(\frac{x+l-u\xi}{\sqrt{4a\xi}}\right) - \phi\left(\frac{x-l-u\xi}{\sqrt{4a\xi}}\right)}{2} \frac{\phi\left(\frac{y+m}{\sqrt{4a\xi}}\right) - \phi\left(\frac{y-m}{\sqrt{4a\xi}}\right)}{2} d\xi \quad 1-5-5.$$

When m is sufficiently large, the variation of $q(\varsigma)$ is slow and the instant of concentration measurement is sufficiently large, the concentration is given by

$$C \doteq q \int_0^\infty \frac{e^{-\frac{z}{B}}}{B} \frac{\phi\left(\frac{x+l-u\xi}{\sqrt{A}}\right) - \phi\left(\frac{x-l-u\xi}{\sqrt{A}}\right)}{2} d\xi \quad 1-5-6.$$

I-6 3-dimensional source

The initial distribution of concentration in 3-dimensional sources are various, but following two cases are common: the case in which the initial concentrations distribute uniformly within a certain domain in horizontal planes but are decreasing exponentially in vertical direction, and the case in which they distribute as Gaussian law in horizontal planes, but are decreasing exponentially in vertical direction. Generally in the former case, the sources are instantaneous, but in the latter case, they are not only instantaneous but also continuous.

I-6-1 Instantaneous 3-dimensional source(A)

This case corresponds to that in which a single large gas vessel is exploded. The initial concentrations distribute uniformly in horizontal planes within a circle radius R and are decreasing as $\exp(-\sigma/r)$ in vertical direction. The current coordinates in that domain be φ, ψ and σ . (Fig. 1-4).

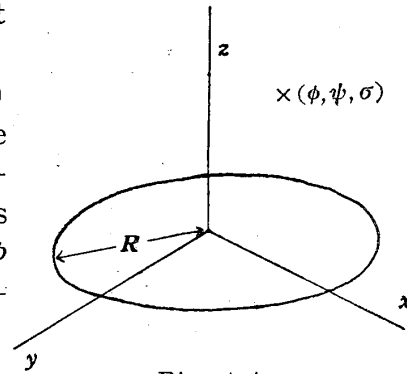


Fig. 1-4

The concentration is given by

$$C = \int_0^\infty d\sigma \int \int q(\varphi, \psi, \sigma) \frac{1}{4ab\pi t^2} e^{-\frac{(x-ut-\varphi)^2 + (y-\psi)^2}{4at} - \frac{z+\sigma}{bt}} J_0\left(i \frac{2\sqrt{\sigma z}}{bt}\right) d\varphi d\psi.$$

Putting $q = Q \exp(-\sigma/r)$, where Q is a constant,

$$C = Q \int_0^\infty e^{-\frac{\sigma}{r} - \frac{z+\sigma}{bt}} J_0\left(i \frac{2\sqrt{\sigma z}}{bt}\right) d\sigma \int \int \frac{e^{-\frac{(x-ut-\varphi)^2 + (y-\psi)^2}{4at}}}{4ab\pi t^2} d\varphi d\psi.$$

The integral of concentration over the whole space should be equal to the total amount M which is independent of time.

$$\begin{aligned} M &= \int_{-\infty}^\infty dy \int_{-\infty}^\infty dx \int_0^\infty C dz \\ &= Q \int_0^\infty d\sigma \int_0^\infty \frac{e^{-\frac{\sigma}{r} - \frac{z+\sigma}{bt}} J_0\left(i \frac{2\sqrt{\sigma z}}{B}\right)}{B} dz \int \int d\varphi d\psi \int_{-\infty}^\infty \frac{e^{-\frac{(x-ut-\varphi)^2}{A}}}{\sqrt{A\pi}} dx \\ &\quad \times \int_{-\infty}^\infty \frac{e^{-\frac{(y-\psi)^2}{A}}}{\sqrt{A\pi}} dy \end{aligned} \quad 1-6-1.$$

Using the formula

$$\int_0^\infty e^{-ax} J_0(bx) dx = \frac{1}{\sqrt{a^2 + b^2}} \quad (5) \quad 1-6-2,$$

we get

$$\int_0^\infty \frac{e^{-\frac{\sigma}{\gamma} - \frac{\sigma}{B} - \frac{z}{B}}}{B} J_0\left(i \frac{2\sqrt{\sigma z}}{B}\right) dz = \frac{e^{-\left(\frac{1}{\gamma} + \frac{1}{B}\right)\sigma}}{B} \int_0^\infty e^{-\frac{z}{B}} J_0\left(i \frac{2\sqrt{\sigma z}}{B}\right) dz.$$

Putting $\sqrt{z} = \nu$,

$$= \frac{e^{-\left(\frac{1}{\gamma} + \frac{1}{B}\right)\sigma}}{B} 2 \int_0^\infty e^{-\frac{\nu^2}{B}} J_0\left(i \frac{2\sqrt{\sigma}}{B} \nu\right) \nu d\nu \quad 1-6-3,$$

while

$$\int_0^\infty e^{-a^2 t^2} J_0(bt) t dt = \frac{1}{2a^2} e^{-\frac{b^2}{4a^2}} \quad (6) \quad 1-6-4,$$

so we get

$$\int_0^\infty e^{-\frac{\nu^2}{B}} J_0\left(i \frac{2\sqrt{\sigma}}{B} \nu\right) \nu d\nu = \frac{B}{2} e^{-\frac{B}{4} \left(-\frac{4\sigma}{B^2}\right)} = \frac{B}{2} e^{\frac{\sigma}{B}}.$$

Then (1-6-3) reduces to

$$= \frac{e^{-\left(\frac{1}{\gamma} + \frac{1}{B}\right)\sigma}}{B} 2 \frac{B}{2} e^{\frac{\sigma}{B}} = e^{-\left(\frac{1}{\gamma} + \frac{1}{B} - \frac{1}{B}\right)\sigma} = e^{-\frac{\sigma}{\gamma}} \quad 1-6-5.$$

So we obtain finally

$$M = Q \int_0^\infty e^{-\frac{\sigma}{\gamma}} d\sigma \int \int d\varphi d\psi = Q \gamma \pi R^2.$$

Therefore,

$$Q = M / (\gamma \pi R^2),$$

$$C = \frac{M}{\gamma \pi R^2} e^{-\frac{z}{B}} \int_0^\infty e^{-\left(\frac{1}{\gamma} + \frac{1}{B}\right)\sigma} J_0\left(i \frac{2\sqrt{\sigma z}}{B}\right) d\sigma \int_{-R}^R \frac{e^{-\frac{(x-ut-\varphi)^2}{A}}}{AB\pi} d\varphi \\ \times \int_{-\sqrt{R^2-\varphi^2}}^{\sqrt{R^2-\varphi^2}} e^{-\frac{(y-\psi)^2}{A}} d\psi \quad 1-6-6.$$

Putting then $\sqrt{\sigma} = \nu$ and using (1-6-4), we get

$$\int_0^\infty e^{-\left(\frac{1}{\gamma} + \frac{1}{B}\right)\sigma} J_0\left(i \frac{2\sqrt{\sigma z}}{B}\right) d\sigma = 2 \int_0^\infty e^{-\left(\frac{1}{\gamma} + \frac{1}{B}\right)\nu^2} J_0\left(i \frac{2\sqrt{z}}{B} \nu\right) d\nu \\ = 2 \frac{1}{2\left(\frac{1}{\gamma} + \frac{1}{B}\right)} e^{-\frac{\frac{4z}{B}}{4\left(\frac{1}{\gamma} + \frac{1}{B}\right)}} = \frac{\gamma B}{\gamma + B} e^{\frac{\gamma z}{B(\gamma + B)}} \quad 1-6-7.$$

On the other hand,

$$\int_{-\sqrt{R^2-\varphi^2}}^{\sqrt{R^2-\varphi^2}} e^{-\frac{(y-\psi)^2}{A}} d\psi = \frac{\sqrt{A\pi}}{2} \left[\phi\left(\frac{y+\sqrt{R^2-\varphi^2}}{\sqrt{A}}\right) - \phi\left(\frac{y-\sqrt{R^2-\varphi^2}}{\sqrt{A}}\right) \right] \quad 1-6-8,$$

so we get

$$\begin{aligned}
C &= \frac{M}{\gamma\pi R^2} e^{-\frac{z}{B}} \frac{\gamma B}{\gamma+B} e^{-\frac{\gamma z}{B(\gamma+B)}} \frac{1}{B} \int_{-R}^R \frac{e^{-\frac{(x-ut-\varphi)^2}{A}}}{\sqrt{A\pi}} \\
&\quad \times \frac{\phi\left(\frac{y+\sqrt{R^2-\varphi^2}}{A}\right) - \phi\left(\frac{y-\sqrt{R^2-\varphi^2}}{A}\right)}{2} d\varphi \\
&= \frac{M}{\pi R^2} \frac{1}{\gamma+B} e^{-\frac{z}{\gamma+B}} \int_{-R}^R \frac{e^{-\frac{(x-ut-\varphi)^2}{A}}}{\sqrt{A\pi}} \frac{\phi\left(\frac{y+\sqrt{R^2-\varphi^2}}{A}\right) - \phi\left(\frac{y-\sqrt{R^2-\varphi^2}}{A}\right)}{2} d\varphi
\end{aligned}$$

1-6-9.

$$Ct = \frac{M}{60\gamma\pi R^2} \int_0^\infty dt \int_0^\infty e^{-\frac{\sigma}{\gamma} - \frac{\sigma+z}{B}} J_0\left(i \frac{2\sqrt{\sigma z}}{B}\right) d\sigma \int_{\bigcirc} \frac{e^{-\frac{(x-ut-\varphi)^2 + (y-\psi)^2}{A}}}{AB\pi} d\varphi d\psi.$$

Putting $t_0 = \sqrt{\left(\frac{x-\varphi}{u}\right)^2 + \left(\frac{4a}{u^2}\right)^2} - \frac{4a}{u^2}$, $4at_0 = A_0$ and $bt_0 = B_0$ 1-6-10,

$$Ct = \frac{M}{60\gamma\pi R^2} \int_0^\infty e^{-\frac{\sigma}{\gamma} - \frac{z+\sigma}{B}} J_0\left(i \frac{2\sqrt{\sigma z}}{B}\right) d\sigma \int_{\bigcirc} \frac{e^{-\frac{(y-\psi)^2}{A_0}}}{A_0 B_0 \pi} d\varphi d\psi \int_e^\infty e^{-\frac{(x-ut-\varphi)^2}{A_0}} dt.$$

Putting $\sqrt{\sigma} = \nu$ and using (1-6-4),

$$\begin{aligned}
Ct &= \frac{M}{60\gamma\pi R^2} \int_{\bigcirc} \frac{\gamma B_0}{\gamma+B_0} e^{-\frac{z}{\gamma+B_0}} \frac{1}{A_0 B_0 \pi} e^{-\frac{(y-\psi)^2}{A_0}} \frac{\sqrt{A_0 \pi}}{u} \frac{1 + \phi\left(\frac{x-\varphi}{\sqrt{A_0}}\right)}{2} d\varphi d\psi \\
&= \frac{M}{60\pi R^2} \int_{-R}^R \frac{e^{-\frac{z}{\gamma+B_0}}}{\gamma+B_0} \frac{1}{u} \frac{1 + \phi\left(\frac{x-\varphi}{\sqrt{A_0}}\right)}{2} \frac{\phi\left(\frac{y+\sqrt{R^2-\varphi^2}}{\sqrt{A_0}}\right) - \phi\left(\frac{y-\sqrt{R^2-\varphi^2}}{\sqrt{A_0}}\right)}{2} d\varphi
\end{aligned}$$

1-6-11.

When we adopt the approximate value for the term $\phi\left(\frac{x-\varphi}{\sqrt{A_0}}\right)$, we obtain

$$x \geq R, \quad Ct = \frac{M}{60\pi R^2} \int_{-R}^R \frac{e^{-\frac{z}{\gamma+B_0}}}{\gamma+B_0} \frac{1}{u} \frac{\phi\left(\frac{y+\sqrt{R^2-\varphi^2}}{\sqrt{A_0}}\right) - \phi\left(\frac{y-\sqrt{R^2-\varphi^2}}{\sqrt{A_0}}\right)}{2} d\varphi$$

1-6-12,

$$R \geq x \geq -R, \quad Ct = \frac{M}{60\pi R^2} \int_{-R}^x \frac{e^{-\frac{z}{\gamma+B_0}}}{\gamma+B_0} \frac{1}{u} \frac{\phi\left(\frac{y+\sqrt{R^2-\varphi^2}}{\sqrt{A_0}}\right) - \phi\left(\frac{y-\sqrt{R^2-\varphi^2}}{\sqrt{A_0}}\right)}{2} d\varphi$$

1-6-13.

The term $\exp\{-z/(\gamma+B_0)\}/(\gamma+B_0)$ does not vary so rapidly as the other terms, so we put

$$t_0' = \sqrt{\left(\frac{x}{u}\right)^2 + \left(\frac{4a}{u^2}\right)^2} - \frac{4a}{u^2}, \quad 4at_0' = A_0' \text{ and } bt_0' = B_0' \quad 1-6-14,$$

$$x \geq R, \quad Ct = \frac{M}{60\pi R^2} \frac{e^{-\frac{z}{\gamma+B_0'}}}{\gamma+B_0'} \frac{1}{u} \int_{-R}^R \frac{\phi\left(\frac{y+\sqrt{R^2-\varphi^2}}{\sqrt{A_0}}\right) - \phi\left(\frac{y-\sqrt{R^2-\varphi^2}}{\sqrt{A_0}}\right)}{2} d\varphi.$$

Putting $\varphi = R \sin \theta$,

$$= \frac{M}{60\pi R^2} \frac{e^{-\frac{z}{\gamma+B_0'}}}{\gamma+B_0'} \frac{1}{u} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\phi\left(\frac{y+R\cos\theta}{\sqrt{A_0}}\right) - \phi\left(\frac{y-R\cos\theta}{\sqrt{A_0}}\right)}{2} d\theta \quad 1-6-15,$$

$$R \geq x \geq -R, \quad Ct = \frac{M}{60\pi R^2} \frac{e^{-\frac{z}{\gamma+B_0'}}}{\gamma+B_0'} \frac{1}{u} \int_{-\frac{\pi}{2}}^{\theta} \frac{\phi\left(\frac{y+R\cos\theta}{\sqrt{A_0}}\right) - \phi\left(\frac{y-R\cos\theta}{\sqrt{A_0}}\right)}{2} d\theta \quad 1-6-16,$$

where $x = R \sin \bar{\theta}$.

In many of the experiments with these sources, we could obtain results which were considerably accurate.

1-6-2 Instantaneous 3-dimensional source (B)

This case corresponds to that in which many gas vessels are simultaneously fallen to one object concentratively. The initial distributions of concentrations $q(\varphi, \psi, \sigma)$ are assumed to be

$$q(\varphi, \psi, \sigma) = Q \exp\left(-\frac{\varphi^2}{\mu} - \frac{\psi^2}{\nu} - \frac{\sigma}{\gamma}\right) \quad 1-6-17,$$

where μ, ν, γ are constants and the former two of them are given by

$$\mu = 1.099 H_{\parallel}^2 \text{ and } \nu = 1.099 H_{\perp}^2 \quad 1-6-18,$$

where H_{\parallel} and H_{\perp} are twice as large as the probable deviations of the scattering of vessels in the direction parallel to the wind and that perpendicular to the wind respectively.

The concentration is given by

$$C = \frac{Q}{4ab\pi} \int_0^\infty d\sigma \int_{-\infty}^\infty d\psi \int_{-\infty}^\infty d\varphi \frac{e^{-\frac{(x-ut-\varphi)^2}{4at} - \frac{\varphi^2}{\mu} - \frac{(y-\psi)^2}{4at} - \frac{\psi^2}{\nu} - \frac{\sigma+z}{B} - \frac{\sigma}{\gamma}}}{t^2} J_0\left(i\frac{2\sqrt{\sigma z}}{bt}\right) d\varphi \quad 1-6-19.$$

The total amount be M , so we get

$$M = Q \int_0^\infty dz \int_{-\infty}^\infty dy \int_{-\infty}^\infty dx \int_0^\infty d\sigma \int_{-\infty}^\infty d\psi \int_{-\infty}^\infty d\varphi \frac{e^{-\frac{(x-ut-\varphi)^2}{A} - \frac{\varphi^2}{\mu} - \frac{(y-\psi)^2}{A} - \frac{\psi^2}{\nu} - \frac{\sigma+z}{B} - \frac{\sigma}{\gamma}}}{AB\pi} \times J_0\left(i\frac{2\sqrt{\sigma z}}{B}\right) d\varphi.$$

Putting $\sqrt{z} = \nu$ and using (1-6-4),

$$M = Q \int_0^\infty d\sigma \int_{-\infty}^\infty d\psi \int_{-\infty}^\infty d\varphi e^{-\frac{\varphi^2}{\mu} - \frac{\psi^2}{\nu} - \frac{\sigma}{\gamma}} d\varphi = Q\sqrt{\mu\pi}\sqrt{\nu\pi}\gamma = Q\pi\gamma\sqrt{\mu\nu}.$$

Therefore,

$$Q = M/(\pi\sqrt{\mu\nu}\gamma) \quad 1-6-20.$$

Then we obtain

$$C = \frac{M}{\pi\sqrt{\mu\nu}\gamma} \int_0^\infty d\sigma \int_{-\infty}^\infty d\psi \int_{-\infty}^\infty d\varphi \frac{e^{-\frac{(x-ut-\varphi)^2}{A} - \frac{\varphi^2}{\mu} - \frac{(y-\psi)^2}{A} - \frac{\psi^2}{\nu} - \frac{\sigma+z}{B} - \frac{\sigma}{\gamma}}}{AB\pi} \times J_0\left(i\frac{2\sqrt{\sigma z}}{B}\right) d\varphi \quad 1-6-21.$$

After some simple calculations, we get

$$C = \frac{M}{\pi\sqrt{\mu\nu\gamma}} \frac{1}{AB\pi} e^{-\frac{(x-ut)^2}{\mu+A}} \sqrt{\frac{\mu A}{\mu+A}} \pi e^{-\frac{y^2}{\nu+A}} \sqrt{\frac{\nu A}{\nu+A}} \pi e^{-\frac{z}{\gamma+B}} \sqrt{\frac{\gamma B}{\gamma+B}}$$

$$= \frac{Me^{-\frac{(x-ut)^2}{\mu+A} - \frac{y^2}{\nu+A} - \frac{z}{\gamma+B}}}{\pi\sqrt{\mu+A}\sqrt{\nu+A}(\gamma+B)} \quad 1-6-22.$$

$$Ct = \frac{1}{60} \int_0^\infty \frac{M}{\pi\sqrt{\mu\nu\gamma}} dt \int_0^\infty e^{-\frac{\sigma+z}{B} - \frac{z}{\gamma}} J_0\left(i \frac{2\sqrt{\sigma z}}{B}\right) d\sigma \int_{-\infty}^\infty e^{-\frac{(y-\psi)^2}{A} - \frac{\psi^2}{\nu}} d\psi$$

$$\times \int_{-\infty}^\infty e^{-\frac{(x-ut-\varphi)^2}{A} - \frac{\varphi^2}{\mu}} d\varphi \quad 1-6-23.$$

Putting $t_0 = \sqrt{\left(\frac{x-\varphi}{u}\right)^2 + \left(\frac{4a}{u^2}\right)^2} - \frac{4a}{u^2}$, $4at_0 = A_0$ and $bt_0 = B_0$,

$$Ct = \frac{1}{60} \frac{M}{\pi\sqrt{\mu\nu\gamma}} \int_0^\infty e^{-\frac{\sigma+z}{B_0} - \frac{z}{\gamma}} J_0\left(i \frac{2\sqrt{\sigma z}}{B_0}\right) d\sigma \int_{-\infty}^\infty e^{-\frac{(y-\psi)^2}{A_0} - \frac{\psi^2}{\nu}} d\psi$$

$$\times \int_{-\infty}^\infty \frac{e^{-\frac{\varphi^2}{\mu}}}{\sqrt{A_0}\pi B_0 u} \frac{1 + \phi\left(\frac{x-\varphi}{\sqrt{A_0}}\right)}{2} d\varphi$$

$$= \frac{M}{60\pi\sqrt{\mu u}} \int_{-\infty}^\infty \frac{1}{\sqrt{\nu+A_0}} \frac{1}{\gamma+B_0} e^{-\frac{y^2}{\gamma+A_0} - \frac{z}{\gamma+B_0} - \frac{\varphi^2}{\mu}} \frac{1 + \phi\left(\frac{x-\varphi}{\sqrt{A_0}}\right)}{2} d\varphi.$$

Adopting the approximate value for the term $\phi\left(\frac{x-\varphi}{\sqrt{A_0}}\right)$, we get

$$Ct = \frac{M}{60\pi\sqrt{\mu u}} \int_{-\infty}^x \frac{1}{\sqrt{\nu+A_0}(\gamma+B_0)} e^{-\frac{y^2}{\gamma+A_0} - \frac{z}{\gamma+B_0} - \frac{\varphi^2}{\mu}} d\varphi.$$

Putting further $t_0' = \sqrt{\left(\frac{x}{u}\right)^2 + \left(\frac{4a}{u^2}\right)^2} - \frac{4a}{u^2}$

$$Ct = \frac{M}{60\pi\sqrt{\mu u}} \frac{e^{-\frac{y^2}{\nu+A_0'} - \frac{z}{\gamma+B_0'}}}{\sqrt{\nu+A_0'}(\gamma+B_0')} \frac{1 + \phi\left(\frac{x}{\sqrt{\mu}}\right)}{2} \quad 1-6-24.$$

I-6-3 Instantaneous 3-dimensional source (C)

This case corresponds to that in which many gas vessels are exploded uniformly over a certain region. The initial distributions of concentration are assumed to be uniform in horizontal planes within a rectangular region ($2l \times 2l'$) and decreasing as $\exp(-\sigma/r)$ in vertical direction. (Fig. 1-5). The concentration is given by

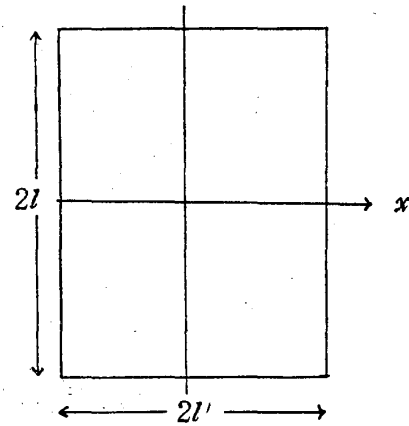


Fig. 1-5

$$C = \int_0^\infty d\sigma \int_{\square} q(\varphi, \psi, \sigma) \frac{1}{4ab\pi t^2} e^{-\frac{(x-ut-\varphi)^2 + (y-\psi)^2}{4at} - \frac{z+\sigma}{bt}} J_0\left(i \frac{2\sqrt{\sigma z}}{bt}\right) d\varphi d\psi.$$

The total amount be M , so we get

$$M = Q \int_0^\infty e^{-\left(\frac{1}{\gamma} + \frac{1}{B}\right)\sigma} d\sigma \int_{-l}^l d\psi \int_{-l'}^{l'} d\varphi \int_0^\infty \frac{e^{-\frac{z}{B}}}{B} J_0\left(i \frac{2\sqrt{\sigma z}}{B}\right) dz \int_{-\infty}^\infty \frac{e^{-\frac{(x-ut-\varphi)^2}{A}}}{\sqrt{A\pi}} dx \\ \times \int_{-\infty}^\infty \frac{e^{-\frac{(y-\psi)^2}{A}}}{\sqrt{A\pi}} dy = Q \int_0^\infty e^{-\frac{\sigma}{\gamma} - \frac{\sigma}{B} + \frac{\sigma}{B}} d\sigma \int_{-l}^l d\psi \int_{-l'}^{l'} d\varphi = Q \int_0^\infty e^{-\frac{\sigma}{\gamma}} 2l \cdot 2l' d\sigma \\ = 2l \cdot 2l' \cdot \gamma \cdot Q$$

Therefore,

$$Q = M / (2l \cdot 2l' \cdot \gamma) \quad 1-6-26.$$

Then we get

$$C = \frac{M}{2l \cdot 2l' (\gamma + B)} e^{-\frac{z}{\gamma + B}} \frac{\phi\left(\frac{x-ut+l}{\sqrt{A}}\right) - \phi\left(\frac{x-ut-l}{\sqrt{A}}\right)}{2} \frac{\phi\left(\frac{y+l}{\sqrt{A}}\right) - \phi\left(\frac{y-l}{\sqrt{A}}\right)}{2} \quad 1-6-27.$$

$$Ct = \frac{Q}{60} \int_0^\infty dt \int_0^\infty d\sigma \int_{-l}^l d\psi \int_{-l'}^{l'} d\varphi \frac{e^{-\frac{(x-ut-\varphi)^2 + (y-\psi)^2}{A} - \frac{z+\sigma}{B} - \frac{\sigma}{\gamma}}}{AB\pi} J_0\left(i \frac{2\sqrt{\sigma z}}{B}\right) d\varphi.$$

Putting $t_0 = \sqrt{\left(\frac{x-\varphi}{u}\right)^2 + \left(\frac{4a}{u^2}\right)^2} - \frac{4a}{u^2}$ 1-6-28,

$$Ct = \frac{Q}{60} \int_{-l'}^{l'} d\varphi \int_0^\infty \frac{e^{-\frac{z+\sigma}{B_0} - \frac{\sigma}{\gamma}}}{B_0} J_0\left(i \frac{2\sqrt{\sigma z}}{B_0}\right) d\sigma \int_{-l}^l \frac{e^{-\frac{(y-\psi)^2}{A_0}}}{\sqrt{A_0\pi}} d\psi \int_0^\infty \frac{e^{-\frac{(x-ut-\varphi)^2}{A_0}}}{\sqrt{A_0\pi}} dt \\ = \frac{M}{60 \cdot 2l \cdot 2l' u} \int_{-l'}^{l'} \frac{e^{-\frac{z}{\gamma+B_0}}}{\gamma+B_0} \frac{\phi\left(\frac{y+l}{\sqrt{A_0}}\right) - \phi\left(\frac{y-l}{\sqrt{A_0}}\right)}{2} \frac{1 + \phi\left(\frac{x-\varphi}{\sqrt{A_0}}\right)}{2} d\varphi.$$

Adopting the approximate value for the term $\phi\left(\frac{x-\varphi}{\sqrt{A_0}}\right)$, we get

$$x \geq l' \quad Ct = \frac{1}{60} \frac{M}{2l \cdot 2l' u} \int_{-l'}^{l'} \frac{e^{-\frac{z}{\gamma+B_0}}}{\gamma+B_0} \frac{\phi\left(\frac{y+l}{\sqrt{A_0}}\right) - \phi\left(\frac{y-l}{\sqrt{A_0}}\right)}{2} d\varphi \quad 1-6-29,$$

$$l' \geq x \geq -l' \quad Ct = \frac{1}{60} \frac{M}{2l \cdot 2l' u} \int_{-l'}^x \frac{e^{-\frac{z}{\gamma+B_0}}}{\gamma+B_0} \frac{\phi\left(\frac{y+l}{\sqrt{A_0}}\right) - \phi\left(\frac{y-l}{\sqrt{A_0}}\right)}{2} d\varphi \quad 1-6-30.$$

Putting further $t_0' = \sqrt{\left(\frac{x}{u}\right)^2 + \left(\frac{4a}{u^2}\right)^2} - \frac{4a}{u^2}$, we get

$$x \geq l' \quad Ct = \frac{M}{60 \cdot 2l \cdot 2l' u} \frac{e^{-\frac{z}{\gamma+B_0'}}}{\gamma+B_0'} \frac{\phi\left(\frac{y+l}{\sqrt{A_0'}}\right) - \phi\left(\frac{y-l}{\sqrt{A_0'}}\right)}{2} \times 2l' \quad 1-6-31,$$

$$l' \geq x \geq -l' \quad Ct = \frac{M}{60 \cdot 2l \cdot 2l' u} \frac{e^{-\frac{z}{\gamma+B_0'}}}{\gamma+B_0'} \frac{\phi\left(\frac{y+l}{\sqrt{A_0'}}\right) - \phi\left(\frac{y-l}{\sqrt{A_0'}}\right)}{2} \times (l' + x) \quad 1-6-32.$$

I-6-4 Continuous 3-dimensional source

This case corresponds to that in which many gas vessels are thrown and exploded concentratively to one object during a certain period. The initial distributions of concentration at every instant $q(\varphi, \psi, \sigma, \varsigma)$ is assumed to be

$$q(\varphi, \psi, \sigma, \varsigma) = Q(\varsigma) \exp\left(-\frac{\varphi^2}{\mu} - \frac{\psi^2}{\nu} - \frac{\sigma}{\gamma}\right), \text{ where}$$

$$\mu = 1.099 H_0^2 \quad \text{and} \quad \nu = 1.099 H_1^2.$$

$$C = \frac{1}{4ab\pi} \int_0^\infty d\sigma \int_{-\infty}^\infty d\psi \int_{-\infty}^\infty d\varphi \int_0^t q(\varphi, \psi, \sigma, \varsigma) \frac{e^{-\frac{(x-u(t-\varsigma)-\varphi)^2 + (y-\psi)^2}{4a(t-\varsigma)} - \frac{\sigma+z}{b(t-\varsigma)}}}{(t-\varsigma)^2} \times J_0\left(i \frac{2\sqrt{\sigma z}}{b(t-\varsigma)}\right) d\varsigma \quad 1-6-33.$$

The integrated value of C over the whole space must be equal to $\int_0^t P(\varsigma) d\varsigma$ for every t , where $P(\varsigma)$ is the amount emitted at every instant.

$$\begin{aligned} \int_0^t P(\varsigma) d\varsigma &= \int_0^t Q(\varsigma) d\varsigma \int_0^\infty d\sigma \int_{-\infty}^\infty d\psi \int_{-\infty}^\infty d\varphi \int_{-\infty}^\infty \frac{e^{-\frac{(x-u(t-\varsigma)-\varphi)^2}{4a(t-\varsigma)} - \frac{\varphi^2}{\mu}}}{\sqrt{4a(t-\varsigma)\pi}} \frac{e^{-\frac{(y-\psi)^2}{4a(t-\varsigma)} - \frac{\psi^2}{\nu}}}{\sqrt{4a(t-\varsigma)\pi}} \frac{e^{-\frac{\sigma+z}{b(t-\varsigma)} - \frac{\sigma}{\gamma}}}{b(t-\varsigma)} J_0\left(i \frac{2\sqrt{\sigma z}}{b(t-\varsigma)}\right) dx \\ &\times \int_{-\infty}^\infty \frac{e^{-\frac{(y-\psi)^2}{4a(t-\varsigma)} - \frac{\psi^2}{\nu}}}{\sqrt{4a(t-\varsigma)\pi}} dy \int_0^\infty \frac{e^{-\frac{\sigma+z}{b(t-\varsigma)} - \frac{\sigma}{\gamma}}}{b(t-\varsigma)} J_0\left(i \frac{2\sqrt{\sigma z}}{b(t-\varsigma)}\right) dz \\ &= \int_0^t Q(\varsigma) d\varsigma \int_0^\infty d\sigma \int_{-\infty}^\infty d\psi \int_{-\infty}^\infty e^{-\frac{\varphi^2}{\mu} - \frac{\psi^2}{\nu} - \frac{\sigma}{\gamma}} d\varphi = \sqrt{\mu\nu\gamma\pi} \int_0^t Q(\varsigma) d\varsigma. \end{aligned}$$

Therefore,

$$Q(\varsigma) = P(\varsigma) / (\pi\gamma\sqrt{\mu\nu}) \quad 1-6-34.$$

Then we get

$$\begin{aligned} C &= \int_0^t \frac{P(\varsigma)}{\sqrt{\mu\nu\pi\gamma}} d\varsigma \int_{-\infty}^\infty \frac{e^{-\frac{(x-u(t-\varsigma)-\varphi)^2}{4a(t-\varsigma)} - \frac{\varphi^2}{\mu}}}{\sqrt{4a(t-\varsigma)\pi}} d\varphi \int_{-\infty}^\infty \frac{e^{-\frac{(y-\psi)^2}{4a(t-\varsigma)} - \frac{\psi^2}{\nu}}}{\sqrt{4a(t-\varsigma)\pi}} d\psi \\ &\times \int_0^\infty \frac{e^{-\frac{\sigma+z}{b(t-\varsigma)} - \frac{\sigma}{\gamma}}}{b(t-\varsigma)} J_0\left(i \frac{2\sqrt{\sigma z}}{b(t-\varsigma)}\right) d\sigma \\ &= \int_0^t \frac{P(\varsigma)}{\pi\sqrt{\mu+A}\sqrt{\nu+A}(\gamma+B)} e^{-\frac{(x-u(t-\varsigma))^2}{\mu+A} - \frac{y^2}{\nu+A} - \frac{z}{\gamma+B}} d\varsigma, \end{aligned}$$

where $A=4a(t-\varsigma)$, $B=b(t-\varsigma)$.

Putting $t-\varsigma=\xi$,

$$C = \int_0^t \frac{P(t-\xi)}{\pi\sqrt{\mu+A}\sqrt{\nu+A}(\gamma+B)} e^{-\frac{(x-u\xi)^2}{\mu+A} - \frac{y^2}{\nu+A} - \frac{z}{\gamma+B}} d\xi \quad 1-6-35.$$

The duration of the explosion be λ (sec.) and we assume that $P(\varsigma)$ is constant ($=D$) during that period,

$$\begin{aligned} P(\varsigma) &= D & 0 \leq \varsigma \leq \lambda, \\ P(\varsigma) &= 0 & \lambda \leq \varsigma. \end{aligned}$$

So we get

$$C = \frac{D}{\pi} \int_0^t \frac{1}{\sqrt{\mu+A}\sqrt{\nu+A}(\gamma+B)} e^{-\frac{(x-u\xi)^2}{\mu+A} - \frac{y^2}{\nu+A} - \frac{z}{\gamma+B}} d\xi, \quad 0 \leq t \leq \lambda \quad 1-6-36,$$

$$= \frac{D}{\pi} \int_{t-\lambda}^t \frac{1}{\sqrt{\mu+A}\sqrt{\nu+A}(\gamma+B)} e^{-\frac{(x-u\xi)^2}{\mu+A} - \frac{y^2}{\nu+A} - \frac{z}{\gamma+B}} d\xi, \quad \lambda \leq t \quad 1-6-37.$$

Putting that

$$\begin{aligned} \xi_0 &= x/u & x \geq 0, \\ \xi_0 &= 0 & x \leq 0, \end{aligned}$$

and $A_0 = 4a\xi_0$, $B_0 = b\xi_0$, we get

$$C = \frac{D}{u\sqrt{\pi} \sqrt{\nu+A_0}(\gamma+B_0)} e^{-\frac{y^2}{\nu+A_0} - \frac{z}{\gamma+B_0}} \frac{\phi\left(\frac{x}{\sqrt{\mu+A_0}}\right) - \phi\left(\frac{x-ut}{\sqrt{\mu+A_0}}\right)}{2} \quad 0 \leq t \leq \lambda \quad 1-6-38,$$

$$= \frac{D}{u\sqrt{\pi} \sqrt{\nu+A_0}(\gamma+B_0)} e^{-\frac{y^2}{\nu+A_0} - \frac{z}{\gamma+B_0}} \frac{\phi\left(\frac{x-ut-u\lambda}{\sqrt{\mu+A_0}}\right) - \phi\left(\frac{x-ut}{\sqrt{\mu+A_0}}\right)}{2} \quad \lambda \leq t \quad 1-6-39.$$

$$Ct = \frac{1}{60} \int_0^\infty dt \int_0^\infty d\sigma \int_{-\infty}^\infty d\psi \int_{-\infty}^\infty d\varphi \int_0^\infty \frac{P(\varsigma)}{\sqrt{\mu\nu}\gamma\pi} \\ \times \frac{e^{-\frac{(x-u(t-\varsigma)-\varphi)^2}{A} - \frac{\varphi^2}{\mu} e^{-\frac{(y-\psi)^2}{A} - \frac{\psi^2}{\nu} e^{-\frac{\sigma+z}{B} - \frac{\sigma}{\gamma}}}}}{AB\pi} J_0\left(i\frac{2\sqrt{\sigma z}}{B}\right) d\varsigma.$$

Changing the order of integration for ς and t and putting $t-\varsigma = \xi$, we get

$$Ct = \frac{1}{60} \int_0^\infty d\sigma \int_{-\infty}^\infty d\psi \int_{-\infty}^\infty d\varphi \int_0^\infty P(\varsigma) d\varsigma \int_\varsigma^\infty ((t-\varsigma)) dt \\ = \frac{1}{60} \int_0^\infty d\sigma \int_{-\infty}^\infty d\psi \int_{-\infty}^\infty d\varphi \int_0^\infty P(\varsigma) d\varsigma \int_0^\infty ((\xi)) d\xi \\ = \frac{1}{60} \int_0^\infty P(\varsigma) d\varsigma \int_0^\infty d\sigma \int_{-\infty}^\infty d\varphi \int_{-\infty}^\infty d\psi \int_0^\infty \frac{1}{\sqrt{\mu\nu}\gamma\pi} \\ \times \frac{e^{-\frac{(x-u\xi-\varphi)^2}{A} - \frac{\varphi^2}{\mu} e^{-\frac{(y-\psi)^2}{A} - \frac{\psi^2}{\nu} e^{-\frac{\sigma+z}{B} - \frac{\sigma}{\gamma}}}}}{AB\pi} J_0\left(i\frac{2\sqrt{\sigma z}}{B}\right) d\xi.$$

Putting $\xi_0 = \sqrt{\left(\frac{x-\varphi}{u}\right)^2 + \left(\frac{4a}{u^2}\right)} - \frac{4a}{u^2}$ and the total amount be M ,

$$M = D\lambda = \int_0^\infty P(\varsigma) d\varsigma,$$

so,

$$Ct = \frac{M}{60} \frac{1}{\sqrt{\mu\nu}\gamma\pi} \int_{-\infty}^\infty e^{-\frac{\varphi^2}{\mu}} \frac{1}{u} \frac{1 + \phi\left(\frac{x-\varphi}{\sqrt{A_0}}\right)}{2} d\varphi \int_{-\infty}^\infty \frac{e^{-\frac{(y-\psi)^2}{A_0} - \frac{\psi^2}{\nu}}}{\sqrt{A_0}\pi} d\psi \\ \times \int_0^\infty \frac{e^{-\frac{\sigma+z}{B_0} - \frac{\sigma}{\gamma}}}{B_0} J_0\left(i\frac{2\sqrt{\sigma z}}{B}\right) d\sigma \\ = \frac{M}{60} \frac{1}{\sqrt{\mu}\pi u} \int_{-\infty}^\infty e^{-\frac{\varphi^2}{\mu}} \frac{1 + \phi\left(\frac{x-\varphi}{\sqrt{A_0}}\right)}{2} \frac{e^{-\frac{y^2}{\nu+A_0}} e^{-\frac{z}{\gamma+B_0}}}{\sqrt{\nu+A_0} \gamma+B_0} d\varphi.$$

Adopting the approximate value for the term $\phi\left(\frac{x-\varphi}{\sqrt{A_0}}\right)$,

$$Ct = \frac{M}{60} \frac{1}{\sqrt{\mu}\pi u} \int_{-\infty}^\infty e^{-\frac{\varphi^2}{\mu}} \frac{e^{-\frac{y^2}{\nu+A_0}} e^{-\frac{z}{\gamma+B_0}}}{\sqrt{\nu+A_0}(\gamma+B_0)} d\varphi.$$

Putting further

$$t_0' = \sqrt{\left(\frac{x}{u}\right)^2 + \left(\frac{4a}{u^2}\right)} - \frac{4a}{u^2} \quad x \geq 0, \\ t_0' = 0 \quad x \leq 0,$$

$$Ct = \frac{M}{60} \frac{1}{\sqrt{\mu\pi u}} \frac{e^{-\frac{y^2}{v+A_0'}} e^{-\frac{z}{\gamma+B_0'}}}{\sqrt{v+A_0'}(\gamma+B_0')} \int_{-\infty}^x e^{-\frac{\varphi^2}{\mu}} d\varphi$$

$$= \frac{M}{60} \frac{1}{\sqrt{\pi u}} \frac{e^{-\frac{y^2}{v+A_0'}} e^{-\frac{z}{\gamma+B_0'}}}{v+A_0' \gamma+B_0'} \frac{1+\Phi\left(\frac{x}{\sqrt{\mu}}\right)}{2} \quad 1-6-41. \quad 4)$$

II EXPERIMENT

II-1 Field experiment

The object of our research was to determine not only the spatial distributions of concentration but also its time changes. Though there were apparatus which determine the time changes of concentration directly, it was difficult to place scores or several hundreds of them in observation areas, from the economical point of view and because of their delicate characters. So we placed only few of them at some special places and at many other places we measured only the values of Ct . Comparing each experimental result with theoretical one, we examined the adequateness of the theory and determined experimental constants included in the theoretical formulae.

The quantities which we must determine are 1) the diffusion coefficients and their relations to the meteorological data, 2) the initial concentrations in the sources and 3) the efficiency of gasification.⁵⁾

The scales of the field experiments were very different according to purpose of the experiments; in a certain case the observation area was several meters square, and in other case it was several kilometers square, but generally the area was 100~300 meters square.

The measuring apparatus were placed in every 1 m. in the most dense cases, but were placed in every 10~20 m. for the region where the diffusion occurred considerably. As the diffusion occurs 3-dimensionally, we also measured the concentration at several heights (e.g. 0.5, 2 and 5 m.) from the ground.

II-2 The method of experiment appropriate to examine the theory

The experiments with instantaneous point sources are the most favorable to examine the adequateness of the theory, because it simplifies the phenomena. However, in order to regard the size of source as a point compared with the observation area, we can treat only a relatively small amount of matter. This causes necessarily that the concentration becomes smaller, and that in order to keep the accuracy of measurement in a certain degree, the observation area becomes narrower. This requires to make the size of source still smaller. On the other hand, for the measurement of the atmospheric diffusion, the observation area must be wide to some extent, so the amount of matter must be

4) This formula does not contain λ , namely it is independent of the duration of explosion.

5) The materials to be diffused partly decompose or polymerize when they are scattered by explosives or fuming agent, and are partly soaked into the ground, or partly fall on the ground as liquid-drops when they are emitted. In these cases, the efficiency of gasification is not 100%.

large. Generally, in order to disperse it, the necessary amount of the explosives becomes large and the size of source becomes unable to be regarded as a point. Therefore, the instantaneous point source is difficult to be realized.

When we use continuous point source, the concentrations at every instant are not generally so high that the apparatus to measure the concentrations can be placed only in a very near region from the source. At many other places, we can only measure Ct . As in this case the source is continuous, so the working period of gas-aspirators must be long, and this causes appearance of a factor, i.e. 'the shifting of the wind direction'.

In a short time interval as gas-cloud passes through the observation area, mean value of wind direction changes more slowly compared with period of its fluctuations. Concerning this effect, we want to discuss later in detail. (II-6-4 and II-6-5)

For the case of 3-dimensional sources, we must consider the initial concentration distributions in the sources.

II-3 Measuring apparatus

We used various measuring apparatus.⁶⁾ The method of determination of concentration was to catch the air loaded with gas and then determined the amount of gas by quantitative chemical analysis or electro-chemical analysis. In early period of the research, test papers were also used, but as they were not adequate for quantitative use, they were used only supplementarily in later period.

II-3-1 Accuracy of aspiration and conditions of design of aspirators.

When we aspirate air whose gas concentration is $C(t)$ (mg/m³) with the speed of $v(t)$ (l/min.) from the instant t_1 till t_2 , and absorption efficiency of absorbent in the sampling vessels be α , the amount of gas m (mg.) in the vessels are given by

$$m = \frac{\alpha}{60 \times 1000} \int_{t_1}^{t_2} C(t) v(t) dt \quad 2-1-1.$$

We choose the absorbent and the speed v so that α becomes almost equal to 1, and we make $v(t)$ constant as possible as we can, so we assume that v is constant and $\alpha=1$. Then we get

$$m = \frac{v}{60 \times 1000} \int_{t_1}^{t_2} C(t) dt \quad 2-1-2.$$

As m is determined by chemical analysis, the value of Ct during the period between t_1 and t_2 can be determined. When $t_1=0$ and $t_2=\infty$, we obtain the total value of Ct .

As there were many observation points in the field experiments, the number of sample was very large (400~600), so we adopted the volumetric analysis,

6) As the chemical quantitative micro-chemical analysis of aerosol was difficult and its accuracy was not high, we generally carried on experiments by using gas. So we want to describe here only the measuring apparatus for gas.

and for titration, we used micro-bullets, then the results of m were expressed by integer multiples of the smallest quantity which corresponded to one drop of the titrating solution. This smallest quantity becomes smaller as the normality of the solution becomes smaller, but in this case the end-point of the titration becomes indistinct. Therefore, in order to obtain accurate results of Ct , we must make m large, and for this purpose we must choose larger value of v .

For example, we consider the analysis of SO_2 . SO_2 gas is absorbed in a absorbent and is titrated with 1/50 N NaOH. One drop of the NaOH solution from the micro-bullets is 1/33 cc. and this corresponds to 0.019 mg. of SO_2 . As

$$Ct = m \cdot 1000/v \quad 2-1-3,$$

one drop corresponds to $Ct=10$, when we assume $v=2$ (l/min.). So the values of Ct are expressed only the integer multiples of 10. These values of Ct are inversely proportional to v , so it is very necessary to make v large.

The method of electric titration, that with photo-electrical colorimeter and that of electric conductivity were not necessarily adequate for field use.

In the experiments in wide areas, it took considerable long time till the concentration became negligible at leeward points, so the working period of aspirator had to be sufficiently long.

Therefore, the necessary conditions for the aspirator are 1) the constancy of the aspirating speed, 2) the larger aspirating speed and 3) the sufficiently long working period.

II-3-2 Motor-aspirator

Electrically driven aspirators were not tractable when we used them more than 10, and in a wide area, we had to make considerable wiring work, and we could not use them when the wire happened to be cut.

II-3-3 Water vessel aspirator

As is shown in Fig. 2-1, 10~15 liters of water is filled in vessel A, and connecting rubber tube is clamped by a pinch cock. When the cock is released by a relay, so the water in A flows into B, therefore the air loaded with gas is flowed into a sampling vessel which is connected with E. The aspirating speed is regulated by an orifice inserted in the tube C (2~8 l/min.). This was used only in the early period, because it did not satisfy the 3 conditions, and moreover, troubles caused by dust in water easily happened.

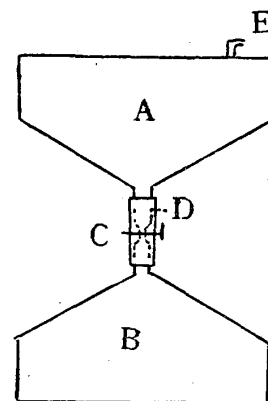


Fig. 2-1

II-3-4 Bellows-type aspirator

This aspirator has a bellows-type bag 100 liters in volume made of rubber, with rings of piano wire as inner supports, and at its bottom a weight of 20 kg. is attached. (Fig. 2-2) Initially the bag is folded and supported by a lever. When the lever is released by a relay, the bag extends owing to the weight, so air is aspirated through the orifice A. The aspirating speed is regulated by

an orifice and ranged from 1 to 10 l/min. Strictly speaking, the speed of this aspirator varies with time, but in the interval of 1 min. to 15~20 min. from the beginning of action, the variations of the speed are only within $\pm 5\%$, when the speed is 3~5 l/min., so if we use only in this period, this aspirator is regarded to satisfy the 3 conditions.

II-3-5-1 The speed regulation device (A)

For each set of a certain aspirator and a certain sampling vessel, we measured the time variations of the aspirating speed empirically, and in an early stage when the speed is larger, we insert a larger resistance, then gradually decrease the resistance according to the experimental results, by the way that a bar B in tube A is drawn gradually as the bag of the aspirator extends, thus we can conserve the constant speed. (Fig. 2-3).

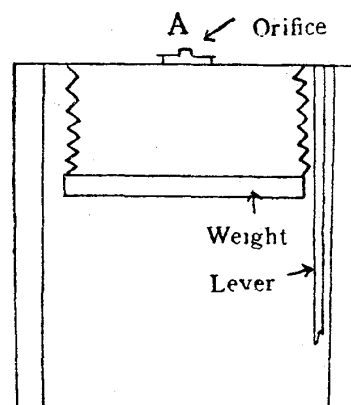


Fig. 2-2

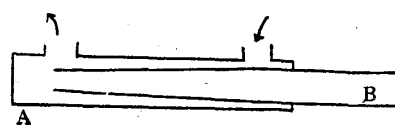


Fig. 2-3

II-3-5-2 The speed regulation device (B)

In Fig. 2-4, when air flows from A to B, as the gap between D and C acts as a capillary part of a capillary flowmeter because the neck of D is in tube C, so the right side level of water E goes down and the left side level rises. For example, if the speed increases, the left side level rises all the more and the neck of D get into C still more, then the resistance increases. As in this case the degree of depression of the aspirator is constant, the speed necessarily decreases. The diameter and angle of taper of the neck D were determined empirically. This device was very useful, but as water causes some troubles, we intended to design some other devices which did not use water, but we could not obtain an adequate one.

II-3-6 Vacuum vessel

A glass vessel 2~3 liters in volume is evacuated (1 mm. Hg) and its inlet is opened in gas. The period of aspiration is very short (2 sec. ca.), so we can obtain the most approximate value of concentration at that moment. But if we intend to measure the concentration during a certain period, the necessary number becomes very large.

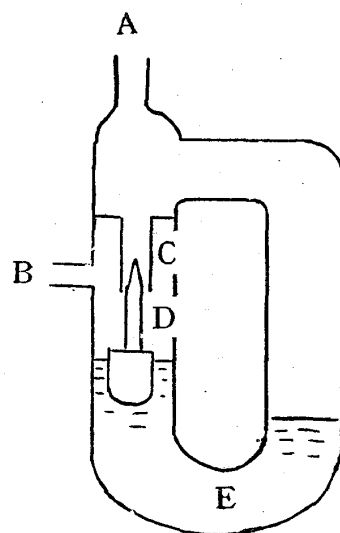


Fig. 2-4

II-3-7 Aspirating state of aspirator

Now we consider the case when the speed of aspirator is v (l/min.) and the wind velocity is w (m/sec.). During 1 sec. the aspirator begins to inhale a por-

tion of air adjacent to the inlet, and at last it inhales a portion of air w meters apart from the inlet in up-wind direction, or a volume of air in a cylindrical form of $v/60$ l., whose axis is the line connecting these two points. If we assume that it has a circular cylindrical form and that its radius is r cm., we get

$$\pi r^2 = \frac{1000}{60} v / 100w = v/6w \quad 2-3-1,$$

$$r = \sqrt{\frac{v}{6\pi w}} \quad 2-3-2.$$

The relation between r , v and w is given in Table 2.

Table 2 r (mm.)

$w \backslash v$	1	3	5	10
1	2.30	3.98	5.14	7.27
2	1.63	2.82	3.64	5.14
3	1.33	2.30	2.97	4.20
4	1.15	1.99	2.57	3.63
5	1.03	1.78	2.30	3.25

II-3-8 Rotating 12-way cock

This apparatus is set between aspirator and sampling vessels and changes the connection between the aspirator and each vessel in order in every definite time. (Fig. 2-5). The inner part of a 12-way cock is rotated by a clock-device. This apparatus was very reliable one.

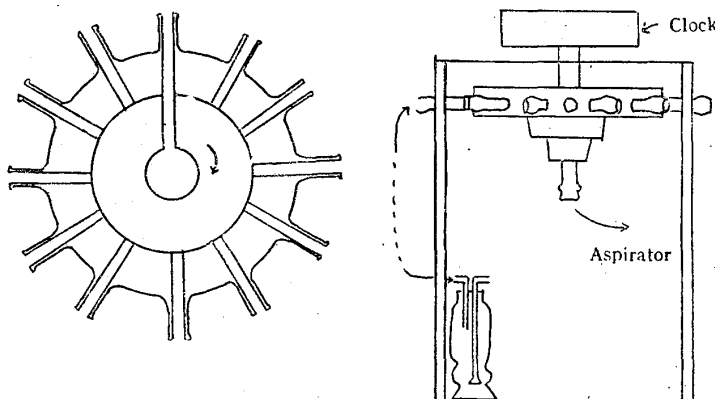


Fig. 2-5

II-3-9 Flowmeter

In order to determine the accurate speed of aspirators, we must use a flowmeter with small resistance. Rotameter is one of the smallest resistance flowmeters, but troubles often occur in a damp place such as field. We made a flowmeter to which a hot-wire anemometer was applied. As the wind velocity at a fixed position in a tube is proportional to the total flow speed, we made a portable flowmeter which is shown in Fig. 2-6 diagrammatically.

II-3-10 Concentration recorder

We made a concentration recorder using the change of electric conductivity. This consisted of a spraying part, a bridge circuit and a recorder. (Fig. 2-7 and 8). The spraying part is made of glass, and constant flow of air from a compressed air bombe is blown out through A, so the absorbent which passes one electrode D is sprinkled at B and gathered by a collector C. At that time

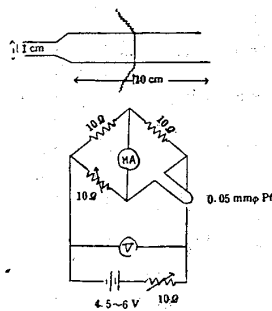


Fig. 2-6

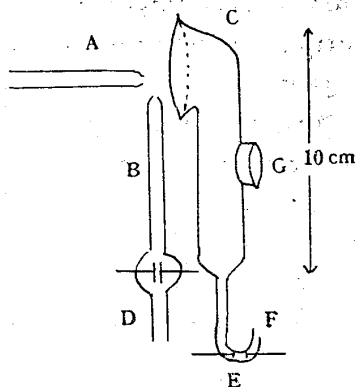


Fig. 2-7

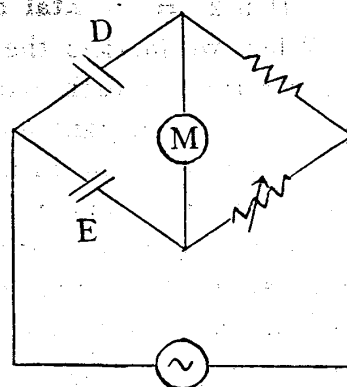


Fig. 2-8

the absorbent absorbs gas and is gathered in E where another electrode is set. As the liquid is blown out continually from B, the liquid in E overflows from F, so the absorbent in E is continually replaced. The electrodes D and E are inserted in the bridge circuit as arms, and amplifying the unbalanced current, it is recorded by a recording milli-ammeter. The absorbent was adequately chosen for each gas.

As the part E had a certain volume, the time lag was more than 10 sec., therefore, there might occur a considerable error when the concentration changed rapidly within 20~30 sec. It was very sensitive for external conditions and was precious and, moreover, needed some expertness for its use, so it needed much more improvements.

II-4 The precision of results of field experiments

As described already, the scales of the field experiments were very various, so the precision of the experiments could not be estimated indiscriminately. By the field experiment in an area of several meters square, by which only few sampling vessels and aspirators with speed regulators were used, the precision was almost equal to that of the experiment in a laboratory. However, in usual field experiments the observation area was several hundreds meters square and several hundreds equipments were used, so there might occur many errors or even many mistakes. Therefore, excepting the special small scale experiment, the results of usual field experiment should be regarded as those with somewhat statistical natures, so the number of the samples should be larger.

II-5 Procedure of analysis of the data

II-5-1 Smoothing

As described above, the data had statistical natures, so they should be smoothed in a certain sense. As the observation area was generally divided into rectangular meshes, at first the results were plotted against the positions along longitudinal lines and each curve was smoothed by inspection, then we follow same procedures along lateral lines. Therefore, two results were obtained for each position. We made smoothing of both results for each position so as to coincide each other. Thus obtained results were considerably reliable.

II-5-2 Horizontal diffusion coefficient $4a$

When we marked the positions of the highest concentration of each curve, they were usually arranged on a straight line. This line shows the mean wind direction during the diffusion and we may call it the 'main line'. Draw perpendiculars from several points on this line XY and we get points A_i, B_i, C_i, \dots (Fig. 2-9). When the values of $\log Ct$ of every points are plotted against their distances from the main line, these curves show usually a parabola-like shape. (Fig. 2-10). Adopting the formula for the considered source, we can determine $A_0 = 4at_0$ from these curves. Every A_0 for each leeward distance x is determined, and then from the graph showing $A_0 - x$, the value of $4a$ is obtained.

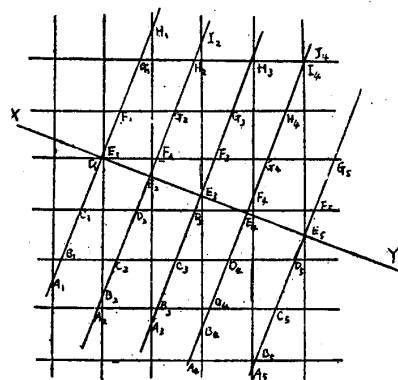


Fig. 2-9

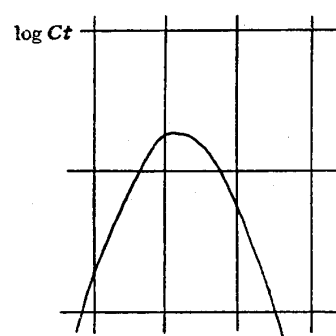


Fig. 2-10

II-5-3 Vertical diffusion coefficient b

In some positions in the observation area, the concentrations at some heights were measured. Generally vertical distribution of Ct showed the form of $\exp(-z/B_0)$ or $\exp(-z/(\gamma+B_0))$, where $B_0 = bt_0$. From these results, we obtained B_0 or $\gamma+B_0$ for each leeward distance x . Then with these values and the distance x , we could determine the value of b and γ .

II-5-4 Curve of Ct on the main line

By the above procedures, we could obtain the value of $4a, b$ and γ , then we calculate the value of Ct_c by the theoretical formulae for every x on the main line ($y=0$). The curve of $\log Ct_c - x$ was compared with that of $\log Ct_e - x$, where Ct_e were the experimental values. These two curves usually did not coincide each other, because the efficiency of gasification was not 100%. However, as the efficiency ought to be constant for every x , if the theory is valid, both curves may coincide each other, when the curve of $\log Ct_e - x$ is translated along y -axis. The degree of the coincidence shows the degree of adequateness of the theory.

II-5-5 Efficiency of gasification

The amount of translation of the curve in the above procedure shows the efficiency of gasification. It was about 90% at most and was usually 75 or 50%.

II-6 General results of the field experiment

The field experiments had been carried on for more than 20 years till the End of the War, so the number of results was numerous, but the experiments in early period were not so exact and had rather qualitative natures.

However, as the accurate measurements began to be carried on only a few years before the End of War, the number of the reliable experiments was about 50~60.

As already described, all reports and data were burnt at the End of the

War, so we cannot report in detail; we want to describe only general results.

II-6-1 Summary

When we analysed the data with the above theory, we have not experienced any trouble, and the agreement of both theoretical and experimental results was fairly good, at least in the first approximation.

II-6-2 Relation between the diffusion coefficients and the wind velocity

Consequences of analysis of the data showed that

$$4a = 4\alpha u \quad \text{and} \quad b = \beta u \quad 2-6-1,$$

where α and β are constants. Besides, we made following experiments.

We placed 10 aspirators with rotating cock devices in every 1 m. in the position 10 meters apart leeward from continuous point sources, and changed the connection of sampling vessels in every 10 sec. When we plotted the mean concentrations in every position at the same instant against each position, we got curves like Fig. 2-11. In these experiments the sources and the aspirators were placed at three heights, i. e. 0.5, 1.0 and 2.0 m. from the ground. According to the observational results, the wind velocity followed the logarithmic law⁷⁾, so the wind velocity at 2 m. was almost twice as much high as that at 0.5 m. However, the values of $4at_0$ at all heights were equal within experimental errors, namely $4at_0$ was independent of u . In this case

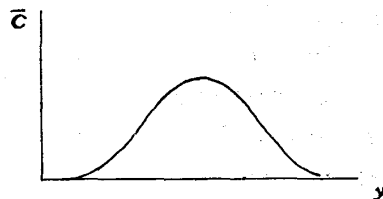


Fig. 2-11

$$\bar{C} \propto \exp(-y^2/4at_0) \quad \text{and} \quad t_0 = \sqrt{\frac{x^2}{u^2} + \left(\frac{4a}{u^2}\right)^2} - \frac{4a}{u^2} \quad 2-6-2.$$

Therefore,

$$4at_0 = 4a \left\{ \sqrt{\frac{x^2}{u^2} + \left(\frac{4a}{u^2}\right)^2} - \frac{4a}{u^2} \right\} = \frac{4a}{u} \left\{ \sqrt{x^2 + \left(\frac{4a}{u}\right)^2} - \frac{4a}{u} \right\} \quad 2-6-3.$$

became independent of u , consequently we could conclude that $4a = 4\alpha u$.

Similar experiments for b , we did not undertake.

II-6-3 Relation between diffusion coefficients and temperature gradient.

Though 4α and β were independent of u , they might have relations with temperature gradient. In order to obtain the relation between β and the temperature gradient, we made next experiments. In a rectangular area 2 m. square, a certain amount of a volatile liquid was scattered uniformly, and in the middle of the area inlets of the aspirators were set at the heights of 10, 20, 30, 40 and 50 cm. from the ground, and aspirated 15 min. and we measured the mean concentration in respective positions. (Fig. 2-12). In this case the source was 2-dimensional, so the vertical distribution of concentration at A was determined principally by the vertical diffusion. Therefore, in order to obtain the relation between β

7) $u = u_0 \log(z/z_0)$, where z_0 is a certain height and u_0 is the wind velocity at that height.

and the temperature gradient, we observed the temperature profiles together with these experiments. We made the measurements at 12h, 14h, 16h, 18h, 20h, 24h and 4h, 6h, 8h and 10h of the next day. These experiments were carried

on at two different places and seasons (Sekiyaama, Niigata Prefecture, in May and Numata, Gumma Prefecture, in July). The wind velocities ranged from 0.5 to 2.5m/sec. and the vertical temperature gradients ranged from -1.0°C (lapse) to $+1.2^{\circ}\text{C}$ (inversion) between the heights of 0.5 and 3 m., so we might conclude that the meteorological conditions varied considerably. The curves of $\log Ct - z$ were lines slightly convex upward, and contrary to our expectations, all curves had almost the same inclination, inspite of considerable variations of the meteorological conditions; the rate of vertical decrement of concentrations should be always the same.

On the other hand, the analysed results of various field experiments in various meteorological conditions showed that 4α and β were almost always constant and that 4α ranged from 1.2 to 1.8 and β ranged from 0.015 to 0.025, and moreover, these variations did not show any clear correlation with the temperature profiles. Therefore, we may conclude that, at least in the first approximation, $4\alpha=1.5$ and $\beta=0.02$, and they are independent of the temperature profiles in such experiments in the regions in which our research was carried on.

II-6-4 Shifting of wind direction

Hitherto, the diffusion has been regarded to subside considerably in the condition of inversion. This contradicts with the conclusion that 4α and β are independent of the temperature profiles. However, here we must bear in mind that the object of measurement hitherto was usually not C but Ct .

In some cases the mean wind direction gradually shifts in a short time interval compared with that in which gas passes through the area. This shifting occurs more slowly compared with the fluctuation of wind direction. If this occurs, the cross-wind distribution of Ct deforms from the full line curve in Fig. 2-13 to the dotted line, so that the breadth of dispersion of gas increases. Generally this shifting seldom occurs in the condition of inversion, so the diffusion seems to subside considerably.

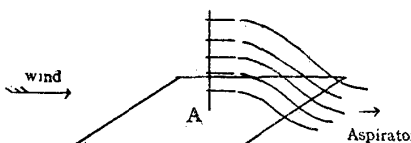


Fig. 2-12

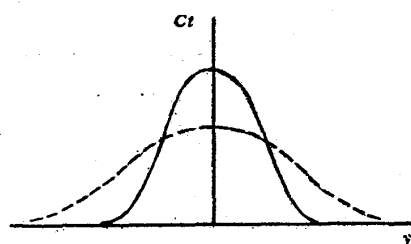


Fig. 2-13

II-6-5 The shifting of wind direction as a factor of the diffusion phenomena

We must consider whether this shifting should be regarded as a factor of the diffusion phenomena. First of all, in the experiment with instantaneous 3-dimensional sources the larger the wind velocity was, the smaller became the time of passage of gas through the observation area, therefore, the lesser

became the shifting of the wind direction. So the horizontal diffusion coefficients obtained by the data of Ct became larger when the wind velocity was smaller than they were when it was larger. This is a physically unreasonable result.

Secondly, when we put the values of 4α and b obtained from the data of Ct in the formulae of Ct for every x , and when we plot the calculated Ct against x , this curve shows steep decreament as x increases, and does not show any resemblance with the experimental curves. This was caused by the fact that the value of 4α was estimated exaggerately, because it included not only the results of the essential diffusion phenomena but also the effects of the shifting of wind direction. If we considered the effect of the shifting, we could usually obtain resonable results.

Thirdly, this shifting does not always occur, and its degree is various without any definite relation to the meteorological conditions at that time and the variety of the experiments.

Finally, the effect of the shifting appeared more remarkably in the experiments of smaller scale, and in the large scale experiments this effect may be negligible in the chief wide domain.

Judging from these conclusions, the effect of the shifting might not be considered as a factor of the diffusion phenomena.

II-6-6 Effective height of source

When we emit a considerable amount of gas from one point, we usually emit it in liquid state and sprinkle it in order to easily gasify, then liquid drops deprive the surrounding air of the heat of vaporization and cool it.⁸⁾ As the result, the air loaded with gas becomes heavier and creeps over the ground. In these cases the height of the source is regarded effectively zero.

On the other hand, when we emit smokes from a smoke candle, the air temperature rises and the smoke cloud ascends, therefore, the effective height differs from the actual one. Detailed investigations concerning to the effective heights have not yet been accomplished.

II-6-7 Initial distributions of the concentration in an instantaneous 3-dimensional source.

In the case of concentrated explosion of gas vessels at one object, the initial distributions of concentration were regarded to follow the Gaussian distributions; and for the experiments with a single large gas vessel, the initial concentrations in horizontal direction seemed to be uniform within a certain area when we saw air-photographs or usual one, and that was also ascertained by direct measurements of the concentrations. However, it was difficult to assume its vertical distribution.

First of all, the experimental results of the vertical distributions of Ct for the concentrated explosion always took the form of $\exp(-z/\gamma)$. The photographs of the initial gas clouds of large gas vessels showed the form like Fig. 2-14, so

8) In a case, the temperature descent was 10°C at a distance of 100 m. leeward from the source.

we imagined that the concentration was uniform even in vertical direction up to a certain height of H . If this be valid, the vertical distributions of Ct near the source would be those shown in Fig. 2-15. However, the experimental results showed that the curve of $\log Ct - z$ were always straight lines. So we conclude that the vertical distributions should take the form of $\exp(-z/r)$, just as in the case of the concentrated explosion in spite of the shapes of initial clouds in the photographs.

II-6-8 Vaporization of persisting gas from the ground

When persisting gas, (and even considerably volatile liquid as HCN also,) is sprinkled on the ground, it does not vaporize at a time but they remain for a certain period, so it becomes continuous source. Using a equipment which is shown in Fig. 2-16, we measured the rate of vaporization of various kinds of liquids in a wind-tunnel. In the equipment soil or sand is put in A and the volatile liquid to be tested is filled in B , so the sand or soil is always in wet state. If the temperature of the soil or sand is $T^\circ\text{C}$ and the saturated concentration of this liquid at $T^\circ\text{C}$ is ω and wind velocity at the surface of A is u , then we get empirically for the rate of vaporization dM/dt (M : mass of liquid) next equation:

$$\frac{dM}{dt} = -K\omega(1+ku) \quad 2-6-4.$$

However, even when we scatter liquid of considerable amount, it is usual that it does not form puddles but is scattered as numerous drops on the ground. This is not the same state as in the above experiment.

When we let fall a liquid drop on soil or sand and investigate the wetting part of it at various times elapsed, we can see that in a very early stage, it takes the form shown (1) in Fig. 2-17; in a short time, it takes the form as shown (2), after then, preserving a similar shape, it takes the form (3), (4) and so on. The time during the stages (1) and (2) is about $1/15 \sim 1/20$ of the total vaporization time, so we can neglect this period. As the surface area which contacts with air is proportional to $M^{2/3}$, where M is the liquid quantity in soil, so the factor K in (2-6-4) becomes

$$K = K' \cdot M^{2/3} \quad 2-6-5,$$

and then we get

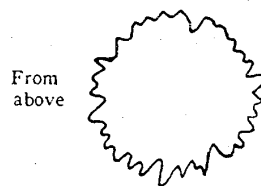


Fig. 2-14

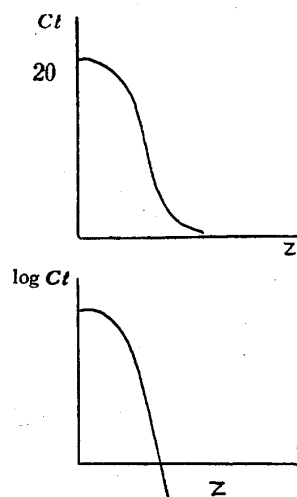


Fig. 2-15

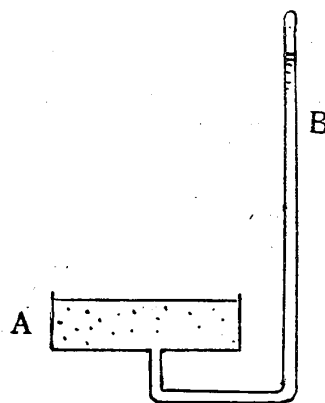


Fig. 2-16

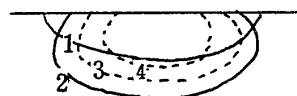


Fig. 2-17

$$\frac{dM}{dt} = -K'M^{2/3}\omega(1+ku) \quad 2-6-6.$$

Therefore, we obtain

$$M^{1/3} = M_0^{1/3} - K' \int_0^t \omega(T)(1+ku) dt \quad 2-6-7.$$

We made many samples of blocks of soil on which a definite amount of a liquid was dropped, and some of them were analysed chemically at definite times elapsed and measured the remaining amount. These results were tolerably expressed by (2-6-7), when we adopted the data of surface temperature of the ground and wind velocity.

III ESTIMATION BY CALCULATION

It is difficult to carry out various kinds of experiments in various scales and in every possible meteorological conditions, so, using the results hitherto obtained, we want to estimate the results of diffusion by calculation and to examine in detail the effect of various factors which take part in the phenomena. We want to report some examples. In these calculations we intended to determine the contours for $Ct=2000$, and to deduce some conclusions.

III-1 Continuous line source

In the experiments of a continuous line source, when a large amount of volatile liquid is emitted, the effective height of the source may be regarded as zero, so the value of Ct can be given in (1-4-11). The place to be covered with gas be a distance x and its breadth be $2s$, so the necessary length of the source $2r$ is given by (1-4-13) (Table. 1), and the emitting amount M per unit length which expects $Ct=2000$ at the end of the place is given by (1-4-14). These results can be expressed by a nomograph which is shown in Fig. 3-1, in which we put $z=0.5$ m. By the right half portion of Fig. 3-1, when x and s are given, $\sigma=s/\sqrt{A_0}$ is obtained. The corresponding value of $\rho(=r/\sqrt{A_0})$ is obtained at the same time. The value of r can be obtained, when the value of ρ just obtained is read on the scale of σ and using x , we get the value of r on the scale of s .⁹⁾

The value of M' (I-4-3) is obtained by the left half portion of the nomograph, by connecting x and u . With the point on the scale of M' and the value of σ , we can get final result, i.e. the emitting amount per unit length.¹⁰⁾

III-2 Continuous 3-dimensional source (Concentrated explosion)

The value of Ct at the concentrated explosion of gas vessels (I-6-4) for one object is given by (1-6-41). Data of calculation are :

Gas content of one vessel = 1.08 kg ; $z=0.5$ m. ; $r=2$; $u=1, 3$ and 5 m/sec. ;

Number of vessels = 100, 200, 300, 400 and 500 ;

$(H_u, H_v) = (20, 42)$; $(28.28, 59.38)$ and $(34.64, 72.65)$.

9) For example, for $x=500$ m. and $s=1000$ m., we obtain $\sigma=36.3$ and $\rho=38.2$. Then 38.2 on the scale of σ , and $x=500$, we get $r=1050$ m.

10) In this case, $\sigma=10$, so $M=M'$, then for $x=500$ m, $u=3$ m/sec. and $r=1050$, we obtain $M=4.02$ kg/m.

The resulted contours are shown in Fig. 3-2~3-4. The synthesized results are shown in Fig. 3-5.

- 1) Such difference of H_{\parallel} and H_{\perp} does not cause considerable difference.
- 2) The areas are on the leeward of the object, and its degree is different according to the wind velocity and the number of vessels.
- 3) The necessary minimum amounts of vessels become larger as H_{\parallel} and H_{\perp} become larger.

III-3 Instantaneous 3-dimensional source (Uniform explosion)

This is the case when gas vessels are exploded uniformly in an area at the same time. (I-6-3). In this case $C\sharp$ is given by (1-6-31) or (1-6-32). The data of the calculation are :

The content of gas in a vessel = 8.0 kg. ;

$z=0.5$ m. ; $r=2$; $2l=2l'=100, 200$ and 300 m. ; $u=2, 4, 6, 8$ and 10 m/sec. ;

$N=25, 50, 100$ and 200 , for $2l=2l'=100$ and 200 m. ;

$=37.5, 70, 150$ and 300 , for $2l=2l'=300$ m.

The resulted areas are shown in Fig. 3-6~3-8. The synthesized results are shown in Fig. 3-9.

- 1) Within this range of l , the areas becomes wider as l becomes larger.
 - 2) The necessary minimum numbers increases as l becomes larger.
 - 3) The resulted areas are on the leeward of the area which is aimed at.
- In some cases, there are places without effect even in the aimed area.

CONCLUSION

We can conclude that the diffusion of matter near the ground in areas within several kilometers can be expected numerically to some extent. These results would be useful for agricultural, sanitary and disaster prevention problems.

Of course, this research is not a complete one, so we want further to investigate the fundamental problems.

This research was carried on by assistances of many persons for several years, the author want to express his deepest thanks to these persons. Especially, the author is grateful to the late Professor S. Fujiwara for his encouragement to write this report and for his kind and detailed instructions.

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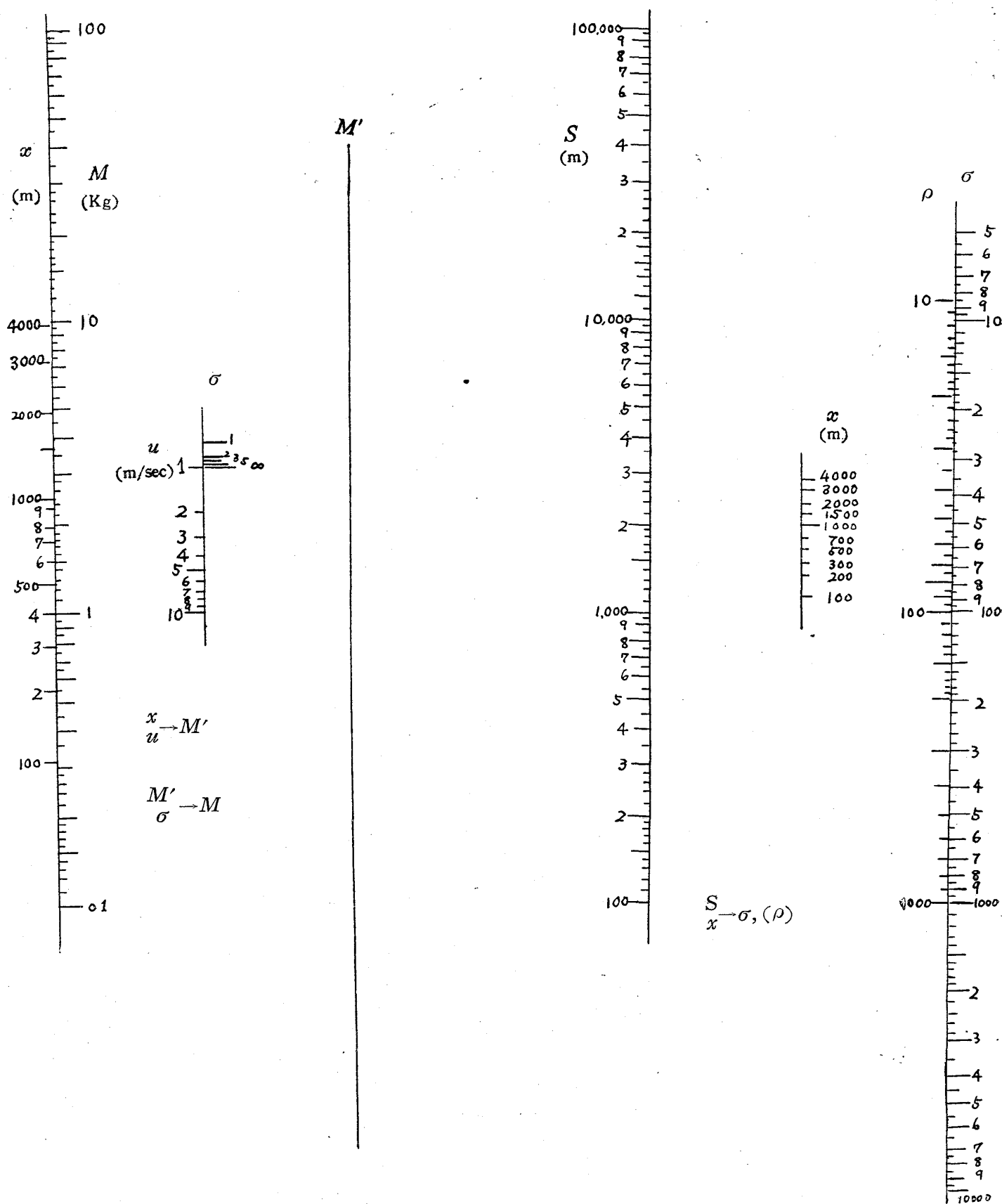


Fig. 3-1

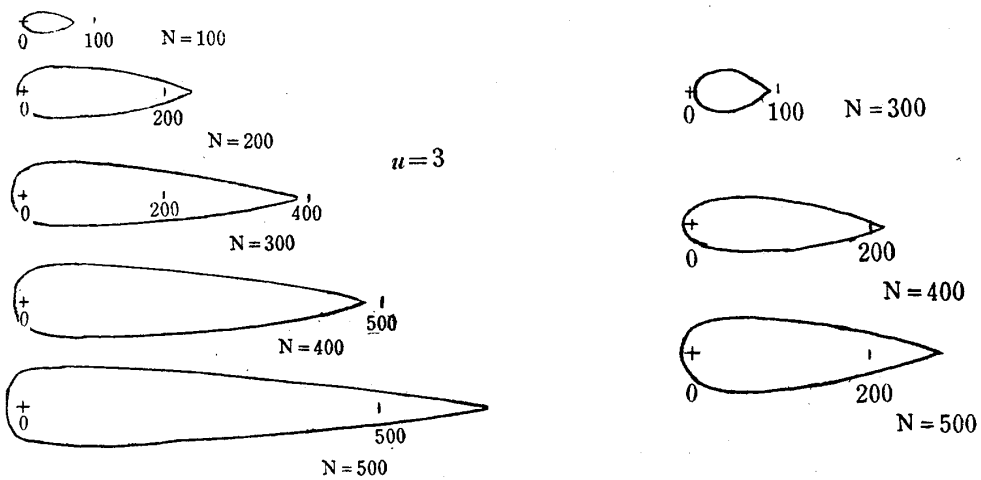
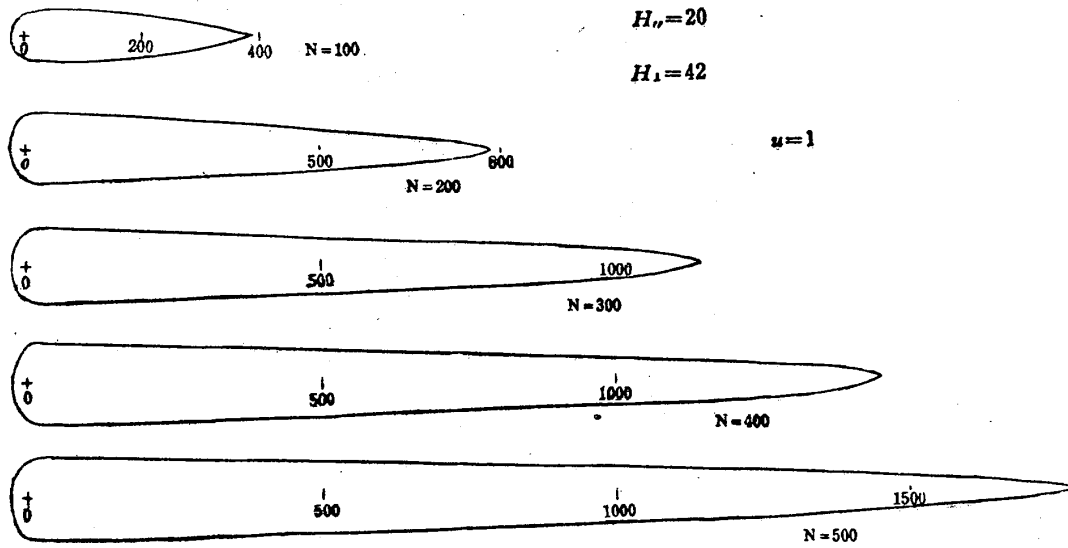


Fig. 3-2

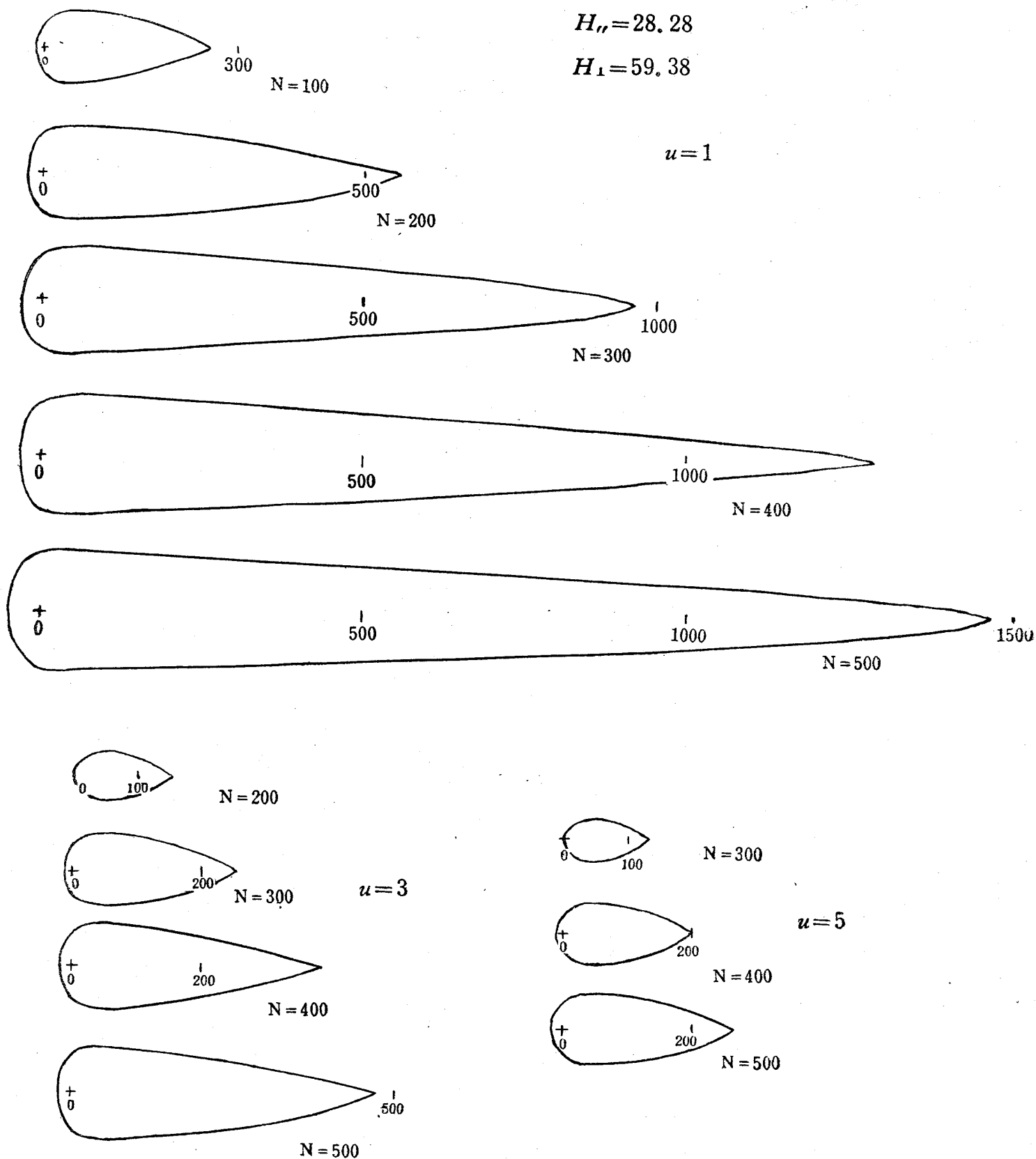


Fig. 3-3

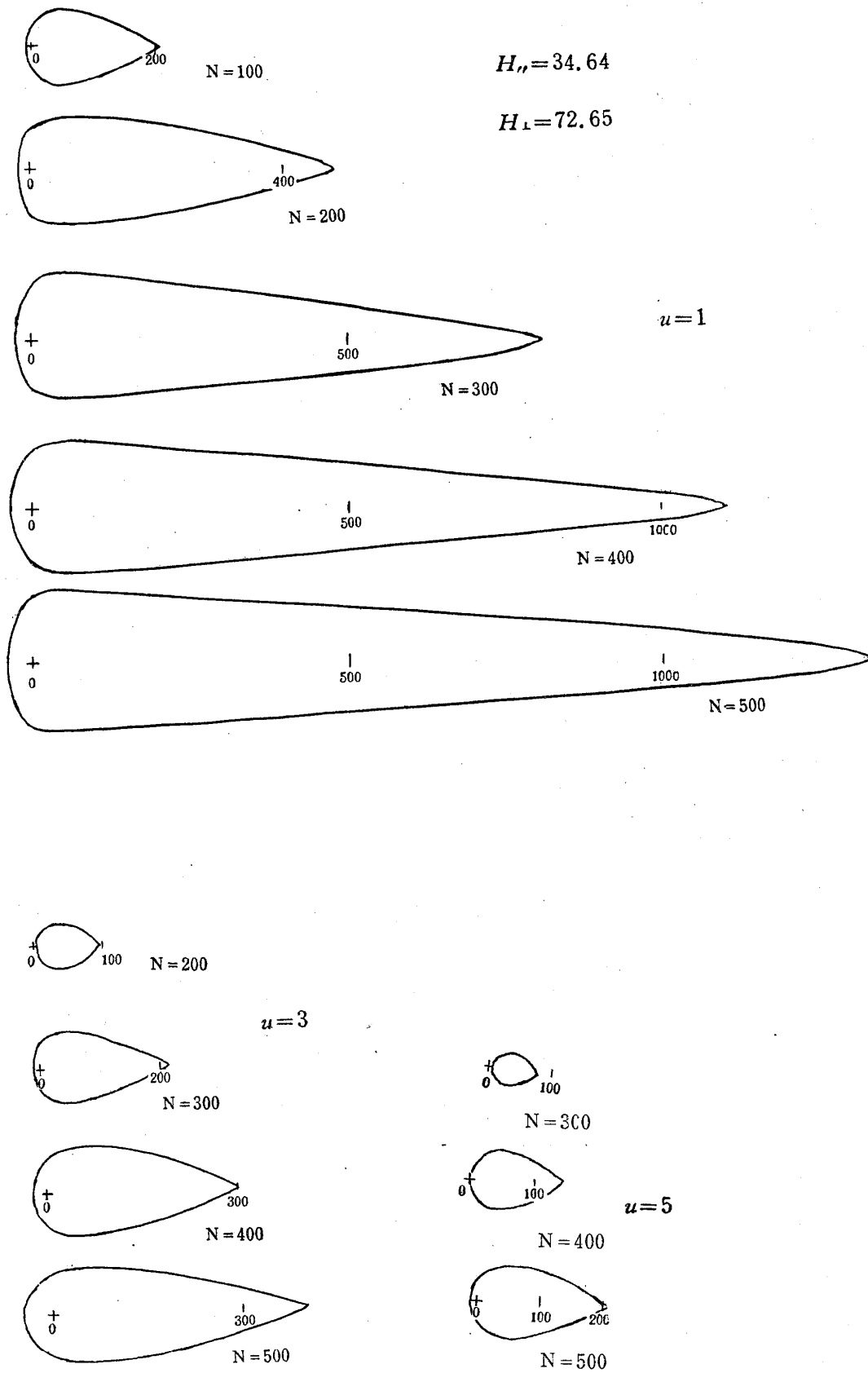


Fig. 3-4

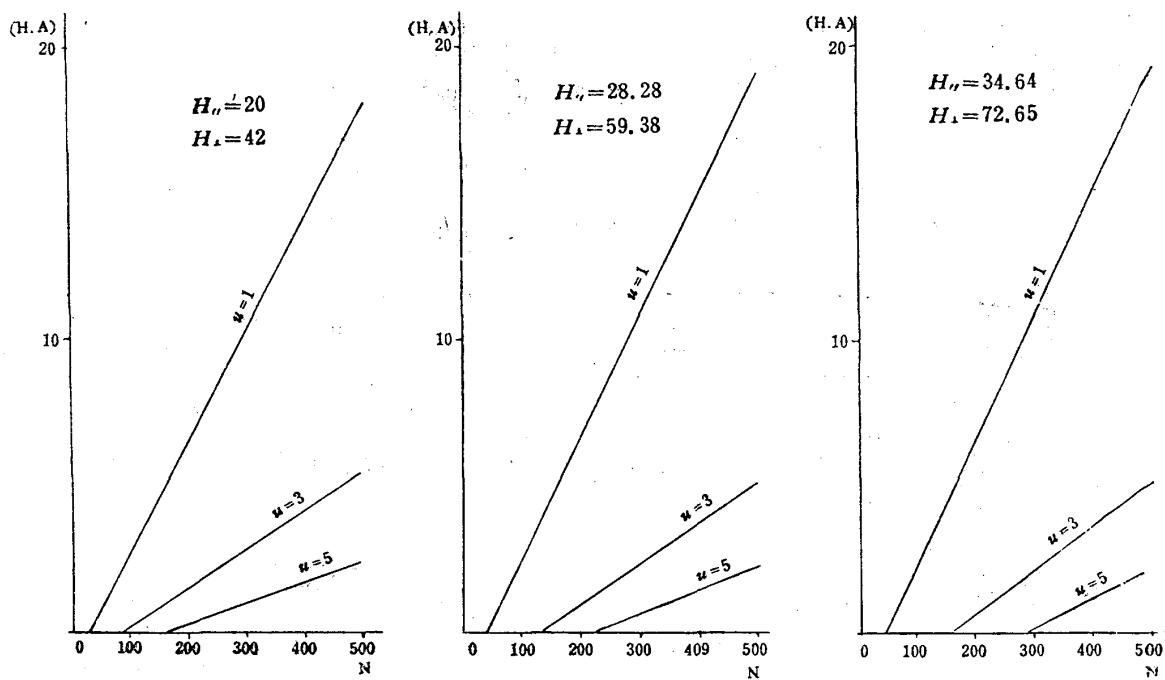


Fig. 3-5

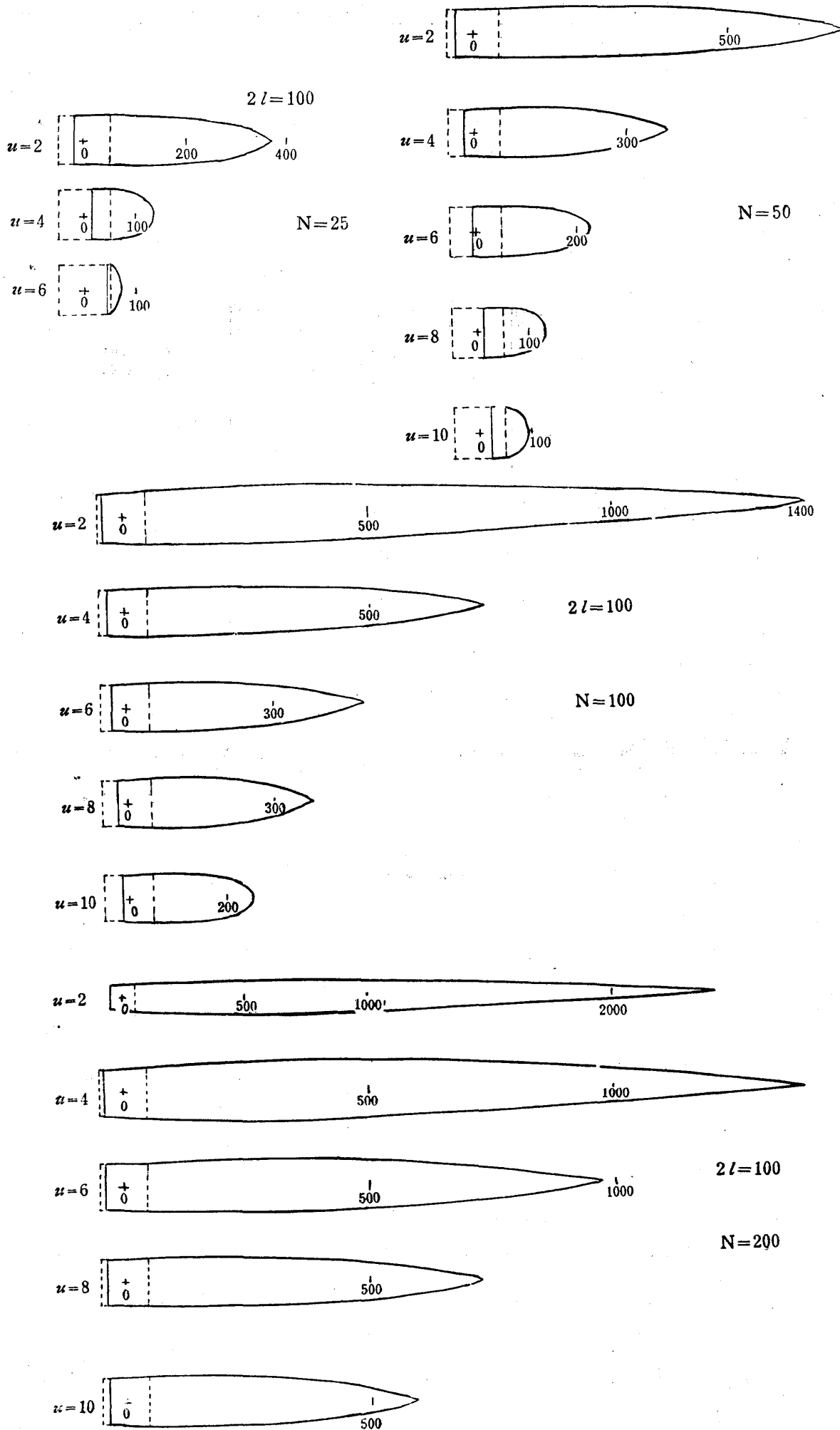


Fig. 3-6

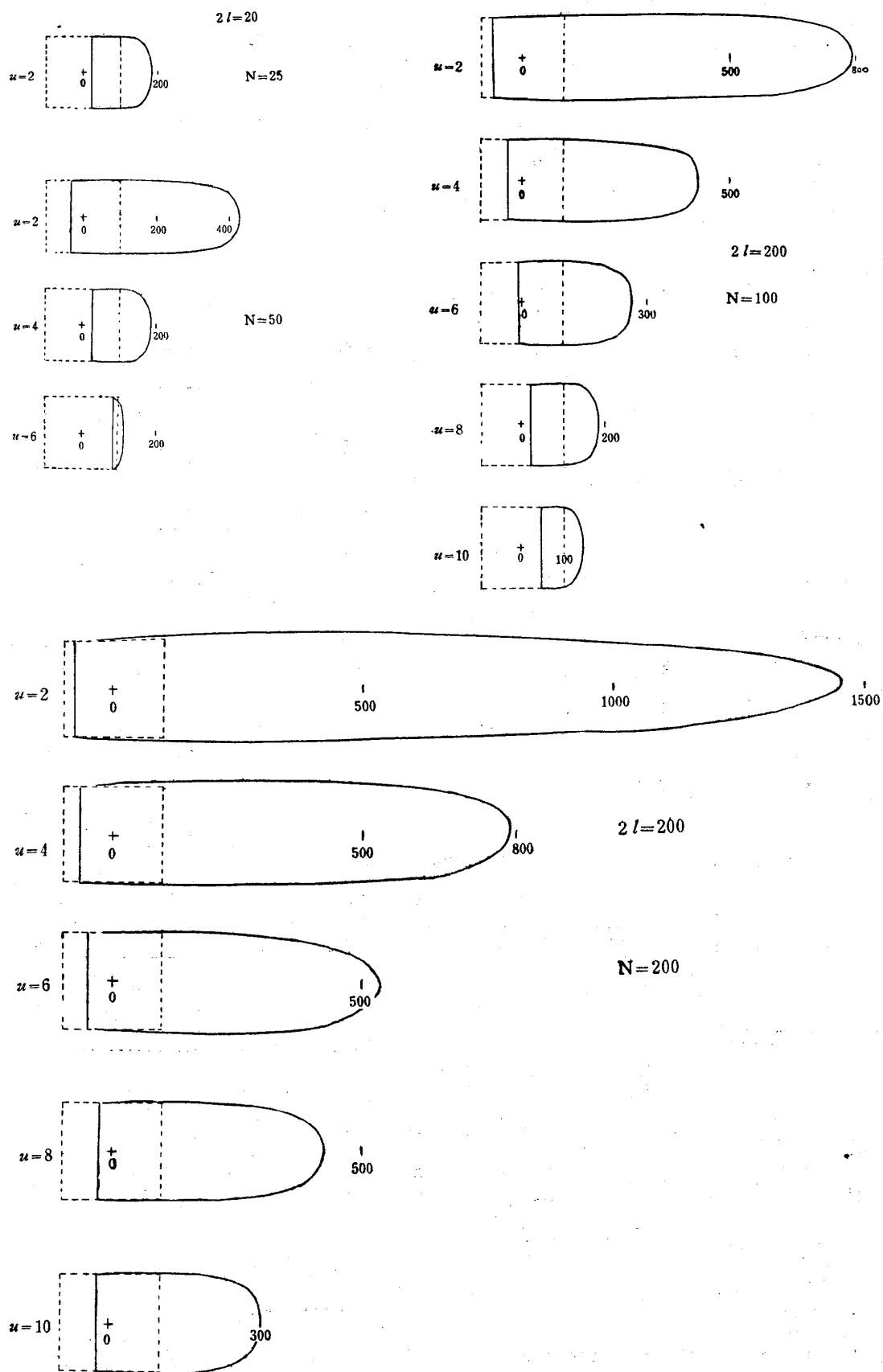


Fig. 3-7

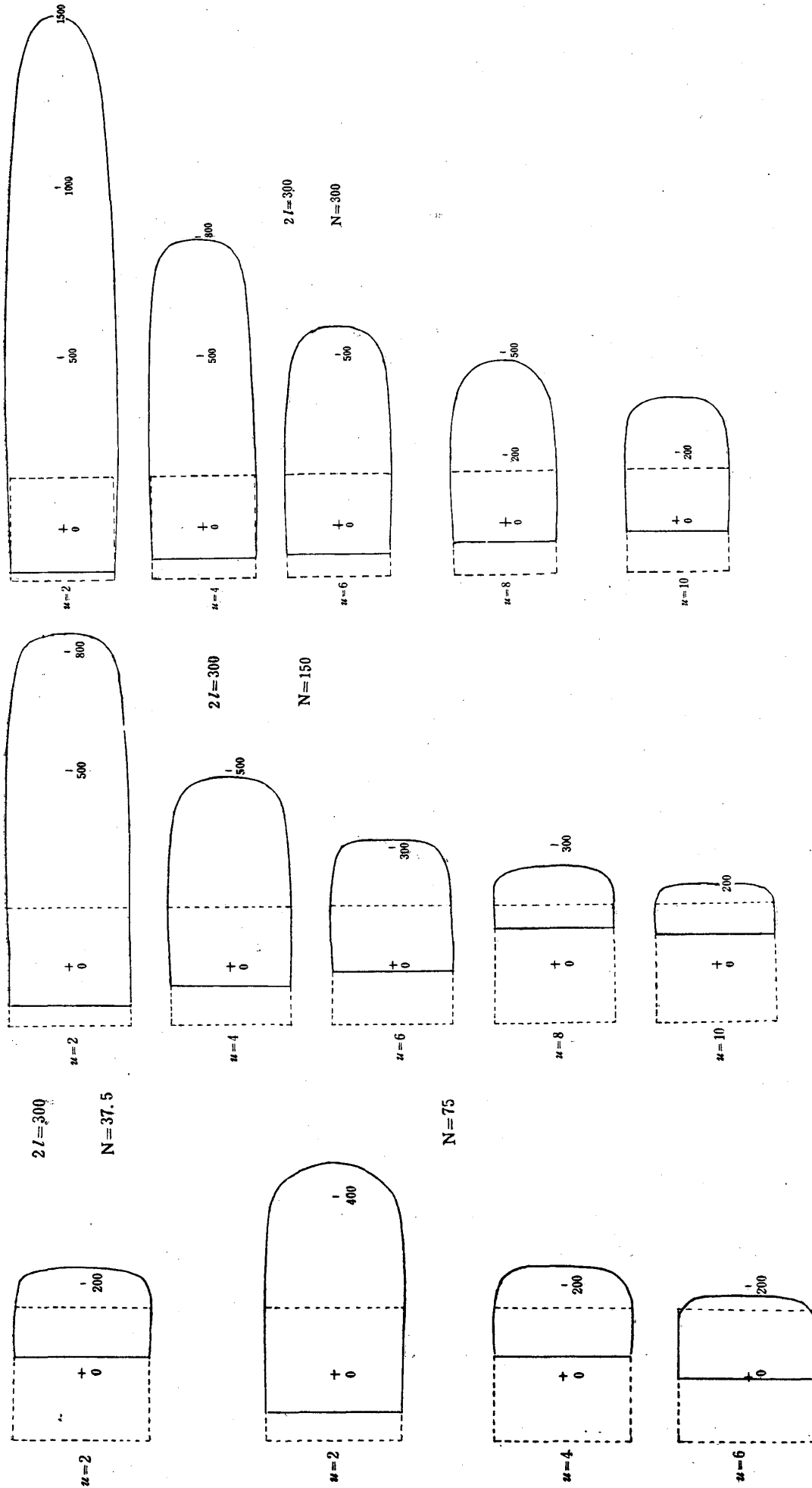


Fig. 3-8

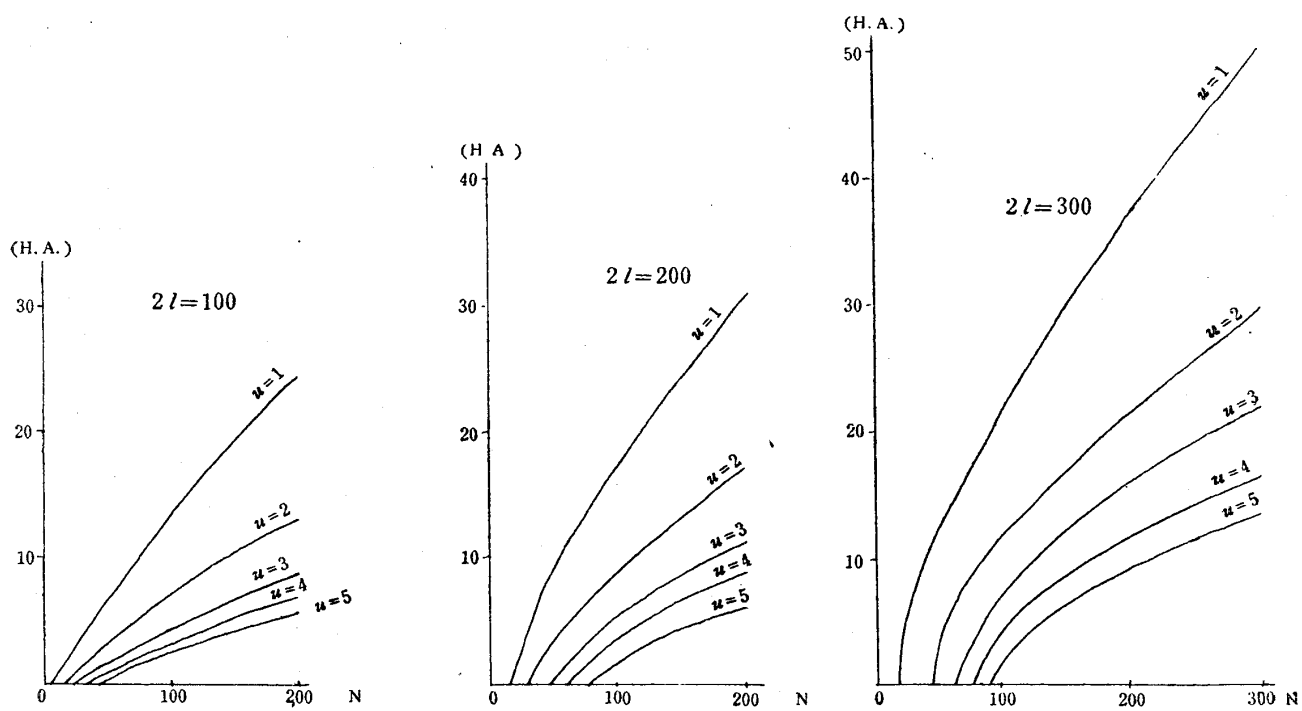


Fig. 3-9