On the Turbulent Diffusion Near the Ground II[1)]

—Diffusion in a region within 6 cm. from the source—

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Introduction

In the previous paper[(1)], one of the authors has reported the research in which, using a particular differential equation for the atmospheric diffusion, compared its solutions with results of rather rough measurements for somewhat larger scale of phenomena.

However, though there have been many reports of accurate researches on the diffusion in steady air flows in wind tunnels, but for the diffusion in the atmosphere, it is difficult to find out objective relations between each result which has been hitherto obtained, because in each case, patterns of turbulence vary for time and location. That is due to the facts that the diffusion coefficient changes its value by passage time which concerns to scales of the diffusion phenomena, and by the observation time (the time to take means) which concerns to the scale of turbulence. So it is necessary to investigate the atmospheric diffusion fundamentally as possible as we can, we took some measurements of diffusion in very small scales.

The measurements of the diffusion hitherto carried out, necessitated the observation time of the order of several minutes due to the measuring technique, so the obtained results were those which were averaged for the period of the order of several minutes.

So, to eliminate the effect of the observation time, we intended to observe the diffusion in a very short observation time.

Method of the observation

In order to measure instantaneous 2-dimensional distributions, we observed by schlieren method[(2)] heat wakes from a line heat source. Between two lenses (Hexar 1 : f=4.5, F=36 cm.), we set a nichrome wire (diameter=0.47 mm., length=6 cm.) and heated it by electric current (3～7 amp.), then we observed the schlieren image of the wake by lattice method (lattice constant=0.586 mm.) or normal schlieren method (focal length of the normal schlieren lens=1270 cm.). (Fig. 1). The exposure was 1/1000 sec.[2)] Near the heat source, we set a small and light wind-vane

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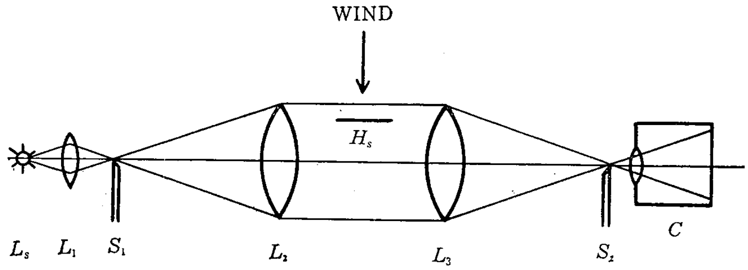
2) True exposure was 1/550 sec.

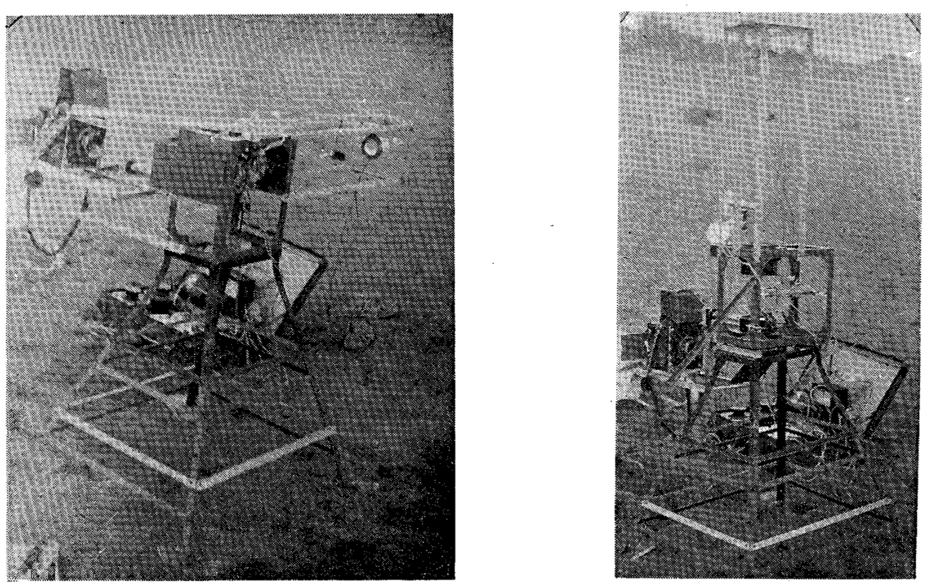
in order to place the apparatus so as the source to be a right angle to the wind direction, and placed a heated wire of a hot-wire anemometer



L_1: Condensor L_2, L_3: Schlieren Lens
S_1: Knife edge or Pinhole S_2: Knife edge or lattice
L_s: Light Source C: Camera
H_s: Heat Source

Fig. 1.

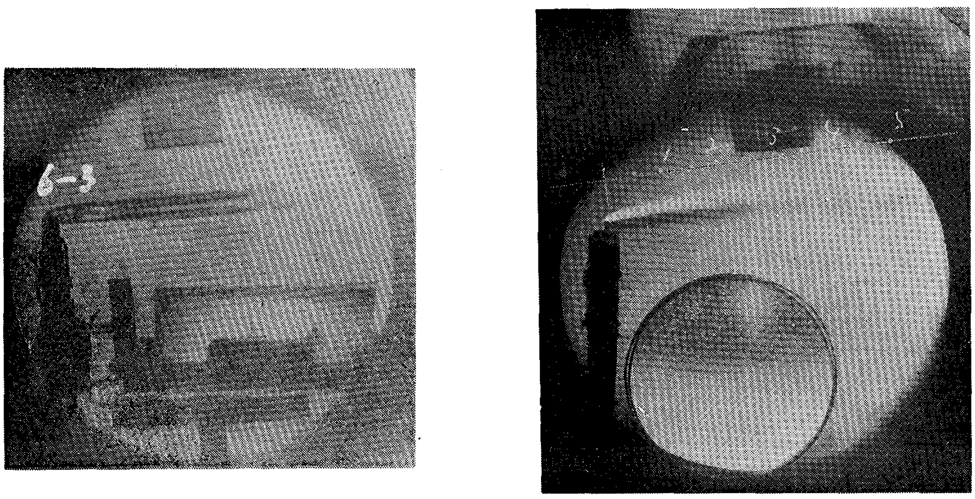
in order to measure the wind velocity at the instant; and designed to be able to photograph the meter-face of the anemometer together with the wake in the same field. We fixed this apparatus in a metal frame and, for the observation of the vertical diffusion (z-direction), we placed the frame so as to level the heat source, and for the horizontal diffusion (y-direction), we placed the frame so as to erect the source vertically. Using adjustable benches, we measured the vertical diffusion at the heights (H) of 20, 40, 60 and 80 cm., and measured the horizontal diffusion at the heights of 35, 55 and 75 cm. from the ground. (Fig. 2).



(a) Vertical Diffusion Observation (b) Horizontal Diffusion

Fig. 2.

The first observation was carried during January and March for the city-wind in an unoccupied area near the University and at the position where there were no large obstacle in the up-wind direction. After that, in August, we carried the observation in sea-wind near the coast of Ninomiya, Kanagawa Prefecture. In the earlier period of the research (for the city-wind), we adopted the lattice method, but by this method, the number of the stripes became so fewer that it became difficult to analyse as the distance (x) from the source became larger. So we adopted the normal schlieren method by the observation for the sea-wind. Some examples of the obtained photographs are shown in Fig. 3, and we can see that the breadths of the wakes are about 1.5 cm. at $x=6$ cm. The



(a) Lattice Method (b) Normal Schlieren Method

Fig. 3.

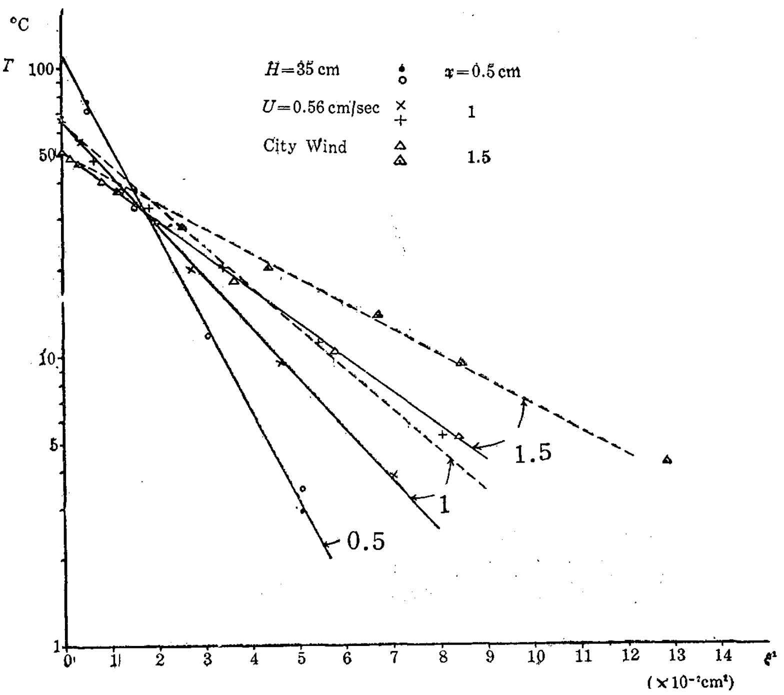


Fig. 4.

analysis was carried up to $x=6$ cm. utmost. When we analysed the temperature distributions for some cases, we could obtain the curves, one of which is shown in Fig. 4. The graphs of $\log T$ and ξ^2 show straight lines, where T is the temperature difference measured from the surrounding air temperature and ξ is the distance from the center line. So we can conclude that the distribution are normal for both horizontal and vertical directions. It can be verified theoretically that the deviation from the normal distribution caused by the effects of the ground[(3)] becomes neglegible for such small breadths of the wake. For a given leeward distance, the temperature distribution of e.g. y-direction can be expressed by

$$T=T_m \exp(-y^2/\alpha) \quad (1),$$

and when we put $y=y_0$ for $\left(\frac{d^2T}{dy^2}\right)_{y=y_0}=0$,

$$\alpha=2y_0^2=2\overline{y^2} \quad (2).$$

By the lattice method, we measured the distance between the stripes of the highest orders, and by the normal schlieren method, we measured the distance between the brightest positions and the darkest ones from the records of the microphotometer. Using the next relation ,

$$\alpha=4Kt=4K\frac{x}{U} \quad (3),$$

we can determine the values of K's, where K is the diffusion coefficient, U is the mean wind velocity and t is the passage time.

Plotting α against x, we get graphs, some of which are shown in Fig. 5[3)]. Then we consider that the variance $\overline{y^2}$ is the sum of the term $\overline{y_m^2}$ which is due to the molecular diffusion and the term $\overline{y_t^2}$ which is due to the turbulent diffusion. The coefficient of the molecular diffusion K_m is given by

$$K_m=\lambda/\rho C_p \quad (4),$$

where λ is the coefficient of the thermal conductivity, ρ is the air density and C_p is the specific heat for constant pressure. The straight lines (α_m-x) in Fig. 5 were calculated by (4). These lines do not pass the origin. This is not only due to the finite diameter of the heat source and the sag of the wire, but also due to the fact that we adopted the lines for the surrounding air temperature, instead of true lines which ought to have steep inclinations for small x where the temperature is very high. We shall denote this intercept as α_0.

Observed points in these figures dispersed considerably, so we drew

3) In Fig. 5, marks z indicate the results of the vertical diffusion, and marks y indicate those in horizontal direction. Units used are cm. for H, m/sec. for U and cm²/sec. for K_t.

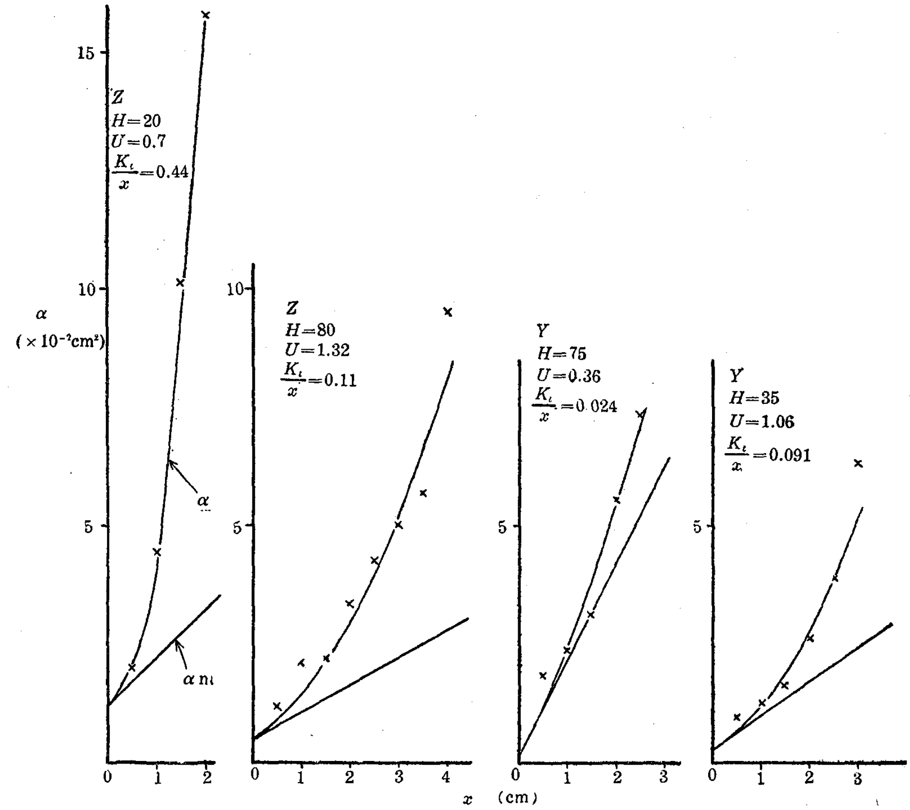


Fig. 5 (a). City-Wind H(cm.), U(m/sec), K_t(cm^2/sec).

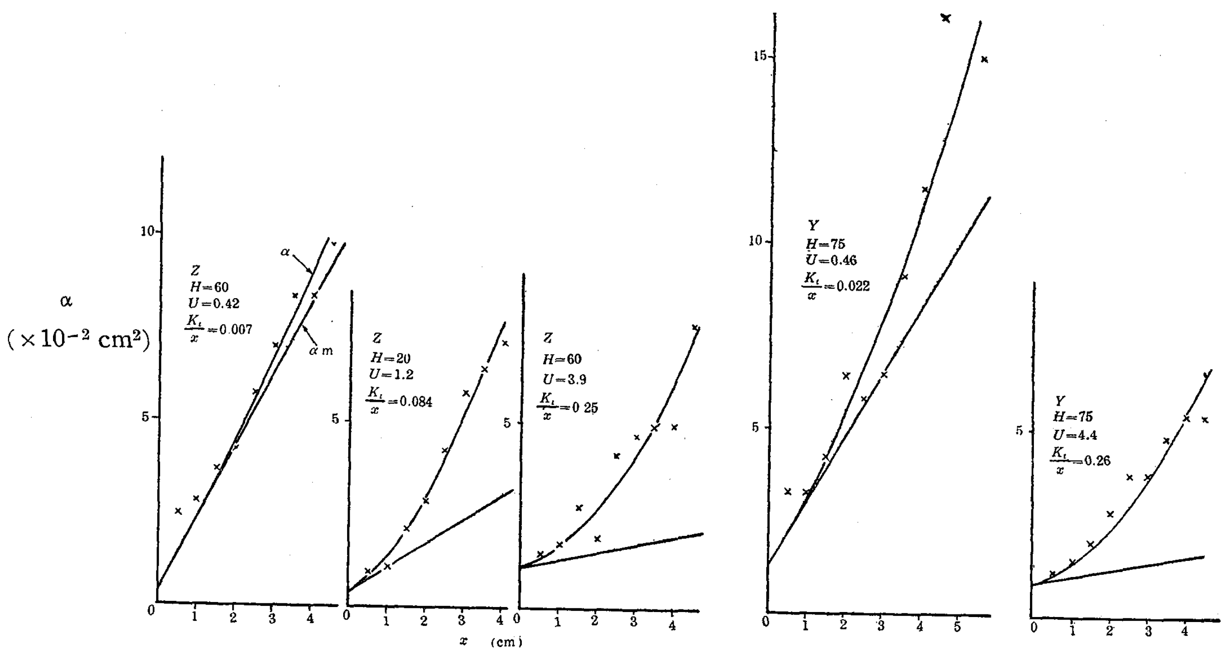


Fig. 5 (b). Sea-Wind.

appropriate curves for these points, then we obtained graphs $\alpha-\alpha_m-\alpha_0=\alpha_t$ against x^2 which show straight lines. So we can get

$$\alpha_t=a_t x^2 \tag{6}$$

Then, from the inclination of these lines, we could determine the value a_t.

From (3), we get

$$a_t = 4K_t x/U \tag{7}$$

and for small t

$$\overline{y_t^2} \approx \overline{v^2} t^2 = (\overline{v^2}/U^2)x^2 \tag{8}$$

where v is the velocity fluctuation[4].

Then we get

$$a_t = \frac{2\overline{v^2}}{U^2} \tag{9}$$

Therefore we can get the turbulent diffusion coefficient and the intensity of turbulence. Relations between K_t and U is shown in Fig. 6.

The distances y_0's measured from the schlieren photographs satisfy that $\partial^2 n/\partial y^2 = 0$, but they do not satisfy $\partial^2 T/\partial y^2 = 0$, where n is the refractive index of air.

The relation between T and n is given by

$$T = \frac{-b\Delta n}{a + \Delta n}, \tag{10}$$

where $a = n_0 - 1 = 2.94 \times 10^{-4}$, n_0: the refractive index of air at 0°C, Δn: the difference of n measured from that of the surrounding air, b: the surrounding air temperature (°K).

So we obtain

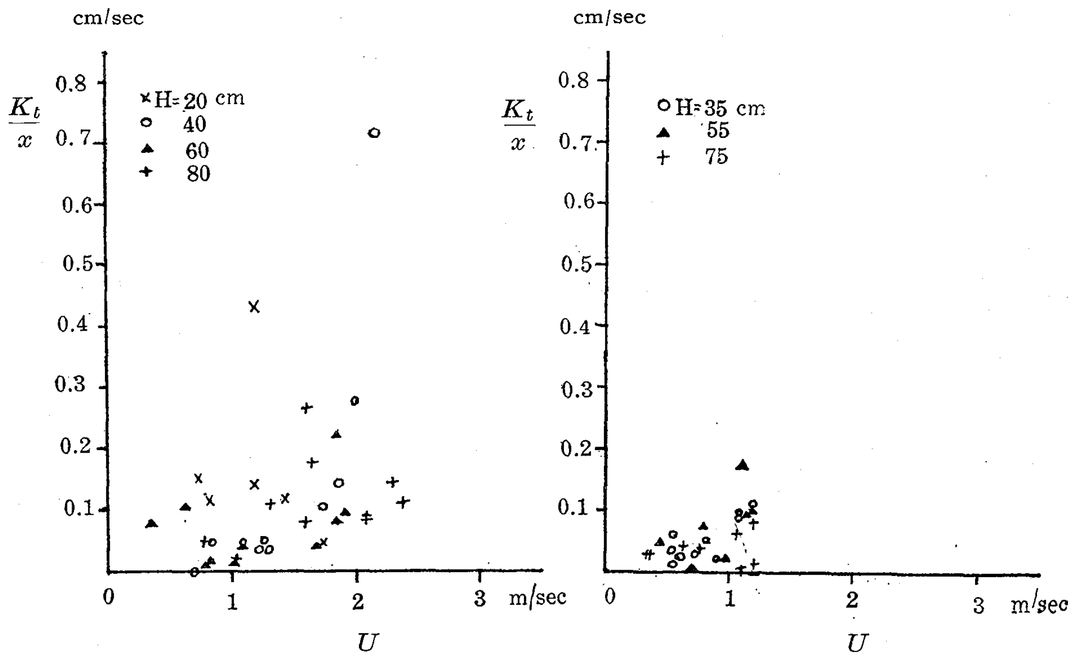


Fig. 6 (a). Diffusion of z-direction. City-Wind. Fig. 6 (b). y-direction. City-Wind.

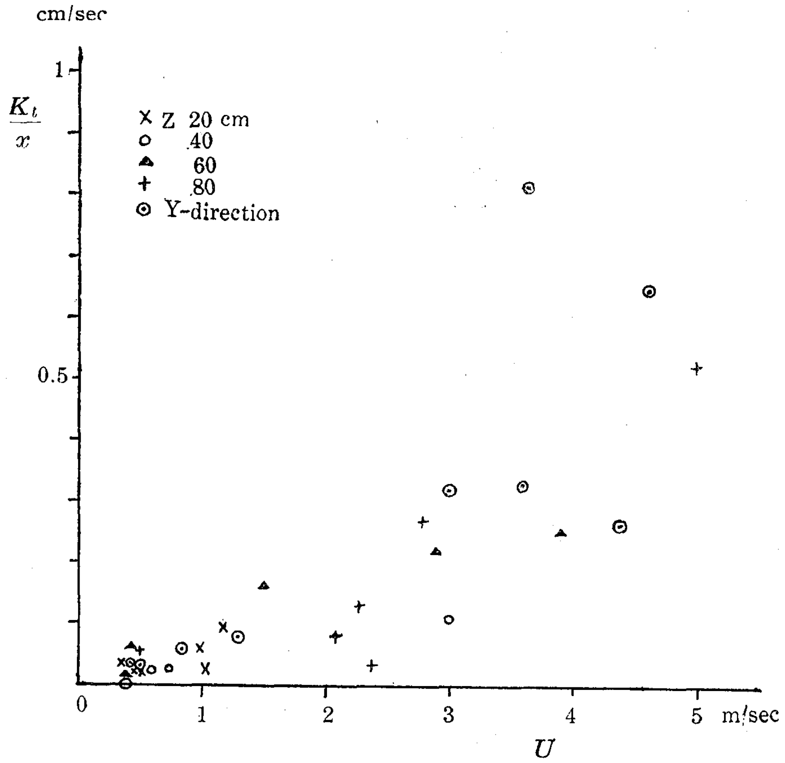


Fig. 6 (c). Sea-Wind.

$$\frac{\partial \Delta n}{\partial y}=\frac{by}{2\alpha}\operatorname{sech}^2\left(\frac{y^2+\beta}{2\alpha}\right), \quad \text{where} \quad \sqrt{\frac{a}{\Delta T_m}}=e^{\frac{\beta}{2\alpha}} \tag{11}$$

$$\frac{\partial^2 \Delta n}{\partial y^2}=\frac{b}{2\alpha}\operatorname{sech}^2\left(\frac{y^2+\beta}{2\alpha}\right)\left[1-\frac{2y^2}{\alpha}\tanh\left(\frac{y^2+\beta}{2\alpha}\right)\right] \tag{12}$$

When we put $y=y_0'$ for $\partial^2\Delta n/\partial y^2=0$ and $(\partial\Delta n/\partial y)_{y=y_0'}=N$,

$$\alpha=2y_0'^2\left[\sqrt{1+\frac{4N^2y_0'^2}{b^2}}-\frac{2Ny_0'}{b}\right] \tag{13}$$

If we use the measured y_0' for $\partial^2\Delta n/\partial y^2=0$ instead of y_0 for $\partial^2T/\partial y^2=0$, we must consider a correcting factor which is expressed by the square bracket in (13) for α. This factor can be calculated by the value N of the extremum values of $\partial\Delta n/\partial y$ and y_0, without analysing the temperature distributions. In practice, this factor is effective only in a small region near the source.

We photographed the heat wakes almost always by the exposure of 1/1000 sec. in order to neglect the effects of the observation time, but in some cases, we photographed them by the exposure of 1/300 or 1/100 sec.[4] However we could not find any clear differences about the states of the diffusion between each case. In order to heighten the sensitivity of the schlieren images and to neglect the end effect of the source, we used a line source of 6 cm. in length, so the images were resulted to be averaged over 6 cm. along the direction of the source. In such a

4) The true exposure times were 1/138 and 1/44 sec. respectively.

short time interval and distance, if the states of turbulence in both directions of wind-ward and cross-wind can be regarded as isotropic, and if the turbulence can be regarded so stationary that the time means can be reduced to the spatial means, the means over 6 cm. have the same significance to the means during 6/100 sec., when U is 1 m/sec. Therefore it is natural that there cannot be found any distinction which is due to such differences of the exposure.

Conclusion

1) The schlieren photographs of the wakes seemed to be smooth and any spotty feature could not noticed. So we can conclude that the diffusion may take place sufficiently in such a small interval of time and space.

2) As we can see in Fig. 4, instantaneous temperature distributions are regarded as normal in both directions of y and z.

3) Fig. 6 shows that K_t/x increases with the wind velocity. Plotting $\log(K_t/x)$ against $\log U$, we obtain Fig. 8, and the line in this figure may indicate that

$$K_t/x \sim U^{3/2},$$

though the dispersion of points can not determine the exact relation.

4) By the diffusion in such a small scale, it seemed that there was scarcely any difference between the diffusion in y- and z-direction, and the difference caused by the height from the ground. Furthermore, these data included the results in various meteorological conditions, but the difference due to them could not be recognized.

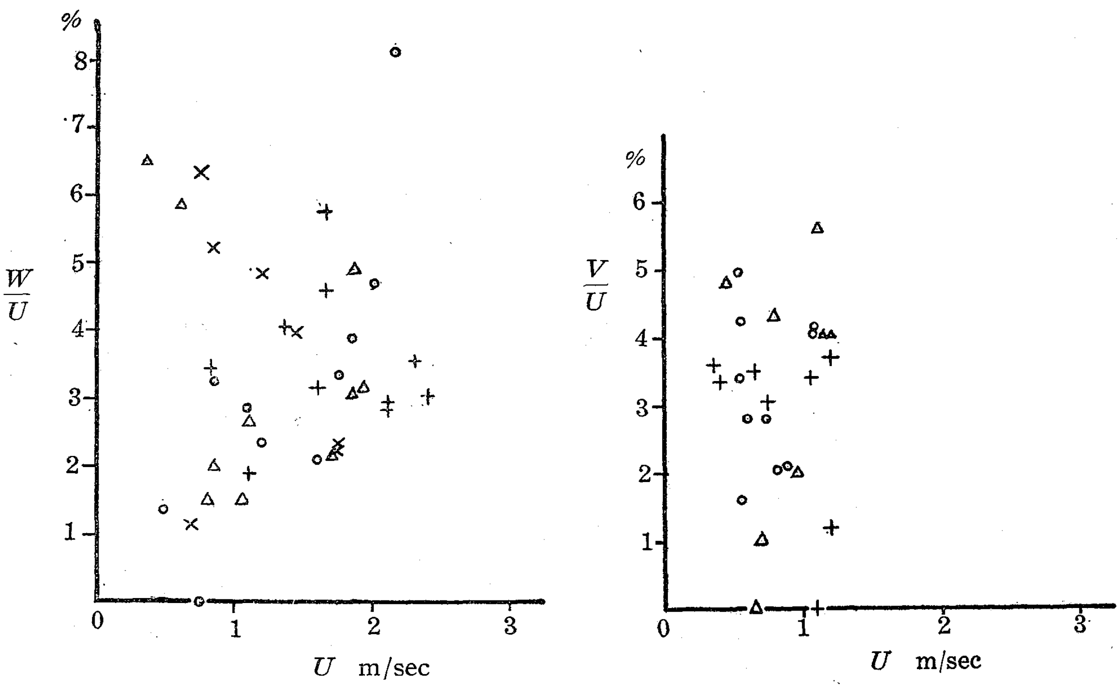


Fig. 7 (a). City-Wind. Fig. 7 (b). City-Wind.

5) Though we cannot decide firmly, because there was hardly any result for larger wind velocity in the city-wind, the diffusion coefficients in the city-wind seem to be ranged to larger values than those in see-wind. Such dispersion of points, which is also noticed in the graph of the intensity of turbulence and U, Fig. 7, indicates that even when the same wind velocity, the instantaneous states of the turbulence vary considerably during the time which is larger compared with such a short time exposure.

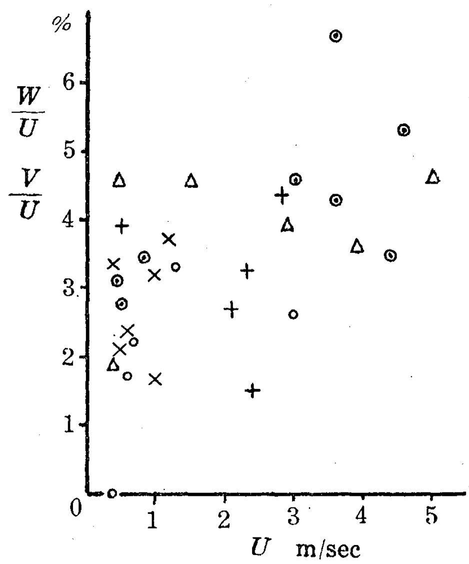


Fig. 7 (c) Sea-Wind.

Future planning

In order to make clear the diffusion mechanism more accurately and

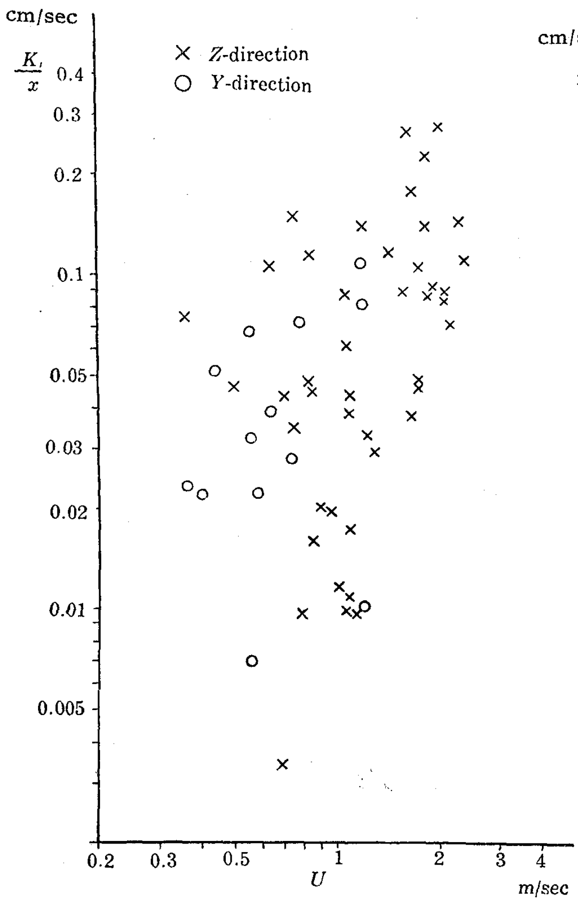


Fig. 8 (a). City-Wind.

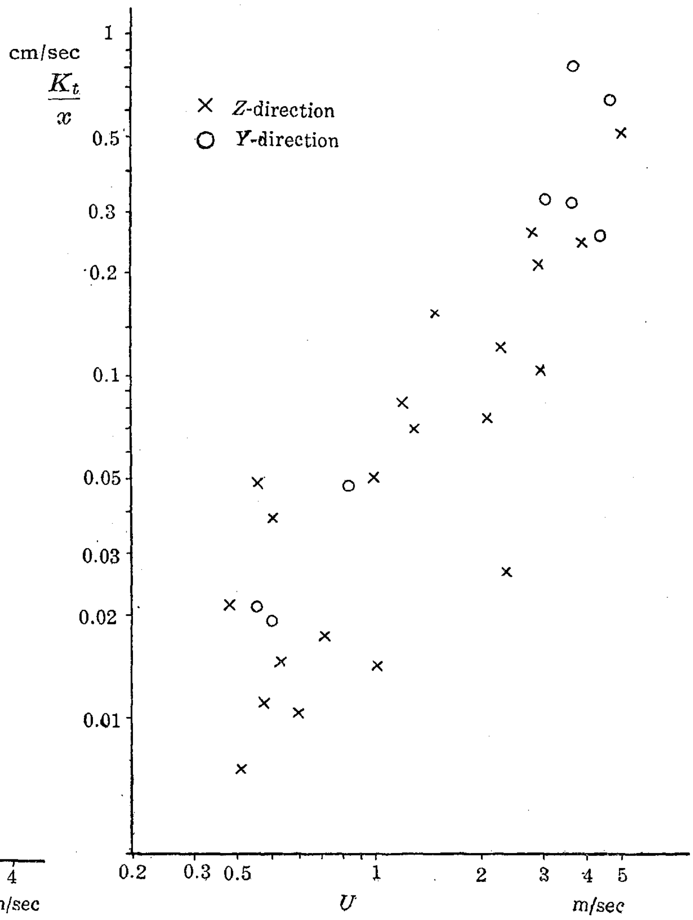


Fig. 8 (b). Sea-Wind.

more concretely, it is necessary to simplify the conditions of the observations, so we are intending to investigate the diffusion from a continuous or instantaneous point source.

On the other hand, the scale of the present observation was so small that we could not find out the effects of the heights from the ground and the difference between the directions, and that we could not observe the diffusion in the region except that in which $\overline{y^2}$ was proportionate to t^2. So we want to observe the diffusion in a more larger scale.

Literature

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