

Tables Useful for the Calculation of the Molecular Integrals VIII

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In Part VIII of this series, we give formulae for the calculation of Coulomb integrals,

$$D_{\alpha\beta\gamma\delta} = (\alpha_a\gamma_a; \beta_b\delta_b) = \int \alpha_a(1)^* \beta_b(2)^* \frac{1}{r_{12}} \gamma_a(1) \delta_b(2) d\tau_1 d\tau_2$$

and ionic integrals,

$$L_{\alpha\beta\gamma\delta} = (\alpha_a\gamma_a; \beta_a\delta_b) = \int \alpha_a(1)^* \beta_a(2)^* \frac{1}{r_{12}} \gamma_a(1) \delta_b(2) d\tau_1 d\tau_2,$$

in the case of homonuclear diatomic molecules. Here α_a, β_a, \dots are the atomic orbitals with their centres at A and $\gamma_b, \delta_b, \dots$ are those with their centres at B. Their explicit forms are:

$$h_a(1) = (1s)_a = \sqrt{\frac{\delta_1^3}{\pi}} e^{-\delta_1 r_{a1}},$$

$$s_a(1) = (2s)_a = \sqrt{\frac{\delta_2^5}{3\pi}} r_{a1} e^{-\delta_2 r_{a1}},$$

$$\sigma_a(1) = (2p\sigma)_a = \sqrt{\frac{\delta_2^5}{\pi}} r_{a1} \cos \theta_{a1} e^{-\delta_2 r_{a1}},$$

$$\pi_a(1) = (2p\pi)_a = \sqrt{\frac{\delta_2^5}{\pi}} r_{a1} \sin \theta_{a1} \cos \varphi_1 e^{-\delta_2 r_{a1}},$$

$$\pi'_a(1) = (2p\pi')_a = \sqrt{\frac{\delta_2^5}{\pi}} r_{a1} \sin \theta_{a1} \sin \varphi_1 e^{-\delta_2 r_{a1}}.$$

In the above, the coordinate system is chosen as follows; the position of electron is denoted by P, $r_{a1} = \overline{AP}$, $r_{b1} = \overline{BP}$, $\theta_{a1} = \angle BAP$, $\theta_{b1} = \angle ABP$. For convenience, we use the parameters

$$\alpha = \delta_1 R, \quad \beta = \delta_2 R,$$

$$\gamma = \frac{1}{2}(\alpha + \beta), \quad \delta = \frac{1}{2}(\alpha - \beta),$$

$$\lambda = \frac{1}{2}(\alpha + 3\beta), \quad \mu = \frac{1}{2}(3\alpha + \beta),$$

where R means internuclear distance \overline{AB} .

$D_{\alpha\beta\gamma\delta}$ are expressed in terms of the auxiliary functions,

$$A_n(\alpha) = \int_1^\infty e^{-\alpha\lambda} \lambda^n d\lambda,$$

$$B_n(\alpha) = \int_{-1}^1 e^{-\alpha\mu} \mu^n d\mu,$$

and are given in Table XXV.

$L_{\alpha\beta\gamma\delta}$ can be reduced with the use of

$$P_{n,l+\frac{1}{2}}(\kappa, \tau) = \int_0^\infty e^{-\kappa t} p_n(1, t; \tau) t^{l+\frac{1}{2}} dt,$$

$$Q_{n,l+\frac{1}{2}}(\kappa, \tau) = \int_0^\infty e^{-\kappa t} q_n(1, t; \tau) t^{l+\frac{1}{2}} dt,$$

and are given in Table XXVI. Formulae for the calculation of $P_{n,l+\frac{1}{2}}(\kappa, \tau)$ and $Q_{n,l+\frac{1}{2}}(\kappa, \tau)$ are explained in Part VI of this series.¹⁾

When orbitals α_a and γ_a are both s type ($1s$ or $2s$), it is shown that $L_{\alpha\beta\gamma\delta}$ can also be expressed in terms of $A_n(\alpha)$ and $B_n(\alpha)$. Since $A_n(\alpha)$ and $B_n(\alpha)$ can be calculated much more easily than $P_{n,l+\frac{1}{2}}(\kappa, \tau)$ and $Q_{n,l+\frac{1}{2}}(\kappa, \tau)$, we shall give in Table XXVII formulae of ionic integrals $L_{\alpha\beta\gamma\delta}$ of this sort expressed in terms of $A_n(\alpha)$ and $B_n(\alpha)$.

Although expressions for $D_{\alpha\beta\gamma\delta}$ and $L_{\alpha\beta\gamma\delta}$ are already given by many authors²⁾⁻⁶⁾ in different forms and each has its own merits and demerits, we believe that our scheme will prove its usefulness among the others.

Errata to Part VI¹⁾ and VII⁷⁾

Part VI	Page	Line	Column			
	57	21	2	3	-1	
	58	4	4-6	(0,26.50)	(0,2800.)	(0,35.50)
Part VII						
	Reprint cover	1	Calculation of the Molecular		
		2	Integrals		
		4		Eiichi Ishiguro		

Literature

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Table XXV. Formulae for Coulomb Integrals $D_{\alpha\beta\gamma\delta}$ *

$$D_{hhhh} = \frac{1}{R} \left[R \times K_{hh} - \frac{\alpha^3}{6} \left\{ 6A_1(2\alpha) + \alpha \left(3A_2(2\alpha) - A_0(2\alpha) \right) \right\} \right]$$

$$D_{hhhs} = \frac{1}{R} \left[R \times K_{hs} - \frac{1}{4} \left(\frac{\alpha^3 \beta^5}{3} \right)^{\frac{1}{2}} \left(P + \frac{\alpha}{2} Q \right) \right]$$

$$P = A_2(\mu)B_0(\delta) - 2A_1(\mu)B_1(\delta) + A_0(\mu)B_2(\delta)$$

$$Q = A_3(\mu)B_0(\delta) - A_2(\mu)B_1(\delta) - A_1(\mu)B_2(\delta) + A_0(\mu)B_3(\delta)$$

$$D_{hh\sigma\sigma} = \frac{1}{R} \left[R \times K_{h\sigma} - \frac{1}{4} (\alpha^3 \beta^5)^{\frac{1}{2}} \left(P + \frac{\alpha}{2} Q \right) \right]$$

$$\begin{cases} P = A_1(\mu)B_0(\delta) - A_0(\mu)B_1(\delta) - A_2(\mu)B_1(\delta) + A_1(\mu)B_2(\delta) \\ Q = A_2(\mu)B_0(\delta) - A_0(\mu)B_2(\delta) - A_3(\mu)B_1(\delta) + A_1(\mu)B_3(\delta) \end{cases}$$

$$D_{hs\hs} = \frac{1}{R} \left[R \times K_{ss} - \frac{1}{24} \beta^5 \left(P + \frac{\alpha}{2} Q \right) \right]$$

$$\begin{cases} P = A_3(2r)B_0(2\delta) - 3A_2(2r)B_1(2\delta) + 3A_1(2r)B_2(2\delta) - A_0(2r)B_3(2\delta) \\ Q = A_4(2r)B_0(2\delta) - 2A_3(2r)B_1(2\delta) + 2A_1(2r)B_3(2\delta) - A_0(2r)B_4(2\delta) \end{cases}$$

$$\begin{cases} P = A_3(2r)B_0(2\delta) - A_2(2r)B_1(2\delta) - A_1(2r)B_2(2\delta) + A_0(2r)B_3(2\delta) - A_4(2r)B_1(2\delta) + A_3(2r)B_2(2\delta) \\ Q = A_2(2r)B_3(2\delta) - A_1(2r)B_4(2\delta) \end{cases}$$

$$D_{hs\sigma\sigma} = \frac{1}{R} \left[R \times K_{s\sigma} - \frac{1}{8\sqrt{3}} \beta^5 \left(P + \frac{\alpha}{2} Q \right) \right]$$

$$\begin{cases} P = A_2(2r)B_0(2\delta) - 2A_1(2r)B_1(2\delta) + A_0(2r)B_2(2\delta) - A_3(2r)B_1(2\delta) + 2A_2(2r)B_2(2\delta) \\ \quad - A_1(2r)B_3(2\delta) \\ Q = A_3(2r)B_0(2\delta) - A_2(2r)B_1(2\delta) - A_1(2r)B_2(2\delta) + A_0(2r)B_3(2\delta) - A_4(2r)B_1(2\delta) + A_3(2r)B_2(2\delta) \\ \quad + A_2(2r)B_3(2\delta) - A_1(2r)B_4(2\delta) \end{cases}$$

$$\begin{cases} P = A_3(2r)B_0(2\delta) - A_2(2r)B_1(2\delta) - A_1(2r)B_2(2\delta) + A_0(2r)B_3(2\delta) - A_4(2r)B_1(2\delta) + A_3(2r)B_2(2\delta) \\ \quad + A_2(2r)B_3(2\delta) - A_1(2r)B_4(2\delta) \\ Q = A_2(2r)B_3(2\delta) - A_1(2r)B_4(2\delta) \end{cases}$$

$$D_{\sigma h\sigma\sigma} = \frac{1}{R} \left[R \times K_{\sigma\sigma} - \frac{1}{8} \beta^5 \left(P + \frac{\alpha}{2} Q \right) \right]$$

$$\begin{cases} P = A_1(2r)B_0(2\delta) - A_0(2r)B_1(2\delta) - 2A_2(2r)B_1(2\delta) + 2A_1(2r)B_3(2\delta) + A_3(2r)B_2(2\delta) \\ \quad - A_2(2r)B_3(2\delta) \\ Q = A_2(2r)B_0(2\delta) - A_0(2r)B_2(2\delta) - 2A_3(2r)B_1(2\delta) + 2A_1(2r)B_3(2\delta) + A_4(2r)B_2(2\delta) \\ \quad - A_2(2r)B_4(2\delta) \end{cases}$$

* In the following $K_{\alpha\beta}$ is defined by $\int \alpha_a(1)^* \frac{1}{r_{b1}} \beta_a(1) d\tau_1$. Formulae for $K_{\alpha\beta}$ are given in Ref. 6).

$$D_{h\pi h\pi} = \frac{3}{2}D_{hshs} - \frac{1}{2}D_{h\sigma h\sigma} = \frac{1}{R} \left[R \times K_{\pi\pi} - \frac{1}{16}\beta^5 \left(P + \frac{\alpha}{2}Q \right) \right]$$

$$\begin{cases} P = A_3(2r)B_0(2\delta) - A_3(2r)B_2(2\delta) - A_1(2r)B_0(2\delta) + A_1(2r)B_2(2\delta) - A_0(2r)B_3(2\delta) + A_0(2r)B_1(2\delta) \\ \quad + A_2(2r)B_3(2\delta) - A_2(2r)B_1(2\delta) \\ Q = A_4(2r)B_0(2\delta) - A_0(2r)B_4(2\delta) - A_2(2r)B_0(2\delta) + A_0(2r)B_2(2\delta) - A_4(2r)B_2(2\delta) + A_2(2r)B_4(2\delta) \end{cases}$$

$$D_{hhs} = \frac{1}{R} \left[R \times \frac{\sqrt{3}}{2} \frac{(\alpha^3\beta^5)^{\frac{1}{2}}}{r^4} K_{hs} - \frac{\alpha^3\beta^5}{360r^4} (30P + 20rQ + r^2U) \right]$$

$$\begin{cases} P = 3A_2(2r) + A_0(2r) \\ Q = 3A_3(2r) - A_1(2r) \\ U = 15A_4(2r) - 10A_2(2r) + 3A_0(2r) \end{cases}$$

$$D_{hhs\sigma} = \frac{1}{R} \left[R \times \frac{\sqrt{3}}{2} \frac{(\alpha^3\beta^5)^{\frac{1}{2}}}{r^4} K_{h\sigma} - \frac{\sqrt{3}\alpha^3\beta^5}{180r^4} (60P + 10rQ + r^2U) \right]$$

$$\begin{cases} P = A_1(2r) \\ Q = 3A_2(2r) - A_0(2r) \\ U = 5A_3(2r) - A_1(2r) \end{cases}$$

$$D_{hhs\sigma} = \frac{1}{R} \frac{\alpha^3\beta^5}{3r^5} \left[I - \left\{ P + rQ + \frac{r^2}{2}U + \frac{r^3}{8}V \right\} \right]$$

$$\begin{cases} I - P - rQ = 2 \left\{ \frac{3}{r^5} - e^{-2r} - 4A_4(2r) - A_1(2r) \right\} \\ U = 3A_1(2r) - A_3(2r) \\ V = \frac{1}{5} \{ -5A_4(2r) + 18A_2(2r) - 5A_0(2r) \} \end{cases}$$

$$D_{hss} = \frac{1}{R} \left[R \times \frac{\sqrt{3}(\alpha^3\alpha^5)^{\frac{1}{2}}}{2r^4} K_{ss} - \frac{(\alpha^3\beta^{15})^{\frac{1}{2}}}{16\sqrt{3}r^4} \left\{ P + \frac{2}{3}rQ + \frac{r^2}{6}U \right\} \right]$$

$$\begin{cases} P = A_3(\lambda)B_0(\delta) - 3A_2(\lambda)B_1(\delta) + 3A_1(\lambda)B_2(\delta) - A_0(\lambda)B_3(\delta) \\ Q = A_4(\lambda)B_0(\delta) - 2A_3(\lambda)B_1(\delta) + 2A_1(\lambda)B_3(\delta) - A_0(\lambda)B_4(\delta) \\ U = A_5(\lambda)B_0(\delta) - A_4(\lambda)B_1(\delta) - 2A_3(\lambda)B_2(\delta) + 2A_2(\lambda)B_3(\delta) + A_1(\lambda)B_4(\delta) - A_0(\lambda)B_5(\delta) \end{cases}$$

$$D_{hss\sigma} = \frac{1}{R} \left[R \times \frac{\sqrt{3}(\alpha^3\beta^5)^{\frac{1}{2}}}{2r^4} K_{s\sigma} - \frac{(\alpha^3\beta^{15})^{\frac{1}{2}}}{16r^4} \left\{ P + \frac{2}{3}rQ + \frac{r^2}{6}U \right\} \right]$$

$$\begin{cases} P = A_2(\lambda)B_0(\delta) - 2A_1(\lambda)B_1(\delta) + A_0(\lambda)B_2(\delta) - A_3(\lambda)B_1(\delta) + 2A_2(\lambda)B_2(\delta) - A_1(\lambda)B_3(\delta) \\ Q = A_3(\lambda)B_0(\delta) - A_2(\lambda)B_1(\delta) - A_1(\lambda)B_2(\delta) + A_0(\lambda)B_3(\delta) - A_4(\lambda)B_1(\delta) + A_3(\lambda)B_2(\delta) + A_2(\lambda)B_3(\delta) \\ \quad - A_1(\lambda)B_4(\delta) \\ U = A_4(\lambda)B_0(\delta) - 2A_2(\lambda)B_2(\delta) + A_0(\lambda)B_4(\delta) - A_3(\lambda)B_1(\delta) + 2A_3(\lambda)B_3(\delta) - A_1(\lambda)B_5(\delta) \end{cases}$$

$$D_{h\sigma s\sigma} = \frac{1}{R} \left[R \times \frac{\sqrt{3}(\alpha^3\beta^5)^{\frac{1}{2}}}{2r^4} K_{\sigma\sigma} - \frac{\sqrt{3}}{16r^4} (\alpha^3\beta^{15})^{\frac{1}{2}} \left\{ P + \frac{2}{3}rQ + \frac{r^2}{6}U \right\} \right]$$

$$\begin{cases} P = A_1(\lambda)B_0(\delta) - A_0(\lambda)B_1(\delta) - 2A_2(\lambda)B_1(\delta) + 2A_1(\lambda)B_2(\delta) + A_3(\lambda)B_2(\delta) - A_2(\lambda)B_3(\delta) \\ Q = A_2(\lambda)B_0(\delta) - A_0(\lambda)B_2(\delta) - 2A_3(\lambda)B_1(\delta) + 2A_1(\lambda)B_3(\delta) + A_4(\lambda)B_2(\delta) - A_2(\lambda)B_4(\delta) \\ U = A_3(\lambda)B_0(\delta) + A_2(\lambda)B_1(\delta) - A_1(\lambda)B_2(\delta) - A_0(\lambda)B_3(\delta) - 2A_4(\lambda)B_1(\delta) + 2A_1(\lambda)B_4(\delta) \\ \quad - 2A_3(\lambda)B_2(\delta) + 2A_2(\lambda)B_3(\delta) + A_5(\lambda)B_2(\delta) + A_4(\lambda)B_3(\delta) - A_3(\lambda)B_4(\delta) - A_2(\lambda)B_5(\delta) \end{cases}$$

$$D_{h\pi s\pi} = \frac{3}{2}D_{hsss} - \frac{1}{2}D_{h\sigma s\sigma} = \frac{1}{R} \left[R \times \frac{\sqrt{3}}{2r^4} (\alpha^3\beta^5)^{\frac{1}{2}} K_{\pi\pi} - \frac{\sqrt{3}}{32r^4} (\alpha^3\beta^{15})^{\frac{1}{2}} \left\{ P + \frac{2}{3}rQ + \frac{r^2}{6}U \right\} \right]$$

$$\begin{cases} P = A_3(\lambda)B_0(\delta) - A_3(\lambda)B_2(\delta) - A_1(\lambda)B_3(\delta) + A_1(\lambda)B_2(\delta) - A_0(\lambda)B_3(\delta) + A_0(\lambda)B_1(\delta) + A_2(\lambda)B_3(\delta) \\ \quad - A_2(\lambda)B_1(\delta) \\ Q = A_4(\lambda)B_0(\delta) - A_0(\lambda)B_4(\delta) - A_2(\lambda)B_0(\delta) + A_0(\lambda)B_2(\delta) - A_4(\lambda)B_2(\delta) + A_2(\lambda)B_4(\delta) \end{cases}$$

$$\begin{aligned}
 & \left\{ \begin{aligned} U &= A_5(\lambda)B_0(\delta) - A_0(\lambda)B_5(\delta) + A_4(\lambda)B_1(\delta) - A_1(\lambda)B_4(\delta) - A_3(\lambda)B_0(\delta) + A_0(\lambda)B_3(\delta) - A_2(\lambda)B_1(\delta) \\ & + A_1(\lambda)B_2(\delta) - A_5(\lambda)B_2(\delta) + A_2(\lambda)B_5(\delta) - A_4(\lambda)B_3(\delta) + A_3(\lambda)B_4(\delta) \end{aligned} \right. \\
 D_{hs\sigma\sigma} &= \frac{1}{R} \left[R \times K_{h\sigma} - \frac{1}{4} (\alpha^3 \beta^5)^{\frac{1}{2}} \left\{ P + \frac{3}{4} \beta Q + \frac{1}{4} \beta^2 U + \frac{1}{24} \beta^3 V \right\} \right] \\
 & \left\{ \begin{aligned} P &= A_1(\lambda)B_0(\delta) + A_0(\lambda)B_1(\delta) + A_2(\lambda)B_1(\delta) + A_1(\lambda)B_2(\delta) \\ Q &= A_2(\lambda)B_0(\delta) - A_0(\lambda)B_2(\delta) + A_3(\lambda)B_1(\delta) - A_1(\lambda)B_3(\delta) \\ U &= A_3(\lambda)B_0(\delta) - A_1(\lambda)B_2(\delta) - A_2(\lambda)B_1(\delta) + A_0(\lambda)B_3(\delta) + A_4(\lambda)B_1(\delta) - A_2(\lambda)B_3(\delta) - A_3(\lambda)B_2(\delta) \\ & + A_1(\lambda)B_4(\delta) \\ V &= A_4(\lambda)B_0(\delta) - 2A_3(\lambda)B_1(\delta) + 2A_1(\lambda)B_3(\delta) - A_0(\lambda)B_4(\delta) + A_5(\lambda)B_1(\delta) - 2A_4(\lambda)B_2(\delta) \\ & + 2A_2(\lambda)B_4(\delta) - A_1(\lambda)B_5(\delta) \end{aligned} \right. \\
 D_{hs\sigma\sigma} &= \frac{1}{R} \frac{5}{4\sqrt{3}} (\alpha^3 \beta^5)^{\frac{1}{2}} \left[I - \left\{ P + \beta Q + \frac{1}{2} \beta^2 U + \frac{3}{20} \beta^3 V + \frac{1}{40} \beta^4 W \right\} \right] \\
 & \left\{ \begin{aligned} I &= \frac{4}{r^5} - \frac{8}{3} \{ 2A_4(2r) + A_1(2r) \} \\ P + \beta Q &= \left\{ \frac{1}{\lambda^3} + \frac{1}{\delta^3} - 4A_2(2\delta) \right\} e^{-2\beta} - 4A_2(2\lambda)e^{+2\beta} + A_2(\lambda)B_1(\delta) + A_1(\lambda)B_2(\delta) \\ U &= A_1(\lambda)B_0(\delta) + A_0(\lambda)B_1(\delta) - A_3(\lambda)B_2(\delta) - A_2(\lambda)B_3(\delta) \\ V &= A_2(\lambda)B_0(\delta) - A_0(\lambda)B_2(\delta) - A_4(\lambda)B_2(\delta) + A_2(\lambda)B_4(\delta) \\ W &= A_3(\lambda)B_0(\delta) - A_2(\lambda)B_1(\delta) - A_1(\lambda)B_2(\delta) + A_0(\lambda)B_3(\delta) - A_5(\lambda)B_2(\delta) + A_4(\lambda)B_3(\delta) + A_3(\lambda)B_4(\delta) \\ & - A_2(\lambda)B_5(\delta) \end{aligned} \right. \\
 D_{hs\sigma\sigma} &= \frac{1}{R} \frac{1}{4r^5} (\alpha^3 \beta^{15})^{\frac{1}{2}} \left[I - \left\{ P + rQ + \frac{r^2}{2} U + \frac{r^3}{8} V \right\} \right] \\
 & \left\{ \begin{aligned} I &= \frac{36}{\beta^7} + \frac{4}{\beta^5} - \frac{16}{15} \{ 6A_6(2\beta) + 5A_4(2\beta) + 4A_1(2\beta) \} \\ P + rQ &= \left\{ -\frac{3}{\lambda^4} + \frac{3}{\delta^4} - 8A_3(2\delta) - 8B_3(2\delta) \right\} e^{-2r} - 8A_3(2\lambda)e^{2r} \\ & + \frac{1}{2} \{ 3A_3(\lambda)B_2(\delta) - 2A_2(\lambda)B_1(\delta) - A_1(\lambda)B_0(\delta) - 3A_2(\lambda)B_3(\delta) + 2A_1(\lambda)B_2(\delta) + A_0(\lambda)B_1(\delta) \} \\ U &= A_1(\lambda)B_0(\delta) - A_0(\lambda)B_1(\delta) - A_2(\lambda)B_1(\delta) + A_1(\lambda)B_2(\delta) - A_3(\lambda)B_2(\delta) + A_2(\lambda)B_3(\delta) + A_4(\lambda)B_3(\delta) \\ & - A_3(\lambda)B_4(\delta) \\ V &= A_2(\lambda)B_0(\delta) - A_0(\lambda)B_2(\delta) - A_3(\lambda)B_1(\delta) + A_1(\lambda)B_3(\delta) - A_4(\lambda)B_2(\delta) + A_2(\lambda)B_4(\delta) + A_5(\lambda)B_3(\delta) \\ & - A_3(\lambda)B_5(\delta) \end{aligned} \right. \\
 D_{h\pi\sigma\pi} &= \frac{3}{2} D_{hs\sigma\sigma} - \frac{1}{2} D_{h\sigma\sigma\sigma} \\
 D_{hh\pi\pi} &= \frac{1}{R} \frac{\alpha^3 \beta^5}{3r^5} \left[I - \left\{ P + rQ + \frac{r^2}{2} U + \frac{r^3}{8} V \right\} \right] \\
 & \left\{ \begin{aligned} I - P - rQ &= \frac{3}{r^5} - 4A_4(2r) + 2A_1(2r) \\ U &= A_3(2r) - A_1(2r) \\ V &= \frac{1}{5} \{ 5A_4(2r) - 6A_2(2r) + A_0(2r) \} \end{aligned} \right. \\
 D_{hs\pi\pi} &= \frac{1}{R} \frac{(\alpha^3 \beta^{15})^{\frac{1}{2}}}{8\sqrt{3}r^5} \left[I - \left\{ P + rQ + \frac{r^2}{2} U + \frac{r^3}{8} V \right\} \right] \\
 & \left\{ \begin{aligned} I &= \frac{20}{\beta^5} - \frac{32}{3} \{ A_5(2\beta) - A_2(2\beta) \} \\ P + rQ &= 2A_2(\lambda)B_2(\delta) - 2A_1(\lambda)B_1(\delta) - A_3(\lambda)B_1(\delta) - A_1(\lambda)B_3(\delta) + A_2(\lambda)B_0(\delta) + A_0(\lambda)B_2(\delta) \end{aligned} \right.
 \end{aligned}$$

$$\begin{cases}
 U = A_4(\lambda)B_0(\delta) + A_0(\lambda)B_4(\delta) - A_2(\lambda)B_0(\delta) - A_0(\lambda)B_2(\delta) - A_4(\lambda)B_2(\delta) - A_2(\lambda)B_4(\delta) \\
 \quad + 2\{A_1(\lambda)B_1(\delta) + A_2(\lambda)B_2(\delta) + A_3(\lambda)B_3(\delta) - A_3(\lambda)B_1(\delta) - A_1(\lambda)B_3(\delta)\} \\
 V = A_5(\lambda)B_0(\delta) + A_0(\lambda)B_5(\delta) - A_1(\lambda)B_0(\delta) - A_0(\lambda)B_1(\delta) - A_3(\lambda)B_2(\delta) - A_2(\lambda)B_3(\delta) \\
 \quad - A_1(\lambda)B_4(\delta) - A_4(\lambda)B_1(\delta) + A_1(\lambda)B_2(\delta) + A_2(\lambda)B_1(\delta) + A_3(\lambda)B_4(\delta) + A_4(\lambda)B_3(\delta)
 \end{cases}$$

$$D_{h\sigma\alpha\pi} = \frac{1}{R} \frac{(\alpha^2\beta^{15})^{\frac{1}{2}}}{8\gamma^5} \left[I - \left\{ P + \gamma Q + \frac{\gamma^2}{2} U + \frac{\gamma^3}{8} V \right\} \right]$$

$$\begin{cases}
 I = \frac{36}{\beta^7} - \frac{32}{5} \{A_6(2\beta) - A_1(2\beta)\} \\
 P + \gamma Q = \frac{1}{2} \{3A_1(\lambda)B_0(\delta) - A_3(\lambda)B_0(\delta) - 3A_0(\lambda)B_1(\delta) - 3A_2(\lambda)B_1(\delta) + 3A_4(\lambda)B_2(\delta) \\
 \quad + 3A_1(\lambda)B_2(\delta) - 3A_2(\lambda)B_3(\delta) + A_0(\lambda)B_4(\delta)\} \\
 U = A_1(\lambda)B_0(\delta) - A_1(\lambda)B_0(\delta) - A_4(\lambda)B_1(\delta) + A_4(\lambda)B_3(\delta) - A_0(\lambda)B_3(\delta) + A_0(\lambda)B_1(\delta) + A_1(\lambda)B_4(\delta) \\
 \quad - A_3(\lambda)B_4(\delta) \\
 V = A_4(\lambda)B_0(\delta) - A_2(\lambda)B_0(\delta) - A_4(\lambda)B_2(\delta) - A_3(\lambda)B_1(\delta) + A_1(\lambda)B_1(\delta) + A_5(\lambda)B_1(\delta) - A_0(\lambda)B_4(\delta) \\
 \quad + A_0(\lambda)B_2(\delta) + A_2(\lambda)B_4(\delta) + A_1(\lambda)B_5(\delta) - A_1(\lambda)B_3(\delta) - A_1(\lambda)B_5(\delta)
 \end{cases}$$

Table XXVI. Formulae for Ionic Integrals $L_{\alpha\beta\gamma\delta}$ in terms of
$$P_{n, l + \frac{1}{2}}(\kappa, \tau) \text{ and } Q_{n, l + \frac{1}{2}}(\kappa, \tau)$$

$$L_{h\bar{h}h\bar{h}} = \frac{1}{R} 4\alpha^{\frac{1}{2}} \left[P_{0, \frac{1}{2}}(1, \alpha) - \left\{ P_{0, \frac{1}{2}}(3, \alpha) + P_{0, \frac{3}{2}}(3, \alpha) \right\} \right]$$

$$L_{h\bar{h}h\bar{h}} = \frac{1}{R} \frac{4}{\sqrt{3}} \frac{\alpha^{\frac{3}{2}}}{\beta} \left[Q_{0, \frac{1}{2}}\left(\frac{\alpha}{\beta}, \beta\right) - Q_{0, \frac{1}{2}}\left(\frac{3\alpha}{\beta}, \beta\right) - \frac{\alpha}{\beta} Q_{0, \frac{3}{2}}\left(\frac{3\alpha}{\beta}, \beta\right) \right]$$

$$L_{h\bar{h}h\bar{h}} = \frac{1}{R} \frac{4}{\sqrt{3}} \frac{\beta^{\frac{5}{2}}}{\alpha^2} \left[P_{0, \frac{3}{2}}\left(\frac{\beta}{\alpha}, \alpha\right) - P_{0, \frac{3}{2}}\left(2 + \frac{\beta}{\alpha}, \alpha\right) - P_{0, \frac{5}{2}}\left(2 + \frac{\beta}{\alpha}, \alpha\right) \right]$$

$$L_{h\bar{h}h\bar{h}} = \frac{1}{R} \left[4\alpha^{\frac{3}{2}} \left\{ P_{0, \frac{1}{2}}\left(\frac{\alpha}{\beta}, \beta\right) - P_{0, \frac{1}{2}}\left(\frac{3\alpha}{\beta}, \beta\right) - \frac{\alpha}{\beta} P_{0, \frac{3}{2}}\left(\frac{3\alpha}{\beta}, \beta\right) \right\} \right. \\
 \left. - 4\frac{\alpha^{\frac{3}{2}}}{\beta} \left\{ P_{1, \frac{3}{2}}\left(\frac{\alpha}{\beta}, \beta\right) - P_{1, \frac{3}{2}}\left(\frac{3\alpha}{\beta}, \beta\right) - \frac{\alpha}{\beta} P_{1, \frac{5}{2}}\left(\frac{3\alpha}{\beta}, \beta\right) \right\} \right]$$

$$L_{h\sigma h\bar{h}} = \frac{1}{R} 4 \frac{\beta^{\frac{5}{2}}}{\alpha^2} \left[P_{1, \frac{3}{2}}\left(\frac{\beta}{\alpha}, \alpha\right) - P_{1, \frac{3}{2}}\left(\frac{\beta}{\alpha} + 2, \alpha\right) - P_{1, \frac{5}{2}}\left(\frac{\beta}{\alpha} + 2, \alpha\right) \right]$$

$$L_{h\bar{h}h\bar{h}} = \frac{1}{R} \frac{4}{3} \beta^{\frac{1}{2}} \left[Q_{0, \frac{3}{2}}(1, \beta) - Q_{0, \frac{3}{2}}\left(1 + 2\frac{\alpha}{\beta}, \beta\right) - \frac{\alpha}{\beta} Q_{0, \frac{5}{2}}\left(1 + 2\frac{\alpha}{\beta}, \beta\right) \right]$$

$$L_{h\bar{h}h\bar{h}} = \frac{1}{R} \frac{4}{\sqrt{3}} \beta^{\frac{3}{2}} \left[\beta \left\{ P_{0, \frac{3}{2}}(1, \beta) - P_{0, \frac{3}{2}}\left(1 + 2\frac{\alpha}{\beta}, \beta\right) - \frac{\alpha}{\beta} P_{0, \frac{5}{2}}\left(1 + 2\frac{\alpha}{\beta}, \beta\right) \right\} \right. \\
 \left. - \left\{ P_{1, \frac{5}{2}}(1, \beta) - P_{1, \frac{5}{2}}\left(1 + 2\frac{\alpha}{\beta}, \beta\right) - \frac{\alpha}{\beta} P_{1, \frac{7}{2}}\left(1 + 2\frac{\alpha}{\beta}, \beta\right) \right\} \right]$$

$$L_{h\sigma h\bar{h}} = \frac{1}{R} \frac{4}{\sqrt{3}} \beta^{\frac{1}{2}} \left[Q_{1, \frac{3}{2}}(1, \beta) - Q_{1, \frac{3}{2}}\left(1 + 2\frac{\alpha}{\beta}, \beta\right) - \frac{\alpha}{\beta} Q_{1, \frac{5}{2}}\left(1 + 2\frac{\alpha}{\beta}, \beta\right) \right]$$

$$L_{h\sigma h\bar{h}} = \frac{1}{R} \left[4\beta^{\frac{3}{2}} \left\{ P_{1, \frac{3}{2}}(1, \beta) - P_{1, \frac{3}{2}}\left(1 + 2\frac{\alpha}{\beta}, \beta\right) - \frac{\alpha}{\beta} P_{1, \frac{5}{2}}\left(1 + 2\frac{\alpha}{\beta}, \beta\right) \right\} \right. \\
 \left. - \frac{4}{3} \beta^{\frac{1}{2}} \left\{ P_{0, \frac{5}{2}}(1, \beta) - P_{0, \frac{5}{2}}\left(1 + 2\frac{\alpha}{\beta}, \beta\right) - \frac{\alpha}{\beta} P_{0, \frac{7}{2}}\left(1 + 2\frac{\alpha}{\beta}, \beta\right) \right\} \right. \\
 \left. - \frac{8}{3} \beta^{\frac{1}{2}} \left\{ P_{2, \frac{5}{2}}(1, \beta) - P_{2, \frac{5}{2}}\left(1 + 2\frac{\alpha}{\beta}, \beta\right) - \frac{\alpha}{\beta} P_{2, \frac{7}{2}}\left(1 + 2\frac{\alpha}{\beta}, \beta\right) \right\} \right]$$

$$L_{h\pi h\pi} = \frac{1}{R} \frac{4}{3} \beta^{\frac{1}{2}} \left[P_{0, \frac{1}{2}}(1, \beta) - \frac{\alpha}{\beta} P_{0, \frac{1}{2}}\left(2\frac{\alpha}{\beta} + 1, \beta\right) - P_{0, \frac{1}{2}}\left(2\frac{\alpha}{\beta} + 1, \beta\right) - P_{2, \frac{1}{2}}(1, \beta) \right. \\ \left. + \frac{\alpha}{\beta} P_{2, \frac{1}{2}}\left(2\frac{\alpha}{\beta} + 1, \beta\right) + P_{2, \frac{1}{2}}\left(2\frac{\alpha}{\beta} + 1, \beta\right) \right]$$

$$L_{hhs\bar{h}} = \frac{1}{R} \frac{16}{\sqrt{3}} \frac{\alpha^2 \beta^{\frac{3}{2}}}{(2r)^4} \left[6P_{0, \frac{1}{2}}(1, \alpha) - 6P_{0, \frac{1}{2}}\left(2 + \frac{\beta}{\alpha}, \alpha\right) - 4\left(\frac{2r}{\alpha}\right) P_{0, \frac{1}{2}}\left(2 + \frac{\beta}{\alpha}, \alpha\right) \right. \\ \left. - \left(\frac{2r}{\alpha}\right)^2 P_{0, \frac{1}{2}}\left(2 + \frac{\beta}{\alpha}, \alpha\right) \right]$$

$$L_{hhs\bar{s}} = \frac{1}{R} \frac{16}{3} \frac{\alpha^2 \beta^{\frac{3}{2}}}{(2r)^4} \left[6Q_{0, \frac{1}{2}}\left(\frac{\alpha}{\beta}, \beta\right) - 6Q_{0, \frac{1}{2}}\left(1 + \frac{2\alpha}{\beta}, \beta\right) - 4\frac{2r}{\beta} Q_{0, \frac{1}{2}}\left(1 + 2\frac{\alpha}{\beta}, \beta\right) \right. \\ \left. - \left(\frac{2r}{\beta}\right)^2 Q_{0, \frac{1}{2}}\left(1 + 2\frac{\alpha}{\beta}, \beta\right) \right]$$

$$L_{hss\bar{h}} = \frac{1}{R} \frac{16}{3} \frac{\alpha^{-\frac{1}{2}} \beta^{\frac{3}{2}}}{(2r)^4} \left[6P_{0, \frac{1}{2}}\left(\frac{\beta}{\alpha}, \alpha\right) - 6P_{0, \frac{1}{2}}\left(1 + 2\frac{\beta}{\alpha}, \alpha\right) - 4\frac{2r}{\alpha} P_{0, \frac{1}{2}}\left(1 + 2\frac{\beta}{\alpha}, \alpha\right) \right. \\ \left. - \left(\frac{2r}{\alpha}\right)^2 P_{0, \frac{1}{2}}\left(1 + 2\frac{\beta}{\alpha}, \alpha\right) \right]$$

$$L_{hss\bar{s}} = \frac{1}{R} \frac{16}{\sqrt{3}} \frac{\alpha^3 \beta^{\frac{3}{2}}}{(2r)^4} \left[\beta \left\{ 6P_{0, \frac{1}{2}}\left(\frac{\alpha}{\beta}, \beta\right) - 6P_{0, \frac{1}{2}}\left(1 + 2\frac{\alpha}{\beta}, \beta\right) - 4\frac{2r}{\beta} P_{0, \frac{1}{2}}\left(1 + 2\frac{\alpha}{\beta}, \beta\right) \right. \right. \\ \left. \left. - \left(\frac{2r}{\beta}\right)^2 P_{0, \frac{1}{2}}\left(1 + 2\frac{\alpha}{\beta}, \beta\right) \right\} - \left\{ 6P_{1, \frac{1}{2}}\left(\frac{\alpha}{\beta}, \beta\right) - 6P_{1, \frac{1}{2}}\left(1 + 2\frac{\alpha}{\beta}, \beta\right) - 4\frac{2r}{\beta} P_{1, \frac{1}{2}}\left(1 + 2\frac{\alpha}{\beta}, \beta\right) \right. \right. \\ \left. \left. - \left(\frac{2r}{\beta}\right)^2 P_{1, \frac{1}{2}}\left(1 + 2\frac{\alpha}{\beta}, \beta\right) \right\} \right]$$

$$L_{hcs\bar{h}} = \frac{1}{R} \frac{16}{\sqrt{3}} \frac{\alpha^{-\frac{1}{2}} \beta^{\frac{3}{2}}}{(2r)^4} \left[6P_{1, \frac{1}{2}}\left(\frac{\beta}{\alpha}, \alpha\right) - 6P_{1, \frac{1}{2}}\left(1 + 2\frac{\beta}{\alpha}, \alpha\right) - 4\frac{2r}{\alpha} P_{1, \frac{1}{2}}\left(1 + 2\frac{\beta}{\alpha}, \alpha\right) \right. \\ \left. - \left(\frac{2r}{\alpha}\right)^2 P_{1, \frac{1}{2}}\left(1 + 2\frac{\beta}{\alpha}, \alpha\right) \right]$$

$$L_{hss\bar{s}} = \frac{1}{R} \frac{16}{\sqrt{27}} \frac{\alpha^{\frac{3}{2}} \beta^3}{(2r)^4} \left[6Q_{0, \frac{1}{2}}(1, \beta) - 6Q_{0, \frac{1}{2}}\left(2 + \frac{\alpha}{\beta}, \beta\right) - 4\frac{2r}{\beta} Q_{0, \frac{1}{2}}\left(2 + \frac{\alpha}{\beta}, \beta\right) \right. \\ \left. - \left(\frac{2r}{\beta}\right)^2 Q_{0, \frac{1}{2}}\left(2 + \frac{\alpha}{\beta}, \beta\right) \right]$$

$$L_{hss\bar{s}} = \frac{1}{R} \frac{16}{3} \frac{\alpha^{\frac{3}{2}} \beta^3}{(2r)^4} \left[\beta \left\{ 6P_{0, \frac{1}{2}}(1, \beta) - 6P_{0, \frac{1}{2}}\left(2 + \frac{\alpha}{\beta}, \beta\right) - 4\frac{2r}{\beta} P_{0, \frac{1}{2}}\left(2 + \frac{\alpha}{\beta}, \beta\right) \right. \right. \\ \left. \left. - \left(\frac{2r}{\beta}\right)^2 P_{0, \frac{1}{2}}\left(2 + \frac{\alpha}{\beta}, \beta\right) \right\} - \left\{ 6P_{1, \frac{1}{2}}(1, \beta) - 6P_{1, \frac{1}{2}}\left(2 + \frac{\alpha}{\beta}, \beta\right) \right. \right. \\ \left. \left. - 4\frac{2r}{\beta} P_{1, \frac{1}{2}}\left(2 + \frac{\alpha}{\beta}, \beta\right) + \left(\frac{2r}{\beta}\right)^2 P_{1, \frac{1}{2}}\left(2 + \frac{\alpha}{\beta}, \beta\right) \right\} \right]$$

$$L_{hcs\bar{s}} = \frac{1}{R} \frac{16}{3} \frac{\alpha^{\frac{3}{2}} \beta^3}{(2r)^4} \left[6Q_{1, \frac{1}{2}}(1, \beta) - 6Q_{1, \frac{1}{2}}\left(2 + \frac{\alpha}{\beta}, \beta\right) - 4\frac{2r}{\beta} Q_{1, \frac{1}{2}}\left(2 + \frac{\alpha}{\beta}, \beta\right) - \left(\frac{2r}{\beta}\right)^2 Q_{1, \frac{1}{2}}\left(2 + \frac{\alpha}{\beta}, \beta\right) \right]$$

$$L_{hcs\bar{s}} = \frac{1}{R} \frac{16}{\sqrt{3}} \frac{\alpha^{\frac{3}{2}} \beta^3}{(2r)^4} \left[\beta \left\{ 6P_{1, \frac{1}{2}}(1, \beta) - 6P_{1, \frac{1}{2}}\left(2 + \frac{\alpha}{\beta}, \beta\right) - 4\frac{2r}{\beta} P_{1, \frac{1}{2}}\left(2 + \frac{\alpha}{\beta}, \beta\right) \right. \right. \\ \left. \left. - \left(\frac{2r}{\beta}\right)^2 P_{1, \frac{1}{2}}\left(2 + \frac{\alpha}{\beta}, \beta\right) \right\} - \frac{1}{3} \left\{ 6P_{0, \frac{1}{2}}(1, \beta) - 6P_{0, \frac{1}{2}}\left(2 + \frac{\alpha}{\beta}, \beta\right) - 4\frac{2r}{\beta} P_{0, \frac{1}{2}}\left(2 + \frac{\alpha}{\beta}, \beta\right) \right. \right. \\ \left. \left. - \left(\frac{2r}{\beta}\right)^2 P_{0, \frac{1}{2}}\left(2 + \frac{\alpha}{\beta}, \beta\right) \right\} - \frac{2}{3} \left\{ 6P_{2, \frac{1}{2}}(1, \beta) - 6P_{2, \frac{1}{2}}\left(2 + \frac{\alpha}{\beta}, \beta\right) - 4\frac{2r}{\beta} P_{2, \frac{1}{2}}\left(2 + \frac{\alpha}{\beta}, \beta\right) \right. \right. \\ \left. \left. - \left(\frac{2r}{\beta}\right)^2 P_{2, \frac{1}{2}}\left(2 + \frac{\alpha}{\beta}, \beta\right) \right\} \right]$$

$$\begin{aligned}
L_{h\pi s\bar{\pi}} &= \frac{1}{R} \frac{16}{3\sqrt{3}} \frac{\alpha^{\frac{3}{2}}\beta^3}{(2r)^4} \left[6P_{0, \frac{5}{2}}(1, \beta) - \left\{ \left(\frac{2r}{\beta} \right)^2 P_{0, \frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 4 \frac{2r}{\beta} P_{0, \frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right. \right. \\
&\quad \left. \left. + 6P_{0, \frac{5}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right\} - 6P_{2, \frac{5}{2}}(1, \beta) + \left\{ \left(\frac{2r}{\beta} \right)^3 P_{2, \frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 4 \frac{2r}{\beta} P_{2, \frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right. \right. \\
&\quad \left. \left. + 6P_{2, \frac{5}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right\} \right] \\
L_{hh\sigma\bar{h}} &= \frac{1}{R} 128 \frac{\alpha^3\beta^{\frac{5}{2}}}{(2r)^5} \left[P_{1, -\frac{1}{2}}(1, \alpha) - \frac{1}{8} \left\{ \left(\frac{2r}{\alpha} \right)^3 P_{1, \frac{1}{2}} \left(1 + \frac{2r}{\alpha}, \alpha \right) + 4 \left(\frac{2r}{\alpha} \right)^2 P_{1, \frac{3}{2}} \left(1 + \frac{2r}{\alpha}, \alpha \right) \right. \right. \\
&\quad \left. \left. + 8 \frac{2r}{\alpha} P_{1, \frac{1}{2}} \left(1 + \frac{2r}{\alpha}, \alpha \right) + 8P_{1, -\frac{1}{2}} \left(1 + \frac{2r}{\alpha}, \alpha \right) \right\} \right] \\
L_{hh\sigma\bar{s}} &= \frac{1}{R} \frac{128}{\sqrt{3}} \frac{\alpha^3\beta^{\frac{5}{2}}}{(2r)^5} \left[Q_{1, -\frac{1}{2}} \left(\frac{\alpha}{\beta}, \beta \right) - \frac{1}{8} \left\{ \left(\frac{2r}{\beta} \right)^3 Q_{1, \frac{1}{2}} \left(2 \frac{\alpha}{\beta} + 1, \beta \right) + 4 \left(\frac{2r}{\beta} \right)^2 Q_{1, \frac{3}{2}} \left(2 \frac{\alpha}{\beta} + 1, \beta \right) \right. \right. \\
&\quad \left. \left. + 8 \frac{2r}{\beta} Q_{1, \frac{1}{2}} \left(2 \frac{\alpha}{\beta} + 1, \beta \right) + 8Q_{1, -\frac{1}{2}} \left(2 \frac{\alpha}{\beta} + 1, \beta \right) \right\} \right] \\
L_{hs\sigma\bar{h}} &= \frac{1}{R} \frac{128}{\sqrt{3}} \frac{\alpha^{\frac{1}{2}}\beta^5}{(2r)^5} \left[P_{1, \frac{1}{2}} \left(\frac{\beta}{\alpha}, \alpha \right) - \frac{1}{8} \left\{ \left(\frac{2r}{\alpha} \right)^3 P_{1, \frac{3}{2}} \left(2 \frac{\beta}{\alpha} + 1, \alpha \right) + 4 \left(\frac{2r}{\alpha} \right)^2 P_{1, \frac{5}{2}} \left(2 \frac{\beta}{\alpha} + 1, \alpha \right) \right. \right. \\
&\quad \left. \left. + 8 \frac{2r}{\alpha} P_{1, \frac{3}{2}} \left(2 \frac{\beta}{\alpha} + 1, \alpha \right) + 8P_{1, \frac{1}{2}} \left(2 \frac{\beta}{\alpha} + 1, \alpha \right) \right\} \right] \\
L_{hs\sigma\bar{s}} &= \frac{1}{R} \left[128 \frac{\alpha^3\beta^{\frac{5}{2}}}{(2r)^5} \left[P_{1, -\frac{1}{2}} \left(\frac{\alpha}{\beta}, \beta \right) - \frac{1}{8} \left\{ \left(\frac{2r}{\beta} \right)^3 P_{1, \frac{1}{2}} \left(2 \frac{\alpha}{\beta} + 1, \beta \right) + 4 \left(\frac{2r}{\beta} \right)^2 P_{1, \frac{3}{2}} \left(2 \frac{\alpha}{\beta} + 1, \beta \right) \right. \right. \right. \\
&\quad \left. \left. + 8 \left(\frac{2r}{\beta} \right) P_{1, \frac{1}{2}} \left(2 \frac{\alpha}{\beta} + 1, \beta \right) + 8P_{1, -\frac{1}{2}} \left(2 \frac{\alpha}{\beta} + 1, \beta \right) \right\} \right] \\
&\quad - \frac{128}{3} \frac{\alpha^3\alpha^{\frac{5}{2}}}{(2r)^5} \left[P_{0, \frac{1}{2}} \left(\frac{\alpha}{\beta}, \beta \right) - \frac{1}{8} \left\{ \left(\frac{2r}{\beta} \right)^3 P_{0, \frac{3}{2}} \left(2 \frac{\alpha}{\beta} + 1, \beta \right) + 4 \left(\frac{2r}{\beta} \right)^2 P_{0, \frac{5}{2}} \left(2 \frac{\alpha}{\beta} + 1, \beta \right) \right. \right. \\
&\quad \left. \left. + 8 \left(\frac{2r}{\beta} \right) P_{0, \frac{3}{2}} \left(2 \frac{\alpha}{\beta} + 1, \beta \right) + 8P_{0, \frac{1}{2}} \left(2 \frac{\alpha}{\beta} + 1, \beta \right) \right\} + 2P_{2, \frac{1}{2}} \left(\frac{\alpha}{\beta}, \beta \right) - \frac{1}{4} \left\{ \left(\frac{2r}{\beta} \right)^3 P_{2, \frac{3}{2}} \left(2 \frac{\alpha}{\beta} + 1, \beta \right) \right. \right. \\
&\quad \left. \left. + 4 \left(\frac{2r}{\beta} \right)^2 P_{2, \frac{5}{2}} \left(2 \frac{\alpha}{\beta} + 1, \beta \right) + 8 \left(\frac{2r}{\beta} \right) P_{2, \frac{3}{2}} \left(2 \frac{\alpha}{\beta} + 1, \beta \right) + 8P_{2, \frac{1}{2}} \left(2 \frac{\alpha}{\beta} + 1, \beta \right) \right\} \right] \Big] \\
L_{hs\sigma\bar{h}} &= \frac{1}{R} \left[\frac{128}{3} \frac{\alpha^{\frac{1}{2}}\beta^5}{(2r)^5} \left[P_{0, \frac{1}{2}} \left(\frac{\beta}{\alpha}, \alpha \right) - \frac{1}{8} \left\{ \left(\frac{2r}{\alpha} \right)^3 P_{0, \frac{3}{2}} \left(2 \frac{\beta}{\alpha} + 1, \alpha \right) + 4 \left(\frac{2r}{\alpha} \right)^2 P_{0, \frac{5}{2}} \left(2 \frac{\beta}{\alpha} + 1, \alpha \right) \right. \right. \right. \\
&\quad \left. \left. + 8 \frac{2r}{\alpha} P_{0, \frac{3}{2}} \left(2 \frac{\beta}{\alpha} + 1, \alpha \right) + 8P_{0, \frac{1}{2}} \left(2 \frac{\beta}{\alpha} + 1, \alpha \right) \right\} \right] \\
&\quad + \frac{256}{3} \frac{\alpha^{\frac{1}{2}}\beta^5}{(2r)^5} \left[P_{2, \frac{1}{2}} \left(\frac{\beta}{\alpha}, \alpha \right) - \frac{1}{8} \left\{ \left(\frac{2r}{\alpha} \right)^3 P_{2, \frac{3}{2}} \left(2 \frac{\beta}{\alpha} + 1, \alpha \right) + 4 \left(\frac{2r}{\alpha} \right)^2 P_{2, \frac{5}{2}} \left(2 \frac{\beta}{\alpha} + 1, \alpha \right) \right. \right. \\
&\quad \left. \left. + 8 \frac{2r}{\alpha} P_{2, \frac{3}{2}} \left(2 \frac{\beta}{\alpha} + 1, \alpha \right) + 8P_{2, \frac{1}{2}} \left(2 \frac{\beta}{\alpha} + 1, \alpha \right) \right\} \right] \Big] \\
L_{hs\sigma\bar{s}} &= \frac{1}{R} \frac{128}{3} \frac{\alpha^{\frac{3}{2}}\beta^4}{(2r)^5} \left[Q_{1, \frac{1}{2}}(1, \beta) - \frac{1}{8} \left\{ \left(\frac{2r}{\beta} \right)^3 Q_{1, \frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 4 \left(\frac{2r}{\beta} \right)^2 Q_{1, \frac{5}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right. \right. \\
&\quad \left. \left. + 8 \left(\frac{2r}{\beta} \right) Q_{1, \frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 8Q_{1, \frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right\} \right] \\
L_{hs\sigma\bar{o}} &= \frac{1}{R} \left[\frac{128}{\sqrt{3}} \frac{\alpha^{\frac{3}{2}}\beta^5}{(2r)^5} \left[P_{1, \frac{1}{2}}(1, \beta) - \frac{1}{8} \left\{ \left(\frac{2r}{\beta} \right)^3 P_{1, \frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 4 \left(\frac{2r}{\beta} \right)^2 P_{1, \frac{5}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right. \right. \right. \\
&\quad \left. \left. + 8 \left(\frac{2r}{\beta} \right) P_{1, \frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 8P_{1, \frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right\} \right] \Big]
\end{aligned}$$

$$\begin{aligned}
& -\frac{128}{3\sqrt{3}} \frac{\alpha^{\frac{3}{2}}\beta^4}{(2r)^5} \left[P_{0,\frac{3}{2}}(1,\beta) - \frac{1}{8} \left\{ \left(\frac{2r}{\beta} \right)^3 P_{0,\frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 4 \left(\frac{2r}{\beta} \right)^2 P_{0,\frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right. \right. \\
& + 8 \left(\frac{2r}{\beta} \right) P_{0,\frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 8 P_{0,\frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \left. \right\} + 2 P_{2,\frac{3}{2}}(1,\beta) - \frac{1}{4} \left\{ \left(\frac{2r}{\beta} \right)^3 P_{2,\frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right. \\
& + 4 \left(\frac{2r}{\beta} \right)^2 P_{2,\frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 8 \left(\frac{2r}{\beta} \right) P_{2,\frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 8 P_{2,\frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \left. \right\} \left. \right] \\
L_{h\sigma\sigma\bar{s}} &= \frac{1}{R} \frac{128}{3\sqrt{3}} \frac{\alpha^{\frac{3}{2}}\beta^4}{(2r)^5} \left[Q_{0,\frac{1}{2}}(1,\beta) - \frac{1}{8} \left\{ \left(\frac{2r}{\beta} \right)^3 Q_{0,\frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 4 \left(\frac{2r}{\beta} \right)^2 Q_{0,\frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right. \right. \\
& + 8 \left(\frac{2r}{\beta} \right) Q_{0,\frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 8 Q_{0,\frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \left. \right\} + 2 Q_{2,\frac{1}{2}}(1,\beta) - \frac{1}{4} \left\{ \left(\frac{2r}{\beta} \right)^3 Q_{2,\frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right. \\
& + 4 \left(\frac{2r}{\beta} \right)^2 Q_{2,\frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 8 \left(\frac{2r}{\beta} \right) Q_{2,\frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 8 Q_{2,\frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \left. \right\} \left. \right] \\
L_{h\sigma\sigma\sigma\bar{\sigma}} &= \frac{1}{R} \left[\frac{128}{3} \frac{\alpha^{\frac{3}{2}}\beta^5}{(2r)^5} \left[P_{0,\frac{1}{2}}(1,\beta) - \frac{1}{8} \left\{ \left(\frac{2r}{\beta} \right)^3 P_{0,\frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 4 \left(\frac{2r}{\beta} \right)^2 P_{0,\frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right. \right. \right. \\
& + 8 \left(\frac{2r}{\beta} \right) P_{0,\frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 8 P_{0,\frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \left. \right\} + 2 P_{2,\frac{1}{2}}(1,\beta) - \frac{1}{4} \left\{ \left(\frac{2r}{\beta} \right)^3 P_{2,\frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right. \\
& + 4 \left(\frac{2r}{\beta} \right)^2 P_{2,\frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 8 \left(\frac{2r}{\beta} \right) P_{2,\frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 8 P_{2,\frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \left. \right\} \left. \right] \\
& - \frac{128}{5} \frac{\alpha^{\frac{3}{2}}\beta^4}{(2r)^5} \left[3 P_{1,\frac{3}{2}}(1,\beta) - \frac{3}{8} \left\{ \left(\frac{2r}{\beta} \right)^3 P_{1,\frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 4 \left(\frac{2r}{\beta} \right)^2 P_{1,\frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right. \right. \\
& + 8 \left(\frac{2r}{\beta} \right) P_{1,\frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 8 P_{1,\frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \left. \right\} + 2 P_{3,\frac{3}{2}}(1,\beta) - \frac{1}{4} \left\{ \left(\frac{2r}{\beta} \right)^3 P_{3,\frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right. \\
& + 4 \left(\frac{2r}{\beta} \right)^2 P_{3,\frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 8 \left(\frac{2r}{\beta} \right) P_{3,\frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 8 P_{3,\frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \left. \right\} \left. \right] \\
L_{h\pi\sigma\bar{\pi}} &= \frac{1}{R} \frac{128}{5} \frac{\alpha^{\frac{3}{2}}\beta^4}{(2r)^5} \left[P_{1,\frac{3}{2}}(1,\beta) - \frac{1}{8} \left\{ \left(\frac{2r}{\beta} \right)^3 P_{1,\frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 4 \left(\frac{2r}{\beta} \right)^2 P_{1,\frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right. \right. \\
& + 8 \left(\frac{2r}{\beta} \right) P_{1,\frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 8 P_{1,\frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \left. \right\} - P_{3,\frac{3}{2}}(1,\beta) + \frac{1}{8} \left\{ \left(\frac{2r}{\beta} \right)^3 P_{3,\frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right. \\
& + 4 \left(\frac{2r}{\beta} \right)^2 P_{3,\frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 8 \left(\frac{2r}{\beta} \right) P_{3,\frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 8 P_{3,\frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \left. \right\} \left. \right] = L_{h\sigma\pi\bar{\pi}} \\
L_{s\bar{h}s\bar{h}} &= \frac{1}{R} 4\alpha^{\frac{1}{2}} \left[P_{0,\frac{1}{2}}(1,\alpha) - \frac{1}{6} \left\{ 6 P_{0,\frac{1}{2}} \left(1 + 2 \frac{\beta}{\alpha}, \alpha \right) + 9 \frac{\beta}{\alpha} P_{0,\frac{1}{2}} \left(1 + 2 \frac{\beta}{\alpha}, \alpha \right) \right. \right. \\
& + 6 \left(\frac{\beta}{\alpha} \right)^2 P_{0,\frac{1}{2}} \left(1 + 2 \frac{\beta}{\alpha}, \alpha \right) + 2 \left(\frac{\beta}{\alpha} \right)^3 P_{0,\frac{1}{2}} \left(1 + 2 \frac{\beta}{\alpha}, \alpha \right) \left. \right\} \left. \right] \\
L_{s\bar{h}s\bar{s}} &= \frac{1}{R} \frac{4}{\sqrt{3}} \frac{\alpha^{\frac{3}{2}}}{\beta} \left[Q_{0,\frac{1}{2}} \left(\frac{\alpha}{\beta}, \beta \right) - \frac{1}{6} \left\{ 6 Q_{0,\frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 9 Q_{0,\frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right. \right. \\
& + 6 Q_{0,\frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 2 Q_{0,\frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \left. \right\} \left. \right] \\
L_{s\bar{s}s\bar{h}} &= \frac{1}{R} \frac{4}{\sqrt{3}} \frac{\beta^{\frac{3}{2}}}{\alpha^2} \left[P_{0,\frac{3}{2}} \left(\frac{\beta}{\alpha}, \alpha \right) - \frac{1}{6} \left\{ 6 P_{0,\frac{3}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) + 9 \frac{\beta}{\alpha} P_{0,\frac{3}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) + 6 \left(\frac{\beta}{\alpha} \right)^2 P_{0,\frac{3}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) \right. \right. \\
& + 2 \left(\frac{\beta}{\alpha} \right)^3 P_{0,\frac{3}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) \left. \right\} \left. \right] \\
L_{s\bar{h}s\bar{\sigma}} &= \frac{1}{R} \frac{1}{6} \alpha^{\frac{3}{2}} \left[24 P_{0,\frac{1}{2}} \left(\frac{\alpha}{\beta}, \beta \right) - 24 P_{0,\frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) - 36 P_{0,\frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right. \\
& \left. - 24 P_{0,\frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) - 8 P_{0,\frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right]
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{\beta} \left\{ 24P_{1, \frac{3}{2}} \left(\frac{\alpha}{\beta}, \beta \right) - 24P_{1, \frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) - 36P_{1, \frac{5}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) - 24P_{1, \frac{7}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right. \\
& \left. - 8P_{1, \frac{9}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right\} \\
L_{s\sigma s\bar{h}} &= \frac{1}{R} 4 \frac{\beta^{\frac{5}{2}}}{\alpha^2} \left[P_{1, \frac{3}{2}} \left(\frac{\beta}{\alpha}, \alpha \right) - \frac{1}{6} \left\{ 6P_{1, \frac{3}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) + 9 \frac{\beta}{\alpha} P_{1, \frac{5}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) + 6 \left(\frac{\beta}{\alpha} \right)^2 P_{1, \frac{7}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) \right. \right. \\
& \left. \left. + 2 \left(\frac{\beta}{\alpha} \right)^3 P_{1, \frac{9}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) \right\} \right] \\
L_{s s s \bar{s}} &= \frac{1}{R} \frac{1}{18} \beta^{\frac{1}{2}} \left[24Q_{0, \frac{3}{2}}(1, \beta) - \left\{ 24Q_{0, \frac{3}{2}}(3, \beta) + 36Q_{0, \frac{5}{2}}(3, \beta) + 24Q_{0, \frac{7}{2}}(3, \beta) + 8Q_{0, \frac{9}{2}}(3, \beta) \right\} \right] \\
L_{s s s \bar{\sigma}} &= \frac{1}{R} \frac{\beta^{\frac{1}{2}}}{6\sqrt{3}} \left[\beta \left[24P_{0, \frac{3}{2}}(1, \beta) - \left\{ 24P_{0, \frac{3}{2}}(3, \beta) + 36P_{0, \frac{5}{2}}(3, \beta) + 24P_{0, \frac{7}{2}}(3, \beta) + 8P_{0, \frac{9}{2}}(3, \beta) \right\} \right] \right. \\
& \left. - \left[24P_{1, \frac{3}{2}}(1, \beta) - \left\{ 24P_{1, \frac{3}{2}}(3, \beta) + 36P_{1, \frac{5}{2}}(3, \beta) + 24P_{1, \frac{7}{2}}(3, \beta) + 8P_{1, \frac{9}{2}}(3, \beta) \right\} \right] \right] \\
L_{s \sigma s \bar{s}} &= \frac{1}{R} \frac{1}{6\sqrt{3}} \beta^{\frac{1}{2}} \left[24Q_{1, \frac{3}{2}}(1, \beta) - \left\{ 24Q_{1, \frac{3}{2}}(3, \beta) + 36P_{1, \frac{5}{2}}(3, \beta) + 24Q_{1, \frac{7}{2}}(3, \beta) + 8Q_{1, \frac{9}{2}}(3, \beta) \right\} \right] \\
L_{s \sigma s \bar{\sigma}} &= \frac{1}{R} \frac{\beta^{\frac{1}{2}}}{18} \left[3\beta \left[24P_{1, \frac{3}{2}}(1, \beta) - \left\{ 24P_{1, \frac{3}{2}}(3, \beta) + 36P_{1, \frac{5}{2}}(3, \beta) + 24P_{1, \frac{7}{2}}(3, \beta) + 8P_{1, \frac{9}{2}}(3, \beta) \right\} \right] \right. \\
& \left. - \left[24P_{0, \frac{3}{2}}(1, \beta) - \left\{ 24P_{0, \frac{3}{2}}(3, \beta) + 36P_{0, \frac{5}{2}}(3, \beta) + 24P_{0, \frac{7}{2}}(3, \beta) + 8P_{0, \frac{9}{2}}(3, \beta) \right\} \right] \right. \\
& \left. - 2 \left[24P_{2, \frac{3}{2}}(1, \beta) - \left\{ 24P_{2, \frac{3}{2}}(3, \beta) + 36P_{2, \frac{5}{2}}(3, \beta) + 24P_{2, \frac{7}{2}}(3, \beta) + 8P_{2, \frac{9}{2}}(3, \beta) \right\} \right] \right] \\
L_{s\pi s\bar{\pi}} &= \frac{1}{R} \frac{4}{3} \beta^{\frac{1}{2}} \left[P_{0, \frac{3}{2}}(1, \beta) - \frac{1}{6} \left\{ 2P_{0, \frac{11}{2}}(3, \beta) + 6P_{0, \frac{9}{2}}(3, \beta) + 9P_{0, \frac{7}{2}}(3, \beta) + 6P_{0, \frac{5}{2}}(3, \beta) \right\} \right. \\
& \left. - P_{2, \frac{3}{2}}(1, \beta) + \frac{1}{6} \left\{ 2P_{2, \frac{11}{2}}(3, \beta) + 6P_{2, \frac{9}{2}}(3, \beta) + 9P_{2, \frac{7}{2}}(3, \beta) + 6P_{2, \frac{5}{2}}(3, \beta) \right\} \right] \\
L_{\sigma h \sigma \bar{h}} &= \frac{1}{R} \left[4\alpha^{\frac{1}{2}} \left[P_{0, \frac{3}{2}}(1, \alpha) - \frac{1}{6} \left\{ 2 \left(\frac{\beta}{\alpha} \right)^3 P_{0, \frac{7}{2}} \left(1 + 2 \frac{\beta}{\alpha}, \alpha \right) + 6 \left(\frac{\beta}{\alpha} \right)^2 P_{0, \frac{5}{2}} \left(1 + 2 \frac{\beta}{\alpha}, \alpha \right) \right. \right. \right. \\
& \left. \left. + 9 \frac{\beta}{\alpha} P_{0, \frac{3}{2}} \left(1 + 2 \frac{\beta}{\alpha}, \alpha \right) + 6P_{0, \frac{1}{2}} \left(1 + 2 \frac{\beta}{\alpha}, \alpha \right) \right\} \right] \\
& + 12 \frac{\alpha^{\frac{5}{2}}}{\beta^2} \left[P_{2, -\frac{3}{2}}(1, \alpha) - \frac{1}{9} \left\{ 2 \left(\frac{\beta}{\alpha} \right)^5 P_{2, \frac{7}{2}} \left(1 + 2 \frac{\beta}{\alpha}, \alpha \right) + 6 \left(\frac{\beta}{\alpha} \right)^4 P_{2, \frac{5}{2}} \left(1 + 2 \frac{\beta}{\alpha}, \alpha \right) \right. \right. \\
& \left. \left. + 12 \left(\frac{\beta}{\alpha} \right)^3 P_{2, \frac{3}{2}} \left(1 + 2 \frac{\beta}{\alpha}, \alpha \right) + 18 \left(\frac{\beta}{\alpha} \right)^2 P_{2, \frac{1}{2}} \left(1 + 2 \frac{\beta}{\alpha}, \alpha \right) + 18 \frac{\beta}{\alpha} P_{2, -\frac{1}{2}} \left(1 + 2 \frac{\beta}{\alpha}, \alpha \right) \right. \right. \\
& \left. \left. + 9P_{2, -\frac{3}{2}} \left(1 + 2 \frac{\beta}{\alpha}, \alpha \right) \right\} \right] \\
L_{\sigma h \sigma \bar{s}} &= \frac{1}{R} \left[\frac{4}{\sqrt{3}} \frac{\alpha^{\frac{3}{2}}}{\beta} \left[Q_{0, \frac{1}{2}} \left(\frac{\alpha}{\beta}, \beta \right) - \frac{1}{6} \left\{ 2Q_{0, \frac{7}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 6Q_{0, \frac{5}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 9Q_{0, \frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right. \right. \right. \right. \\
& \left. \left. + 6Q_{0, \frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right\} \right] + \frac{12}{\sqrt{3}} \frac{\alpha^{\frac{3}{2}}}{\beta} \left[Q_{2, -\frac{3}{2}} \left(\frac{\alpha}{\beta}, \beta \right) - \frac{1}{9} \left\{ 2Q_{2, \frac{7}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 6Q_{2, \frac{5}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right. \right. \right. \\
& \left. \left. + 12Q_{2, \frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 18Q_{2, \frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 18Q_{2, -\frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 9Q_{2, -\frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right\} \right] \\
L_{\sigma s \sigma \bar{h}} &= \frac{1}{R} \left[\frac{4}{\sqrt{3}} \frac{\beta^{\frac{5}{2}}}{\alpha^2} \left[P_{0, \frac{3}{2}} \left(\frac{\beta}{\alpha}, \alpha \right) - \frac{1}{6} \left\{ 2 \left(\frac{\beta}{\alpha} \right)^3 P_{0, \frac{7}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) + 6 \left(\frac{\beta}{\alpha} \right)^2 P_{0, \frac{5}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) \right. \right. \right. \right. \\
& \left. \left. + 9 \frac{\beta}{\alpha} P_{0, \frac{3}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) + 6P_{0, \frac{1}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) \right\} \right]
\end{aligned}$$

$$\begin{aligned}
& + \frac{12}{\sqrt{3}} \beta^{\frac{1}{2}} \left[P_{2,-\frac{1}{2}} \left(\frac{\beta}{\alpha}, \alpha \right) - \frac{1}{9} \left\{ 2 \left(\frac{\beta}{\alpha} \right)^5 P_{2,\frac{3}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) + 6 \left(\frac{\beta}{\alpha} \right)^4 P_{2,\frac{7}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) \right. \right. \\
& \left. \left. + 12 \left(\frac{\beta}{\alpha} \right)^3 P_{2,\frac{5}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) + 18 \left(\frac{\beta}{\alpha} \right)^2 P_{2,\frac{3}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) + 18 \frac{\beta}{\alpha} P_{2,\frac{1}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) + 9 P_{2,-\frac{1}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) \right\} \right] \\
L_{\sigma h \sigma \bar{\sigma}} = & \frac{1}{R} \left[4 \alpha^{\frac{3}{2}} \left[P_{0,\frac{1}{2}} \left(\frac{\alpha}{\beta}, \beta \right) - \frac{1}{6} \left\{ 2 P_{0,\frac{7}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) + 6 P_{0,\frac{5}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) + 9 P_{0,\frac{3}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) \right. \right. \right. \\
& \left. \left. + 6 P_{0,\frac{1}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) \right\} \right] \\
& - 4 \frac{\alpha^{\frac{3}{2}}}{\beta} \left[P_{1,\frac{3}{2}} \left(\frac{\alpha}{\beta}, \beta \right) - \frac{1}{6} \left\{ 2 P_{1,\frac{5}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) + 6 P_{1,\frac{7}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) + 9 P_{1,\frac{3}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) \right. \right. \\
& \left. \left. + 6 P_{1,\frac{1}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) \right\} \right] \\
& + 12 \alpha^{\frac{3}{2}} \left[P_{2,-\frac{3}{2}} \left(\frac{\alpha}{\beta}, \beta \right) - \frac{1}{9} \left\{ 2 P_{2,\frac{7}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) + 6 P_{2,\frac{5}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) + 12 P_{2,\frac{3}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) \right. \right. \\
& \left. \left. + 18 P_{2,\frac{1}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) + 18 P_{2,-\frac{1}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) + 9 P_{2,-\frac{3}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) \right\} \right] \\
& - \frac{12}{5} \frac{\alpha^{\frac{3}{2}}}{\beta} \left[2 P_{1,-\frac{1}{2}} \left(\frac{\alpha}{\beta}, \beta \right) - \frac{2}{9} \left\{ 2 P_{1,\frac{3}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) + 6 P_{1,\frac{5}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) + 12 P_{1,\frac{7}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) \right. \right. \\
& \left. \left. + 18 P_{1,\frac{1}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) + 18 P_{1,\frac{3}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) + 9 P_{1,-\frac{1}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) \right\} \right] \\
& - \frac{12}{5} \frac{\alpha^{\frac{3}{2}}}{\beta} \left[3 P_{3,-\frac{1}{2}} \left(\frac{\alpha}{\beta}, \beta \right) - \frac{1}{3} \left\{ 2 P_{3,\frac{3}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) + 6 P_{3,\frac{5}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) + 12 P_{3,\frac{7}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) \right. \right. \\
& \left. \left. + 18 P_{3,\frac{1}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) + 18 P_{3,\frac{3}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) + 9 P_{3,-\frac{1}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) \right\} \right] \\
L_{\sigma \sigma \sigma \bar{h}} = & \frac{1}{R} \left[4 \frac{\beta^{\frac{5}{2}}}{\alpha^2} \left[P_{1,\frac{3}{2}} \left(\frac{\beta}{\alpha}, \alpha \right) - \frac{1}{6} \left\{ 2 \left(\frac{\beta}{\alpha} \right)^3 P_{1,\frac{5}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) + 6 \left(\frac{\beta}{\alpha} \right)^2 P_{1,\frac{7}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) \right. \right. \right. \\
& \left. \left. + 9 \left(\frac{\beta}{\alpha} \right) P_{1,\frac{3}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) + 6 P_{1,\frac{1}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) \right\} \right] \\
& + \frac{12}{5} \beta^{\frac{1}{2}} \left[2 P_{1,-\frac{1}{2}} \left(\frac{\beta}{\alpha}, \alpha \right) - \frac{2}{9} \left\{ 2 \left(\frac{\beta}{\alpha} \right)^5 P_{1,\frac{3}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) + 6 \left(\frac{\beta}{\alpha} \right)^4 P_{1,\frac{5}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) \right. \right. \\
& \left. \left. + 12 \left(\frac{\beta}{\alpha} \right)^3 P_{1,\frac{1}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) + 18 \left(\frac{\beta}{\alpha} \right)^2 P_{1,\frac{3}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) + 18 \left(\frac{\beta}{\alpha} \right) P_{1,\frac{5}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) \right. \right. \\
& \left. \left. + 9 P_{1,-\frac{1}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) \right\} + 3 P_{3,-\frac{1}{2}} \left(\frac{\beta}{\alpha}, \alpha \right) - \frac{1}{3} \left\{ 2 \left(\frac{\beta}{\alpha} \right)^5 P_{3,\frac{3}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) + 6 \left(\frac{\beta}{\alpha} \right)^4 P_{3,\frac{5}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) \right. \right. \\
& \left. \left. + 12 \left(\frac{\beta}{\alpha} \right)^3 P_{3,\frac{1}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) + 18 \left(\frac{\beta}{\alpha} \right)^2 P_{3,\frac{3}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) + 18 \left(\frac{\beta}{\alpha} \right) P_{3,\frac{5}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) \right. \right. \\
& \left. \left. + 9 P_{3,-\frac{1}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) \right\} \right] \\
L_{\sigma \sigma \sigma \bar{\sigma}} = & \frac{1}{R} \left[\frac{4}{3} \beta^{\frac{1}{2}} \left[Q_{0,\frac{3}{2}}(1, \beta) - \frac{1}{6} \left\{ 2 Q_{0,\frac{5}{2}}(3, \beta) + 6 Q_{0,\frac{7}{2}}(3, \beta) + 9 Q_{0,\frac{9}{2}}(3, \beta) + 6 Q_{0,\frac{11}{2}}(3, \beta) \right\} \right. \right. \\
& \left. \left. + 4 \beta^{\frac{1}{2}} \left[Q_{2,-\frac{1}{2}}(1, \beta) - \frac{1}{9} \left\{ 2 Q_{2,\frac{3}{2}}(3, \beta) + 6 Q_{2,\frac{5}{2}}(3, \beta) + 12 Q_{2,\frac{7}{2}}(3, \beta) + 18 Q_{2,\frac{9}{2}}(3, \beta) \right. \right. \right. \right. \\
& \left. \left. \left. + 18 Q_{2,\frac{1}{2}}(3, \beta) + 9 Q_{2,-\frac{1}{2}}(3, \beta) \right\} \right] \right] \\
L_{\sigma \sigma \sigma \bar{\sigma}} = & \frac{1}{R} \left[\frac{4}{\sqrt{3}} \beta^{\frac{3}{2}} \left[P_{0,\frac{3}{2}}(1, \beta) - \frac{1}{6} \left\{ 2 P_{0,\frac{5}{2}}(3, \beta) + 6 P_{0,\frac{7}{2}}(3, \beta) + 9 P_{0,\frac{9}{2}}(3, \beta) + 6 P_{0,\frac{11}{2}}(3, \beta) \right\} \right. \right. \\
& \left. \left. + 4 \beta^{\frac{3}{2}} \left[P_{2,-\frac{1}{2}}(1, \beta) - \frac{1}{9} \left\{ 2 P_{2,\frac{3}{2}}(3, \beta) + 6 P_{2,\frac{5}{2}}(3, \beta) + 12 P_{2,\frac{7}{2}}(3, \beta) + 18 P_{2,\frac{9}{2}}(3, \beta) \right. \right. \right. \right. \\
& \left. \left. \left. + 18 P_{2,\frac{1}{2}}(3, \beta) + 9 P_{2,-\frac{1}{2}}(3, \beta) \right\} \right] \right]
\end{aligned}$$

$$\begin{aligned}
& -\frac{4}{\sqrt{3}}\beta^{\frac{1}{2}}\left[P_{1,\frac{5}{2}}(1,\beta)-\frac{1}{6}\left\{2P_{1,\frac{11}{2}}(3,\beta)+6P_{1,\frac{9}{2}}(3,\beta)+9P_{1,\frac{7}{2}}(3,\beta)+6P_{1,\frac{5}{2}}(3,\beta)\right\}\right] \\
& +\frac{12}{\sqrt{3}}\beta^{\frac{3}{2}}\left[P_{2,-\frac{1}{2}}(1,\beta)-\frac{1}{9}\left\{2P_{2,\frac{9}{2}}(3,\beta)+6P_{2,\frac{7}{2}}(3,\beta)+12P_{2,\frac{5}{2}}(3,\beta)+18P_{2,\frac{3}{2}}(3,\beta)\right.\right. \\
& \left.\left.+18P_{2,\frac{1}{2}}(3,\beta)+9P_{2,-\frac{1}{2}}(3,\beta)\right\}\right] \\
& -\frac{12}{5\sqrt{3}}\beta^{\frac{5}{2}}\left[2P_{1,\frac{1}{2}}(1,\beta)-\frac{2}{9}\left\{2P_{1,\frac{11}{2}}(3,\beta)+6P_{1,\frac{9}{2}}(3,\beta)+12P_{1,\frac{7}{2}}(3,\beta)+18P_{1,\frac{5}{2}}(3,\beta)\right.\right. \\
& \left.\left.+18P_{1,\frac{3}{2}}(3,\beta)+9P_{1,\frac{1}{2}}(3,\beta)\right\}\right] \\
& -\frac{12}{5\sqrt{3}}\beta^{\frac{7}{2}}\left[3P_{3,\frac{1}{2}}(1,\beta)-\frac{1}{3}\left\{2P_{3,\frac{11}{2}}(3,\beta)+6P_{3,\frac{9}{2}}(3,\beta)+12P_{3,\frac{7}{2}}(3,\beta)+18P_{3,\frac{5}{2}}(3,\beta)\right.\right. \\
& \left.\left.+18P_{3,\frac{3}{2}}(3,\beta)+9P_{3,\frac{1}{2}}(3,\beta)\right\}\right] \\
L_{\sigma\sigma\sigma\bar{\sigma}} & =\frac{1}{R}\left[\frac{4}{\sqrt{3}}\beta^{\frac{1}{2}}\left[Q_{1,\frac{5}{2}}(1,\beta)-\frac{1}{6}\left\{2Q_{1,\frac{9}{2}}(3,\beta)+6Q_{1,\frac{7}{2}}(3,\beta)+9Q_{1,\frac{5}{2}}(3,\beta)+6Q_{1,\frac{3}{2}}(3,\beta)\right\}\right]\right. \\
& \left.+\frac{12}{5\sqrt{3}}\beta^{\frac{3}{2}}\left[2Q_{1,-\frac{1}{2}}(1,\beta)-\frac{2}{9}\left\{2Q_{1,\frac{9}{2}}(3,\beta)+6Q_{1,\frac{7}{2}}(3,\beta)+12Q_{1,\frac{5}{2}}(3,\beta)+18Q_{1,\frac{3}{2}}(3,\beta)\right.\right.\right. \\
& \left.\left.+18Q_{1,\frac{1}{2}}(3,\beta)+9Q_{1,-\frac{1}{2}}(3,\beta)\right\}+3Q_{3,-\frac{1}{2}}(1,\beta)-\frac{1}{3}\left\{2Q_{3,\frac{9}{2}}(3,\beta)+6Q_{3,\frac{7}{2}}(3,\beta)+12Q_{3,\frac{5}{2}}(3,\beta)\right.\right. \\
& \left.\left.+18Q_{3,\frac{3}{2}}(3,\beta)+18Q_{3,\frac{1}{2}}(3,\beta)+9Q_{3,-\frac{1}{2}}(3,\beta)\right\}\right] \\
L_{\sigma\sigma\sigma\bar{\sigma}} & =\frac{1}{R}\left[4\beta^{\frac{3}{2}}\left[P_{1,\frac{3}{2}}(1,\beta)-\frac{1}{6}\left\{2P_{1,\frac{9}{2}}(3,\beta)+6P_{1,\frac{7}{2}}(3,\beta)+9P_{1,\frac{5}{2}}(3,\beta)+6P_{1,\frac{3}{2}}(3,\beta)\right\}\right]\right. \\
& -\frac{4}{3}\beta^{\frac{5}{2}}\left[P_{0,\frac{5}{2}}(1,\beta)-\frac{1}{6}\left\{2P_{0,\frac{11}{2}}(3,\beta)+6P_{0,\frac{9}{2}}(3,\beta)+9P_{0,\frac{7}{2}}(3,\beta)+6P_{0,\frac{5}{2}}(3,\beta)\right\}+2P_{2,\frac{5}{2}}(1,\beta)\right. \\
& \left.-\frac{1}{3}\left\{2P_{2,\frac{11}{2}}(3,\beta)+6P_{2,\frac{9}{2}}(3,\beta)+9P_{2,\frac{7}{2}}(3,\beta)+6P_{2,\frac{5}{2}}(3,\beta)\right\}\right] \\
& +\frac{12}{5}\beta^{\frac{7}{2}}\left[2P_{1,-\frac{1}{2}}(1,\beta)-\frac{2}{9}\left\{2P_{1,\frac{9}{2}}(3,\beta)+6P_{1,\frac{7}{2}}(3,\beta)+12P_{1,\frac{5}{2}}(3,\beta)+18P_{1,\frac{3}{2}}(3,\beta)\right.\right. \\
& \left.\left.+18P_{1,\frac{1}{2}}(3,\beta)+9P_{1,-\frac{1}{2}}(3,\beta)\right\}+3P_{3,-\frac{1}{2}}(1,\beta)-\frac{1}{3}\left\{2P_{3,\frac{9}{2}}(3,\beta)+6P_{3,\frac{7}{2}}(3,\beta)+12P_{3,\frac{5}{2}}(3,\beta)\right.\right. \\
& \left.\left.+18P_{3,\frac{3}{2}}(3,\beta)+18P_{3,\frac{1}{2}}(3,\beta)+9P_{3,-\frac{1}{2}}(3,\beta)\right\}\right] \\
& -\frac{4}{35}\beta^{\frac{9}{2}}\left[14P_{0,\frac{1}{2}}(1,\beta)-\frac{14}{9}\left\{2P_{0,\frac{11}{2}}(3,\beta)+6P_{0,\frac{9}{2}}(3,\beta)+12P_{0,\frac{7}{2}}(3,\beta)+18P_{0,\frac{5}{2}}(3,\beta)\right.\right. \\
& \left.\left.+18P_{0,\frac{3}{2}}(3,\beta)+9P_{0,\frac{1}{2}}(3,\beta)\right\}+55P_{2,\frac{1}{2}}(1,\beta)-\frac{55}{9}\left\{2P_{2,\frac{11}{2}}(3,\beta)+6P_{2,\frac{9}{2}}(3,\beta)+12P_{2,\frac{7}{2}}(3,\beta)\right.\right. \\
& \left.\left.+18P_{2,\frac{5}{2}}(3,\beta)+18P_{2,\frac{3}{2}}(3,\beta)+9P_{2,\frac{1}{2}}(3,\beta)\right\}+36P_{4,\frac{1}{2}}(1,\beta)-4\left\{2P_{4,\frac{11}{2}}(3,\beta)+6P_{4,\frac{9}{2}}(3,\beta)\right.\right. \\
& \left.\left.+12P_{4,\frac{7}{2}}(3,\beta)+18P_{4,\frac{5}{2}}(3,\beta)+18P_{4,\frac{3}{2}}(3,\beta)+9P_{4,\frac{1}{2}}(3,\beta)\right\}\right] \\
L_{\sigma\pi\sigma\bar{\pi}} & =\frac{1}{R}\left[\frac{4}{3}\beta^{\frac{1}{2}}\left[P_{0,\frac{5}{2}}(1,\beta)-\frac{1}{6}\left\{2P_{0,\frac{11}{2}}(3,\beta)+6P_{0,\frac{9}{2}}(3,\beta)+9P_{0,\frac{7}{2}}(3,\beta)+6P_{0,\frac{5}{2}}(3,\beta)\right\}\right]\right. \\
& \left.-P_{2,\frac{5}{2}}(1,\beta)+\frac{1}{6}\left\{2P_{2,\frac{11}{2}}(3,\beta)+6P_{2,\frac{9}{2}}(3,\beta)+9P_{2,\frac{7}{2}}(3,\beta)+6P_{2,\frac{5}{2}}(3,\beta)\right\}\right] \\
& -\frac{4}{5}\beta^{\frac{3}{2}}\left[P_{0,\frac{1}{2}}(1,\beta)-\frac{1}{9}\left\{2P_{0,\frac{11}{2}}(3,\beta)+6P_{0,\frac{9}{2}}(3,\beta)+12P_{0,\frac{7}{2}}(3,\beta)+18P_{0,\frac{5}{2}}(3,\beta)\right.\right.
\end{aligned}$$

$$\begin{aligned}
& +18P_{0, \frac{3}{2}}(3, \beta) + 9P_{0, \frac{1}{2}}(3, \beta) \Big] \\
& + \frac{20}{7} \beta^{\frac{1}{2}} \left[P_{2, \frac{1}{2}}(1, \beta) - \frac{1}{9} \left\{ 2P_{2, \frac{11}{2}}(3, \beta) + 6P_{2, \frac{3}{2}}(3, \beta) + 12P_{2, \frac{7}{2}}(3, \beta) + 18P_{2, \frac{5}{2}}(3, \beta) \right. \right. \\
& \left. \left. + 18P_{2, \frac{3}{2}}(3, \beta) + 9P_{2, \frac{1}{2}}(3, \beta) \right\} \right] \\
& - \frac{72}{35} \beta^{\frac{1}{2}} \left[P_{4, \frac{1}{2}}(1, \beta) - \frac{1}{9} \left\{ 2P_{4, \frac{11}{2}}(3, \beta) + 6P_{4, \frac{3}{2}}(3, \beta) + 12P_{4, \frac{7}{2}}(3, \beta) + 18P_{4, \frac{5}{2}}(3, \beta) \right. \right. \\
& \left. \left. + 18P_{4, \frac{3}{2}}(3, \beta) + 9P_{2, \frac{1}{2}}(3, \beta) \right\} \right] \\
L_{\pi h \pi \bar{h}} = & \frac{1}{R} \left[4\alpha^{\frac{1}{2}} \left[P_{0, \frac{1}{2}}(1, \beta) - \frac{1}{6} \left\{ 2 \left(\frac{\beta}{\alpha} \right)^3 P_{0, \frac{7}{2}} \left(2 \frac{\beta}{\alpha} + 1, \alpha \right) + 6 \left(\frac{\beta}{\alpha} \right)^2 P_{0, \frac{5}{2}} \left(2 \frac{\beta}{\alpha} + 1, \alpha \right) \right. \right. \right. \\
& \left. \left. + 9 \frac{\beta}{\alpha} P_{0, \frac{3}{2}} \left(2 \frac{\beta}{\alpha} + 1, \alpha \right) + 6 P_{0, \frac{1}{2}} \left(2 \frac{\beta}{\alpha} + 1, \alpha \right) \right\} \right] \\
& - 6 \frac{\alpha^{\frac{5}{2}}}{\beta^2} \left[P_{2, -\frac{3}{2}}(1, \alpha) - \frac{1}{9} \left\{ 2 \left(\frac{\beta}{\alpha} \right)^5 P_{2, \frac{7}{2}} \left(2 \frac{\beta}{\alpha} + 1, \alpha \right) + 6 \left(\frac{\beta}{\alpha} \right)^4 P_{2, \frac{5}{2}} \left(2 \frac{\beta}{\alpha} + 1, \alpha \right) \right. \right. \\
& \left. \left. + 12 \left(\frac{\beta}{\alpha} \right)^3 P_{2, \frac{3}{2}} \left(2 \frac{\beta}{\alpha} + 1, \alpha \right) + 18 \left(\frac{\beta}{\alpha} \right)^2 P_{2, \frac{1}{2}} \left(2 \frac{\beta}{\alpha} + 1, \alpha \right) + 18 \frac{\beta}{\alpha} P_{2, -\frac{1}{2}} \left(2 \frac{\beta}{\alpha} + 1, \alpha \right) \right. \right. \\
& \left. \left. + 9 P_{2, -\frac{3}{2}} \left(2 \frac{\beta}{\alpha} + 1, \alpha \right) \right\} \right] \\
L_{\pi h \pi \bar{s}} = & \frac{1}{R} \left[\frac{4}{\sqrt{3}} \frac{\alpha^{\frac{3}{2}}}{\beta} \left[Q_{0, \frac{1}{2}} \left(\frac{\alpha}{\beta}, \beta \right) - \frac{1}{6} \left\{ 2 Q_{0, \frac{7}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) + 6 Q_{0, \frac{5}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) \right. \right. \right. \\
& \left. \left. + 9 Q_{0, \frac{3}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) + 6 Q_{0, \frac{1}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) \right\} \right] \\
& - \frac{6}{\sqrt{3}} \frac{\alpha^{\frac{3}{2}}}{\beta} \left[Q_{2, -\frac{3}{2}} \left(\frac{\alpha}{\beta}, \beta \right) - \frac{1}{9} \left\{ 2 Q_{2, \frac{7}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) + 6 Q_{2, \frac{5}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) + 12 Q_{2, \frac{3}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) \right. \right. \\
& \left. \left. + 18 Q_{2, \frac{1}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) + 18 Q_{2, -\frac{1}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) + 9 Q_{2, -\frac{3}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) \right\} \right] \\
L_{\pi s \pi \bar{h}} = & \frac{1}{R} \left[\frac{4}{\sqrt{3}} \frac{\beta^{\frac{5}{2}}}{\alpha^2} \left[P_{0, \frac{3}{2}} \left(\frac{\beta}{\alpha}, \alpha \right) - \frac{1}{6} \left\{ 2 \left(\frac{\beta}{\alpha} \right)^3 P_{0, \frac{3}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) + 6 \left(\frac{\beta}{\alpha} \right)^2 P_{0, \frac{1}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) \right. \right. \right. \\
& \left. \left. + 9 \frac{\beta}{\alpha} P_{0, \frac{3}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) + 6 P_{0, \frac{1}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) \right\} \right] \\
& - 2\sqrt{3} \beta^{\frac{1}{2}} \left[P_{2, -\frac{1}{2}} \left(\frac{\beta}{\alpha}, \alpha \right) - \frac{1}{9} \left\{ 2 \left(\frac{\beta}{\alpha} \right)^5 P_{2, \frac{3}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) + 6 \left(\frac{\beta}{\alpha} \right)^4 P_{2, \frac{1}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) \right. \right. \\
& \left. \left. + 12 \left(\frac{\beta}{\alpha} \right)^3 P_{2, \frac{3}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) + 18 \left(\frac{\beta}{\alpha} \right)^2 P_{2, \frac{1}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) + 18 \frac{\beta}{\alpha} P_{2, \frac{1}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) + 9 P_{2, -\frac{1}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) \right\} \right] \\
L_{\pi h \pi \bar{\sigma}} = & \frac{1}{R} \left[4 \alpha^{\frac{3}{2}} \left[P_{0, \frac{1}{2}} \left(\frac{\alpha}{\beta}, \beta \right) - \frac{1}{6} \left\{ 2 P_{0, \frac{7}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) + 6 P_{0, \frac{5}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) + 9 P_{0, \frac{3}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) \right. \right. \right. \\
& \left. \left. + 6 P_{0, \frac{1}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) \right\} \right] \\
& - 4 \frac{\alpha^{\frac{3}{2}}}{\beta} \left[P_{1, \frac{3}{2}} \left(\frac{\alpha}{\beta}, \beta \right) - \frac{1}{6} \left\{ 2 P_{1, \frac{3}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) + 6 P_{1, \frac{1}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) + 9 P_{1, \frac{1}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) \right. \right. \\
& \left. \left. + 6 P_{1, \frac{3}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) \right\} \right] \\
& - 6 \alpha^{\frac{3}{2}} \left[P_{2, -\frac{3}{2}} \left(\frac{\alpha}{\beta}, \beta \right) - \frac{1}{9} \left\{ 2 P_{2, \frac{7}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) + 6 P_{2, \frac{5}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) + 12 P_{2, \frac{3}{2}} \left(\frac{\alpha}{\beta} + 2, \beta \right) \right. \right.
\end{aligned}$$

$$\begin{aligned}
& +18P_{2, \frac{1}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right)+18P_{2, -\frac{1}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right)+9P_{2, -\frac{3}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right)\Big] \\
& +\frac{12}{5}\frac{\alpha^{\frac{3}{2}}}{\beta}\left[P_{1, -\frac{1}{2}}\left(\frac{\alpha}{\beta}, \beta\right)-\frac{1}{9}\left\{2P_{1, \frac{3}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right)+6P_{1, \frac{7}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right)+12P_{1, \frac{5}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right)\right.\right. \\
& \left.\left.+18P_{1, \frac{3}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right)+18P_{1, \frac{1}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right)+9P_{1, -\frac{1}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right)\right\}\right] \\
& +\frac{18}{5}\frac{\alpha^{\frac{3}{2}}}{\beta}\left[P_{3, -\frac{1}{2}}\left(\frac{\alpha}{\beta}, \beta\right)-\frac{1}{9}\left\{2P_{3, \frac{3}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right)+6P_{3, \frac{7}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right)+12P_{3, \frac{5}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right)\right.\right. \\
& \left.\left.+18P_{3, \frac{3}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right)+18P_{3, \frac{1}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right)+9P_{3, -\frac{1}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right)\right\}\right]\Big] \\
L_{\pi\sigma\pi\bar{h}} & =\frac{1}{R}\left[4\frac{\alpha^{\frac{5}{2}}}{\alpha^2}\left[P_{1, \frac{3}{2}}\left(\frac{\beta}{\alpha}, \alpha\right)-\frac{1}{6}\left\{2\left(\frac{\beta}{\alpha}\right)^3P_{1, \frac{3}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right)+6\left(\frac{\beta}{\alpha}\right)^2P_{1, \frac{7}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right)\right.\right.\right. \\
& \left.\left.+9\frac{\beta}{\alpha}P_{1, \frac{5}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right)+6P_{1, \frac{3}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right)\right\}\right] \\
& -\frac{12}{5}\beta^{\frac{1}{2}}\left[P_{1, -\frac{1}{2}}\left(\frac{\beta}{\alpha}, \alpha\right)-\frac{1}{9}\left\{2\left(\frac{\beta}{\alpha}\right)^5P_{1, \frac{3}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right)+6\left(\frac{\beta}{\alpha}\right)^4P_{1, \frac{7}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right)\right.\right. \\
& \left.\left.+12\left(\frac{\beta}{\alpha}\right)^3P_{1, \frac{5}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right)+18\left(\frac{\beta}{\alpha}\right)^2P_{1, \frac{3}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right)+18\left(\frac{\beta}{\alpha}\right)P_{1, \frac{1}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right)+9P_{1, -\frac{1}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right)\right\}\right] \\
& -\frac{18}{5}\beta^{\frac{1}{2}}\left[P_{3, -\frac{1}{2}}\left(\frac{\beta}{\alpha}, \alpha\right)-\frac{1}{9}\left\{2\left(\frac{\beta}{\alpha}\right)^5P_{3, \frac{3}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right)+6\left(\frac{\beta}{\alpha}\right)^4P_{3, \frac{7}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right)\right.\right. \\
& \left.\left.+12\left(\frac{\beta}{\alpha}\right)^3P_{3, \frac{5}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right)+18\left(\frac{\beta}{\alpha}\right)^2P_{3, \frac{3}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right)+18\left(\frac{\beta}{\alpha}\right)P_{3, \frac{1}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right)\right.\right. \\
& \left.\left.+9P_{3, -\frac{1}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right)\right\}\right]\Big] \\
L_{\pi s \pi \bar{s}} & =\frac{1}{R}\left[\frac{4}{3}\beta^{\frac{1}{2}}\left[Q_{0, \frac{3}{2}}(1, \beta)-\frac{1}{6}\left\{2Q_{0, \frac{3}{2}}(3, \beta)+6Q_{0, \frac{7}{2}}(3, \beta)+9Q_{0, \frac{5}{2}}(3, \beta)+6Q_{0, \frac{3}{2}}(3, \beta)\right\}\right]\right. \\
& \left.-2\beta^{\frac{1}{2}}\left[Q_{2, -\frac{1}{2}}(1, \beta)-\frac{1}{9}\left\{2Q_{2, \frac{3}{2}}(3, \beta)+6Q_{2, \frac{7}{2}}(3, \beta)+12Q_{2, \frac{5}{2}}(3, \beta)+18Q_{2, \frac{3}{2}}(3, \beta)\right.\right.\right. \\
& \left.\left.+18Q_{2, \frac{1}{2}}(3, \beta)+9Q_{2, -\frac{1}{2}}(3, \beta)\right\}\right]\Big] \\
L_{\pi s \pi \bar{\sigma}} & =\frac{1}{R}\left[\frac{4}{\sqrt{3}}\beta^{\frac{3}{2}}\left[P_{0, \frac{3}{2}}(1, \beta)-\frac{1}{6}\left\{2P_{0, \frac{3}{2}}(3, \beta)+6P_{0, \frac{7}{2}}(3, \beta)+9P_{0, \frac{5}{2}}(3, \beta)+6P_{0, \frac{3}{2}}(3, \beta)\right\}\right]\right. \\
& \left.-\frac{4}{\sqrt{3}}\beta^{\frac{1}{2}}\left[P_{1, \frac{5}{2}}(1, \beta)-\frac{1}{6}\left\{2P_{1, \frac{11}{2}}(3, \beta)+6P_{1, \frac{3}{2}}(3, \beta)+9P_{1, \frac{7}{2}}(3, \beta)+6P_{1, \frac{5}{2}}(3, \beta)\right\}\right]\right. \\
& \left.-2\sqrt{3}\beta^{\frac{3}{2}}\left[P_{2, -\frac{1}{2}}(1, \beta)-\frac{1}{9}\left\{2P_{2, \frac{3}{2}}(3, \beta)+6P_{2, \frac{7}{2}}(3, \beta)+12P_{2, \frac{5}{2}}(3, \beta)+18P_{2, \frac{3}{2}}(3, \beta)\right.\right.\right. \\
& \left.\left.+18P_{2, \frac{1}{2}}(3, \beta)+9P_{2, -\frac{1}{2}}(3, \beta)\right\}\right] \\
& +\frac{4\sqrt{3}}{5}\beta^{\frac{1}{2}}\left[P_{1, \frac{1}{2}}(1, \beta)-\frac{1}{9}\left\{2P_{1, \frac{11}{2}}(3, \beta)+6P_{1, \frac{3}{2}}(3, \beta)+12P_{1, \frac{7}{2}}(3, \beta)+18P_{1, \frac{5}{2}}(3, \beta)\right.\right. \\
& \left.\left.+18P_{1, \frac{3}{2}}(3, \beta)+9P_{1, \frac{1}{2}}(3, \beta)\right\}\right] \\
& +\frac{6\sqrt{3}}{5}\beta^{\frac{1}{2}}\left[P_{3, \frac{1}{2}}(1, \beta)-\frac{1}{9}\left\{2P_{3, \frac{11}{2}}(3, \beta)+6P_{3, \frac{3}{2}}(3, \beta)+12P_{3, \frac{7}{2}}(3, \beta)+18P_{3, \frac{5}{2}}(3, \beta)\right.\right. \\
& \left.\left.+18P_{3, \frac{3}{2}}(3, \beta)+9P_{3, \frac{1}{2}}(3, \beta)\right\}\right]\Big]
\end{aligned}$$

$$L_{\pi\sigma\pi\bar{\sigma}} = \frac{1}{R} \frac{1}{\sqrt{3}} \beta^{\frac{1}{2}} \left[-6Q_{1,-\frac{1}{2}}(1, \beta) + 4Q_{1,\frac{3}{2}}(1, \beta) + 6Q_{1,-\frac{1}{2}}(3, \beta) + 12Q_{1,\frac{1}{2}}(3, \beta) + 8Q_{1,\frac{3}{2}}(3, \beta) \right. \\ \left. + 2Q_{1,\frac{5}{2}}(3, \beta) + \frac{1}{5} \left\{ 18Q_{1,-\frac{1}{2}}(1, \beta) - 18Q_{1,-\frac{1}{2}}(3, \beta) - 36Q_{1,\frac{1}{2}}(3, \beta) - 36Q_{1,\frac{3}{2}}(3, \beta) \right. \right. \\ \left. \left. - 24Q_{1,\frac{5}{2}}(3, \beta) - 12Q_{1,\frac{7}{2}}(3, \beta) - 4Q_{1,\frac{9}{2}}(3, \beta) - 18Q_{3,-\frac{1}{2}}(1, \beta) + 18Q_{3,-\frac{1}{2}}(3, \beta) + 36Q_{3,\frac{1}{2}}(3, \beta) \right. \right. \\ \left. \left. + 36Q_{3,\frac{3}{2}}(3, \beta) + 24Q_{3,\frac{5}{2}}(3, \beta) + 12Q_{3,\frac{7}{2}}(3, \beta) + 4Q_{3,\frac{9}{2}}(3, \beta) \right\} \right]$$

$$L_{\pi\sigma\pi\bar{\sigma}} = \frac{1}{R} \left[4\beta^{\frac{3}{2}} \left[P_{1,\frac{3}{2}}(1, \beta) - \frac{1}{6} \left\{ 2P_{1,\frac{3}{2}}(3, \beta) + 6P_{1,\frac{7}{2}}(3, \beta) + 9P_{1,\frac{5}{2}}(3, \beta) + 6P_{1,\frac{9}{2}}(3, \beta) \right\} \right] \right. \\ \left. - \frac{4}{3} \beta^{\frac{1}{2}} \left[P_{0,\frac{5}{2}}(1, \beta) - \frac{1}{6} \left\{ 2P_{0,\frac{11}{2}}(3, \beta) + 6P_{0,\frac{9}{2}}(3, \beta) + 9P_{0,\frac{7}{2}}(3, \beta) + 6P_{0,\frac{5}{2}}(3, \beta) \right\} \right. \right. \\ \left. \left. + 2P_{2,\frac{5}{2}}(1, \beta) - \frac{1}{3} \left\{ 2P_{2,\frac{11}{2}}(3, \beta) + 6P_{2,\frac{9}{2}}(3, \beta) + 9P_{2,\frac{7}{2}}(3, \beta) + 6P_{2,\frac{5}{2}}(3, \beta) \right\} \right] \right. \\ \left. - \frac{12}{5} \beta^{\frac{3}{2}} \left[P_{1,-\frac{1}{2}}(1, \beta) - \frac{1}{9} \left\{ 2P_{1,\frac{3}{2}}(3, \beta) + 6P_{1,\frac{7}{2}}(3, \beta) + 12P_{1,\frac{5}{2}}(3, \beta) + 18P_{1,\frac{9}{2}}(3, \beta) \right. \right. \right. \\ \left. \left. + 18P_{1,\frac{1}{2}}(3, \beta) + 9P_{1,-\frac{1}{2}}(3, \beta) \right\} \right] \\ \left. + \frac{4}{5} \beta^{\frac{1}{2}} \left[P_{0,\frac{1}{2}}(1, \beta) - \frac{1}{9} \left\{ 2P_{0,\frac{11}{2}}(3, \beta) + 6P_{0,\frac{9}{2}}(3, \beta) + 12P_{0,\frac{7}{2}}(3, \beta) + 18P_{0,\frac{5}{2}}(3, \beta) \right. \right. \right. \\ \left. \left. + 18P_{0,\frac{3}{2}}(3, \beta) + 9P_{0,\frac{1}{2}}(3, \beta) \right\} \right] \\ \left. + \frac{22}{7} \beta^{\frac{1}{2}} \left[P_{2,\frac{1}{2}}(1, \beta) - \frac{1}{9} \left\{ 2P_{2,\frac{11}{2}}(3, \beta) + 6P_{2,\frac{9}{2}}(3, \beta) + 12P_{2,\frac{7}{2}}(3, \beta) + 18P_{2,\frac{5}{2}}(3, \beta) \right. \right. \right. \\ \left. \left. + 18P_{2,\frac{3}{2}}(3, \beta) + 9P_{2,\frac{1}{2}}(3, \beta) \right\} \right] \\ \left. - \frac{18}{5} \beta^{\frac{3}{2}} \left[P_{3,-\frac{1}{2}}(1, \beta) - \frac{1}{9} \left\{ 2P_{3,\frac{3}{2}}(3, \beta) + 6P_{3,\frac{7}{2}}(3, \beta) + 12P_{3,\frac{5}{2}}(3, \beta) + 18P_{3,\frac{9}{2}}(3, \beta) \right. \right. \right. \\ \left. \left. + 18P_{3,\frac{1}{2}}(3, \beta) + 9P_{3,-\frac{1}{2}}(3, \beta) \right\} \right] \\ \left. + \frac{72}{35} \beta^{\frac{1}{2}} \left[P_{4,\frac{1}{2}}(1, \beta) - \frac{1}{9} \left\{ 2P_{4,\frac{11}{2}}(3, \beta) + 6P_{4,\frac{9}{2}}(3, \beta) + 12P_{4,\frac{7}{2}}(3, \beta) + 18P_{4,\frac{5}{2}}(3, \beta) \right. \right. \right. \\ \left. \left. + 18P_{4,\frac{3}{2}}(3, \beta) + 9P_{4,\frac{1}{2}}(3, \beta) \right\} \right] \right]$$

$$L_{\pi\pi\pi\bar{\pi}} = \frac{1}{R} \left[\frac{1}{12} \beta^{\frac{1}{2}} \left[16P_{0,\frac{5}{2}}(1, \beta) + 32P_{0,\frac{5}{2}}(3, \beta) + 8P_{0,\frac{7}{2}}(3, \beta) - 16P_{2,\frac{5}{2}}(1, \beta) - 32P_{2,\frac{5}{2}}(3, \beta) \right. \right. \\ \left. \left. - 8P_{2,\frac{7}{2}}(3, \beta) \right] \right. \\ \left. + \frac{1}{5} \beta^{\frac{1}{2}} \left[8P_{0,\frac{1}{2}}(1, \beta) - \left\{ 8P_{0,\frac{1}{2}}(3, \beta) + 16P_{0,\frac{3}{2}}(3, \beta) + 36P_{0,\frac{5}{2}}(3, \beta) + 24P_{0,\frac{7}{2}}(3, \beta) \right. \right. \right. \\ \left. \left. + 12P_{0,\frac{9}{2}}(3, \beta) + 4P_{0,\frac{11}{2}}(3, \beta) \right\} - 8P_{2,\frac{1}{2}}(1, \beta) + \left\{ 8P_{2,\frac{1}{2}}(3, \beta) + 16P_{2,\frac{3}{2}}(3, \beta) + 36P_{2,\frac{5}{2}}(3, \beta) \right. \right. \\ \left. \left. + 24P_{2,\frac{7}{2}}(3, \beta) + 12P_{2,\frac{9}{2}}(3, \beta) + 4P_{2,\frac{11}{2}}(3, \beta) \right\} \right] \\ \left. - \frac{3}{35} \beta^{\frac{1}{2}} \left[18P_{2,\frac{1}{2}}(1, \beta) - \left\{ 18P_{2,\frac{1}{2}}(3, \beta) + 36P_{2,\frac{3}{2}}(3, \beta) + 36P_{2,\frac{5}{2}}(3, \beta) + 24P_{2,\frac{7}{2}}(3, \beta) \right. \right. \right. \\ \left. \left. + 12P_{2,\frac{9}{2}}(3, \beta) + 4P_{2,\frac{11}{2}}(3, \beta) \right\} - 18P_{4,\frac{1}{2}}(1, \beta) + \left\{ 18P_{4,\frac{1}{2}}(3, \beta) + 36P_{4,\frac{3}{2}}(3, \beta) \right. \right. \\ \left. \left. + 36P_{4,\frac{5}{2}}(3, \beta) + 24P_{4,\frac{7}{2}}(3, \beta) + 12P_{4,\frac{9}{2}}(3, \beta) + 4P_{4,\frac{11}{2}}(3, \beta) \right\} \right] \right]$$

$$\begin{aligned}
L_{\pi\pi'\pi\bar{\pi}'} = & \frac{1}{R} \left[\frac{1}{12} \beta^{\frac{1}{2}} \left[-24P_{0, \frac{1}{2}}(1, \beta) + 16P_{0, \frac{5}{2}}(1, \beta) + \left\{ 24P_{0, \frac{1}{2}}(3, \beta) + 48P_{0, \frac{3}{2}}(3, \beta) + 32P_{0, \frac{5}{2}}(3, \beta) \right. \right. \right. \\
& + 8P_{0, \frac{7}{2}}(3, \beta) \left. \left. \left. \right\} + 24P_{2, \frac{1}{2}}(3, \beta) - 16P_{2, \frac{5}{2}}(1, \beta) - \left\{ 24P_{2, \frac{1}{2}}(3, \beta) + 48P_{2, \frac{3}{2}}(3, \beta) + 32P_{2, \frac{5}{2}}(3, \beta) \right. \right. \right. \\
& + 8P_{2, \frac{7}{2}}(3, \beta) \left. \left. \left. \right\} \right] \right. \\
& + \frac{1}{120} \beta^{\frac{1}{2}} \left[144P_{0, \frac{1}{2}}(1, \beta) - \left\{ 144P_{0, \frac{1}{2}}(3, \beta) + 288P_{0, \frac{3}{2}}(3, \beta) + 288P_{0, \frac{5}{2}}(3, \beta) + 192P_{0, \frac{7}{2}}(3, \beta) \right. \right. \\
& + 96P_{0, \frac{9}{2}}(3, \beta) + 32P_{0, \frac{11}{2}}(3, \beta) \left. \left. \right\} \right] \\
& - \left[\frac{1}{120} + \frac{1}{280} \right] \beta^{\frac{1}{2}} \left[144P_{2, \frac{1}{2}}(1, \beta) - \left\{ 144P_{2, \frac{1}{2}}(3, \beta) + 288P_{2, \frac{3}{2}}(3, \beta) + 288P_{2, \frac{5}{2}}(3, \beta) \right. \right. \\
& + 192P_{2, \frac{7}{2}}(3, \beta) + 96P_{2, \frac{9}{2}}(3, \beta) + 32P_{2, \frac{11}{2}}(3, \beta) \left. \left. \right\} \right] \\
& + \frac{1}{280} \beta^{\frac{1}{2}} \left[144P_{4, \frac{1}{2}}(1, \beta) - \left\{ 144P_{4, \frac{1}{2}}(3, \beta) + 288P_{4, \frac{3}{2}}(3, \beta) + 288P_{4, \frac{5}{2}}(3, \beta) + 192P_{4, \frac{7}{2}}(3, \beta) \right. \right. \\
& + 96P_{4, \frac{9}{2}}(3, \beta) + 32P_{4, \frac{11}{2}}(3, \beta) \left. \left. \right\} \right] \left. \right]
\end{aligned}$$

$$\begin{aligned}
L_{\pi\pi\pi'\pi'} = & \frac{1}{R} \left[\frac{1}{120} \beta^{\frac{1}{2}} \left[144P_{0, \frac{1}{2}}(1, \beta) - \left\{ 144P_{0, \frac{1}{2}}(3, \beta) + 288P_{0, \frac{3}{2}}(3, \beta) + 288P_{0, \frac{5}{2}}(3, \beta) \right. \right. \right. \\
& + 192P_{0, \frac{7}{2}}(3, \beta) + 96P_{0, \frac{9}{2}}(3, \beta) + 32P_{0, \frac{11}{2}}(3, \beta) \left. \left. \left. \right\} \right] \right. \\
& - \left(\frac{1}{120} + \frac{1}{280} \right) \beta^{\frac{1}{2}} \left[144P_{2, \frac{1}{2}}(1, \beta) - \left\{ 144P_{2, \frac{1}{2}}(3, \beta) + 288P_{2, \frac{3}{2}}(3, \beta) + 288P_{2, \frac{5}{2}}(3, \beta) \right. \right. \\
& + 192P_{2, \frac{7}{2}}(3, \beta) + 96P_{2, \frac{9}{2}}(3, \beta) + 32P_{2, \frac{11}{2}}(3, \beta) \left. \left. \right\} \right] \\
& + \frac{1}{280} \beta^{\frac{1}{2}} \left[144P_{4, \frac{1}{2}}(1, \beta) - \left\{ 144P_{4, \frac{1}{2}}(3, \beta) + 288P_{4, \frac{3}{2}}(3, \beta) + 288P_{4, \frac{5}{2}}(3, \beta) \right. \right. \\
& + 192P_{4, \frac{7}{2}}(3, \beta) + 96P_{4, \frac{9}{2}}(3, \beta) + 32P_{4, \frac{11}{2}}(3, \beta) \left. \left. \right\} \right] \left. \right]
\end{aligned}$$

$$\begin{aligned}
L_{s h \sigma \bar{h}} = & \frac{1}{R} \frac{10}{\sqrt{3}} \frac{\alpha^{\frac{3}{2}}}{\beta} \left[P_{1, -\frac{1}{2}}(1, \alpha) - \frac{1}{5} \left\{ 2 \left(\frac{\beta}{\alpha} \right)^4 P_{1, \frac{7}{2}} \left(1 + 2 \frac{\beta}{\alpha}, \alpha \right) + 6 \left(\frac{\beta}{\alpha} \right)^3 P_{1, \frac{5}{2}} \left(1 + 2 \frac{\beta}{\alpha}, \alpha \right) \right. \right. \\
& + 10 \left(\frac{\beta}{\alpha} \right)^2 P_{1, \frac{3}{2}} \left(1 + 2 \frac{\beta}{\alpha}, \alpha \right) + 10 \frac{\beta}{\alpha} P_{1, \frac{1}{2}} \left(1 + 2 \frac{\beta}{\alpha}, \alpha \right) + 5 P_{1, -\frac{1}{2}} \left(1 + 2 \frac{\beta}{\alpha}, \alpha \right) \left. \left. \right\} \right]
\end{aligned}$$

$$\begin{aligned}
L_{s h \sigma \bar{s}} = & \frac{1}{R} \frac{10}{3} \frac{\alpha^{\frac{3}{2}}}{\beta} \left[Q_{1, -\frac{1}{2}} \left(\frac{\alpha}{\beta}, \beta \right) - \frac{1}{5} \left\{ 2 Q_{1, \frac{7}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 6 Q_{1, \frac{5}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 10 Q_{1, \frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right. \right. \\
& + 10 Q_{1, \frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 5 Q_{1, -\frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \left. \left. \right\} \right]
\end{aligned}$$

$$\begin{aligned}
L_{s s \sigma \bar{h}} = & \frac{1}{R} \frac{10}{3} \frac{\beta^{\frac{3}{2}}}{\alpha} \left[P_{1, \frac{1}{2}} \left(\frac{\beta}{\alpha}, \alpha \right) - \frac{1}{5} \left\{ 2 \left(\frac{\beta}{\alpha} \right)^4 P_{1, \frac{7}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) + 6 \left(\frac{\beta}{\alpha} \right)^3 P_{1, \frac{5}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) \right. \right. \\
& + 10 \left(\frac{\beta}{\alpha} \right)^2 P_{1, \frac{3}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) + 10 \frac{\beta}{\alpha} P_{1, \frac{1}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) + 5 P_{1, \frac{1}{2}} \left(3 \frac{\beta}{\alpha}, \alpha \right) \left. \left. \right\} \right]
\end{aligned}$$

$$\begin{aligned}
L_{s h \sigma \bar{s}} = & \frac{1}{R} \left[\frac{10}{\sqrt{3}} \frac{\alpha^{\frac{3}{2}}}{\beta} \left[P_{1, -\frac{1}{2}} \left(\frac{\alpha}{\beta}, \beta \right) - \frac{1}{5} \left\{ 2 P_{1, \frac{7}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 6 P_{1, \frac{5}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 10 P_{1, \frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right. \right. \right. \\
& + 10 P_{1, \frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 5 P_{1, -\frac{1}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \left. \left. \left. \right\} \right] \right. \\
& - \frac{10}{3\sqrt{3}} \frac{\alpha^{\frac{3}{2}}}{\beta} \left[P_{0, \frac{1}{2}} \left(\frac{\alpha}{\beta}, \beta \right) - \frac{1}{5} \left\{ 2 P_{0, \frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 6 P_{0, \frac{5}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 10 P_{0, \frac{7}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right. \right. \\
& + 10 P_{0, \frac{9}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) + 5 P_{0, \frac{11}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \left. \left. \right\} + 2 P_{2, \frac{1}{2}} \left(\frac{\alpha}{\beta}, \beta \right) - \frac{2}{5} \left\{ 2 P_{2, \frac{3}{2}} \left(2 + \frac{\alpha}{\beta}, \beta \right) \right. \right.
\end{aligned}$$

$$\begin{aligned}
& +6P_{2, \frac{7}{2}}\left(2+\frac{\alpha}{\beta}, \beta\right)+10P_{2, \frac{5}{2}}\left(2+\frac{\alpha}{\beta}, \beta\right)+10P_{2, \frac{3}{2}}\left(2+\frac{\alpha}{\beta}, \beta\right)+5P_{2, \frac{1}{2}}\left(2+\frac{\alpha}{\beta}, \beta\right)\} \\
L_{s\sigma\sigma\bar{h}} &= \frac{1}{R} \frac{10}{3\sqrt{3}} \frac{\beta^{\frac{3}{2}}}{\alpha} \left[P_{0, \frac{1}{2}}\left(\frac{\beta}{\alpha}, \alpha\right) - \frac{1}{5} \left\{ 2\left(\frac{\beta}{\alpha}\right)^4 P_{0, \frac{3}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right) + 6\left(\frac{\beta}{\alpha}\right)^3 P_{0, \frac{7}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right) \right. \right. \\
& + 10\left(\frac{\beta}{\alpha}\right)^2 P_{0, \frac{5}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right) + 10\frac{\beta}{\alpha} P_{0, \frac{3}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right) + 5P_{0, \frac{1}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right) \left. \left. \right\} + 2P_{2, \frac{1}{2}}\left(\frac{\beta}{\alpha}, \alpha\right) \right. \\
& - \frac{2}{5} \left\{ 2\left(\frac{\beta}{\alpha}\right)^4 P_{2, \frac{3}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right) + 6\left(\frac{\beta}{\alpha}\right)^3 P_{2, \frac{7}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right) + 10\left(\frac{\beta}{\alpha}\right)^2 P_{2, \frac{5}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right) \right. \\
& \left. \left. + 10\frac{\beta}{\alpha} P_{2, \frac{3}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right) + 5P_{2, \frac{1}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right) \right\} \right] \\
L_{s\sigma\sigma\bar{s}} &= \frac{1}{R} \frac{10}{3\sqrt{3}} \beta^{\frac{1}{2}} \left[Q_{1, \frac{1}{2}}(1, \beta) - \frac{1}{5} \left\{ 2Q_{1, \frac{3}{2}}(3, \beta) + 6Q_{1, \frac{7}{2}}(3, \beta) + 10Q_{1, \frac{5}{2}}(3, \beta) + 10Q_{1, \frac{3}{2}}(3, \beta) \right. \right. \\
& \left. \left. + 5Q_{1, \frac{1}{2}}(3, \beta) \right\} \right] \\
L_{s\sigma\sigma\bar{o}} &= \frac{1}{R} \left[\frac{10}{3} \beta^{\frac{3}{2}} \left[P_{1, \frac{1}{2}}(1, \beta) - \frac{1}{5} \left\{ 2P_{1, \frac{3}{2}}(3, \beta) + 6P_{1, \frac{7}{2}}(3, \beta) + 10P_{1, \frac{5}{2}}(3, \beta) + 10P_{1, \frac{3}{2}}(3, \beta) \right. \right. \right. \\
& \left. \left. + 5P_{1, \frac{1}{2}}(3, \beta) \right\} \right] \\
& - \frac{10}{9} \beta^{\frac{1}{2}} \left[P_{0, \frac{3}{2}}(1, \beta) - \frac{1}{5} \left\{ 2P_{0, \frac{11}{2}}(3, \beta) + 6P_{0, \frac{9}{2}}(3, \beta) + 10P_{0, \frac{7}{2}}(3, \beta) + 10P_{0, \frac{5}{2}}(3, \beta) \right. \right. \\
& \left. \left. + 5P_{0, \frac{3}{2}}(3, \beta) \right\} + 2P_{2, \frac{3}{2}}(1, \beta) - \frac{2}{5} \left\{ 2P_{2, \frac{11}{2}}(3, \beta) + 6P_{2, \frac{9}{2}}(3, \beta) + 10P_{2, \frac{7}{2}}(3, \beta) \right. \right. \\
& \left. \left. + 10P_{2, \frac{5}{2}}(3, \beta) + 5P_{2, \frac{3}{2}}(3, \beta) \right\} \right] \\
L_{s\sigma\sigma\bar{v}} &= \frac{1}{R} \frac{10}{9} \beta^{\frac{1}{2}} \left[Q_{0, \frac{1}{2}}(1, \beta) - \frac{1}{5} \left\{ 2Q_{0, \frac{3}{2}}(3, \beta) + 6Q_{0, \frac{7}{2}}(3, \beta) + 10Q_{0, \frac{5}{2}}(3, \beta) + 10Q_{0, \frac{3}{2}}(3, \beta) \right. \right. \\
& \left. \left. + 5Q_{0, \frac{1}{2}}(3, \beta) \right\} + 2Q_{2, \frac{1}{2}}(1, \beta) - \frac{2}{5} \left\{ 2Q_{2, \frac{3}{2}}(3, \beta) + 6Q_{2, \frac{7}{2}}(3, \beta) + 10Q_{2, \frac{5}{2}}(3, \beta) \right. \right. \\
& \left. \left. + 10Q_{2, \frac{3}{2}}(3, \beta) + 5Q_{2, \frac{1}{2}}(3, \beta) \right\} \right] \\
L_{s\sigma\sigma\bar{w}} &= \frac{1}{R} \left[\frac{10}{3\sqrt{3}} \beta^{\frac{3}{2}} \left[P_{0, \frac{1}{2}}(1, \beta) - \frac{1}{5} \left\{ 2P_{0, \frac{3}{2}}(3, \beta) + 6P_{0, \frac{7}{2}}(3, \beta) + 10P_{0, \frac{5}{2}}(3, \beta) + 10P_{0, \frac{3}{2}}(3, \beta) \right. \right. \right. \\
& \left. \left. + 5P_{0, \frac{1}{2}}(3, \beta) \right\} + 2P_{2, \frac{1}{2}}(1, \beta) - \frac{2}{5} \left\{ 2P_{2, \frac{3}{2}}(3, \beta) + 6P_{2, \frac{7}{2}}(3, \beta) + 10P_{2, \frac{5}{2}}(3, \beta) \right. \right. \\
& \left. \left. + 10P_{2, \frac{3}{2}}(3, \beta) + 5P_{2, \frac{1}{2}}(3, \beta) \right\} \right] \\
& - \frac{2}{\sqrt{3}} \beta^{\frac{1}{2}} \left[3P_{1, \frac{3}{2}}(1, \beta) - \frac{3}{5} \left\{ 2P_{1, \frac{11}{2}}(3, \beta) + 6P_{1, \frac{9}{2}}(3, \beta) + 10P_{1, \frac{7}{2}}(3, \beta) + 10P_{1, \frac{5}{2}}(3, \beta) \right. \right. \\
& \left. \left. + 5P_{1, \frac{3}{2}}(3, \beta) \right\} + 2P_{3, \frac{3}{2}}(1, \beta) - \frac{2}{5} \left\{ 2P_{3, \frac{11}{2}}(3, \beta) + 6P_{3, \frac{9}{2}}(3, \beta) + 10P_{3, \frac{7}{2}}(3, \beta) \right. \right. \\
& \left. \left. + 10P_{3, \frac{5}{2}}(3, \beta) + 5P_{3, \frac{3}{2}}(3, \beta) \right\} \right] \\
L_{s\pi\sigma\bar{\pi}} &= \frac{1}{R} \frac{2}{\sqrt{3}} \beta^{\frac{1}{2}} \left[P_{1, \frac{3}{2}}(1, \beta) - \frac{1}{5} \left\{ 2P_{1, \frac{11}{2}}(3, \beta) + 6P_{1, \frac{9}{2}}(3, \beta) + 10P_{1, \frac{7}{2}}(3, \beta) + 10P_{1, \frac{5}{2}}(3, \beta) \right. \right. \\
& \left. \left. + 5P_{1, \frac{3}{2}}(3, \beta) \right\} - P_{3, \frac{3}{2}}(1, \beta) + \frac{1}{5} \left\{ 2P_{3, \frac{11}{2}}(3, \beta) + 6P_{3, \frac{9}{2}}(3, \beta) + 10P_{3, \frac{7}{2}}(3, \beta) \right. \right. \\
& \left. \left. + 10P_{3, \frac{5}{2}}(3, \beta) + 5P_{3, \frac{3}{2}}(3, \beta) \right\} \right]
\end{aligned}$$

$$\begin{aligned}
& +6P_{2, \frac{1}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right)+10P_{2, \frac{3}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right)+10P_{2, \frac{5}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right)+5P_{2, \frac{7}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right)\Big] \\
L_{s\pi\pi\bar{h}} = & \frac{1}{R} \frac{10}{3\sqrt{3}} \frac{\beta^{\frac{3}{2}}}{\alpha} \left[P_{0, \frac{1}{2}}\left(\frac{\beta}{\alpha}, \alpha\right) - \frac{1}{5} \left\{ 2\left(\frac{\beta}{\alpha}\right)^4 P_{0, \frac{3}{2}}\left(\frac{3\beta}{\alpha}, \alpha\right) + 6\left(\frac{\beta}{\alpha}\right)^3 P_{0, \frac{5}{2}}\left(\frac{3\beta}{\alpha}, \alpha\right) \right. \right. \\
& + 10\left(\frac{\beta}{\alpha}\right)^2 P_{0, \frac{7}{2}}\left(\frac{3\beta}{\alpha}, \alpha\right) + 10\frac{\beta}{\alpha} P_{0, \frac{9}{2}}\left(\frac{3\beta}{\alpha}, \alpha\right) + 5P_{0, \frac{11}{2}}\left(\frac{3\beta}{\alpha}, \alpha\right) \Big\} \\
& - P_{2, \frac{1}{2}}\left(\frac{\beta}{\alpha}, \alpha\right) + \frac{1}{5} \left\{ 2\left(\frac{\beta}{\alpha}\right)^4 P_{2, \frac{3}{2}}\left(\frac{3\beta}{\alpha}, \alpha\right) + 6\left(\frac{\beta}{\alpha}\right)^3 P_{2, \frac{5}{2}}\left(\frac{3\beta}{\alpha}, \alpha\right) + 10\left(\frac{\beta}{\alpha}\right)^2 P_{2, \frac{7}{2}}\left(\frac{3\beta}{\alpha}, \alpha\right) \right. \\
& \left. \left. + 10\frac{\beta}{\alpha} P_{2, \frac{9}{2}}\left(\frac{3\beta}{\alpha}, \alpha\right) + 5P_{2, \frac{11}{2}}\left(\frac{3\beta}{\alpha}, \alpha\right) \right\} \right] \\
L_{s\sigma\pi\bar{\pi}} = & \frac{1}{R} \frac{10}{9} \beta^{\frac{1}{2}} \left[P_{0, \frac{3}{2}}(1, \beta) - \frac{1}{5} \left\{ 2P_{0, \frac{11}{2}}(3, \beta) + 6P_{0, \frac{9}{2}}(3, \beta) + 10P_{0, \frac{7}{2}}(3, \beta) + 10P_{0, \frac{5}{2}}(3, \beta) \right. \right. \\
& + 5P_{0, \frac{3}{2}}(3, \beta) \Big\} - P_{2, \frac{3}{2}}(1, \beta) + \frac{1}{5} \left\{ 2P_{2, \frac{11}{2}}(3, \beta) + 6P_{2, \frac{9}{2}}(3, \beta) + 10P_{2, \frac{7}{2}}(3, \beta) \right. \\
& \left. \left. + 10P_{2, \frac{5}{2}}(3, \beta) + 5P_{2, \frac{3}{2}}(3, \beta) \right\} \right] \\
L_{s\pi\pi\bar{s}} = & \frac{1}{R} \frac{10}{9} \beta^{\frac{1}{2}} \left[Q_{0, \frac{1}{2}}(1, \beta) - \frac{1}{5} \left\{ 2Q_{0, \frac{3}{2}}(3, \beta) + 6Q_{0, \frac{5}{2}}(3, \beta) + 10Q_{0, \frac{7}{2}}(3, \beta) + 10Q_{0, \frac{9}{2}}(3, \beta) \right. \right. \\
& + 5Q_{0, \frac{1}{2}}(3, \beta) \Big\} - Q_{2, \frac{1}{2}}(1, \beta) + \frac{1}{5} \left\{ 2Q_{2, \frac{3}{2}}(3, \beta) + 6Q_{2, \frac{5}{2}}(3, \beta) + 10Q_{2, \frac{7}{2}}(3, \beta) \right. \\
& \left. \left. + 10Q_{2, \frac{9}{2}}(3, \beta) + 5Q_{2, \frac{1}{2}}(3, \beta) \right\} \right] \\
L_{s\sigma\sigma\bar{\pi}} = & \frac{1}{R} \frac{2}{\sqrt{3}} \beta^{\frac{1}{2}} \left[P_{1, \frac{3}{2}}(1, \beta) - \frac{1}{5} \left\{ 2P_{1, \frac{11}{2}}(3, \beta) + 6P_{1, \frac{9}{2}}(3, \beta) + 10P_{1, \frac{7}{2}}(3, \beta) + 10P_{1, \frac{5}{2}}(3, \beta) \right. \right. \\
& + 5P_{1, \frac{3}{2}}(3, \beta) \Big\} - P_{3, \frac{3}{2}}(1, \beta) + \frac{1}{5} \left\{ 2P_{3, \frac{11}{2}}(3, \beta) + 6P_{3, \frac{9}{2}}(3, \beta) + 10P_{3, \frac{7}{2}}(3, \beta) \right. \\
& \left. \left. + 10P_{3, \frac{5}{2}}(3, \beta) + 5P_{3, \frac{3}{2}}(3, \beta) \right\} \right] \\
L_{s\pi\pi\bar{\sigma}} = & \frac{1}{R} \left[\frac{10}{3\sqrt{3}} \beta^{\frac{3}{2}} \left[P_{0, \frac{1}{2}}(1, \beta) - \frac{1}{5} \left\{ 2P_{0, \frac{7}{2}}(3, \beta) + 6P_{0, \frac{5}{2}}(3, \beta) + 10P_{0, \frac{3}{2}}(3, \beta) + 10P_{0, \frac{1}{2}}(3, \beta) \right. \right. \right. \\
& + 5P_{0, \frac{1}{2}}(3, \beta) \Big\} - P_{2, \frac{1}{2}}(1, \beta) + \frac{1}{5} \left\{ 2P_{2, \frac{3}{2}}(3, \beta) + 6P_{2, \frac{5}{2}}(3, \beta) + 10P_{2, \frac{7}{2}}(3, \beta) \right. \\
& \left. \left. + 10P_{2, \frac{9}{2}}(3, \beta) + 5P_{2, \frac{1}{2}}(3, \beta) \right\} \right] \\
& - \frac{2}{\sqrt{3}} \beta^{\frac{1}{2}} \left[P_{1, \frac{3}{2}}(1, \beta) - \frac{1}{5} \left\{ 2P_{1, \frac{11}{2}}(3, \beta) + 6P_{1, \frac{9}{2}}(3, \beta) + 10P_{1, \frac{7}{2}}(3, \beta) + 10P_{1, \frac{5}{2}}(3, \beta) \right. \right. \\
& + 5P_{1, \frac{3}{2}}(3, \beta) \Big\} - P_{3, \frac{3}{2}}(1, \beta) + \frac{1}{5} \left\{ 2P_{3, \frac{11}{2}}(3, \beta) + 6P_{3, \frac{9}{2}}(3, \beta) + 10P_{3, \frac{7}{2}}(3, \beta) \right. \\
& \left. \left. + 10P_{3, \frac{5}{2}}(3, \beta) + 5P_{3, \frac{3}{2}}(3, \beta) \right\} \right] \Big] \\
L_{\sigma h\pi\bar{\pi}} = & \frac{1}{R} \frac{18}{5} \frac{\alpha^{\frac{3}{2}}}{\beta} \left[P_{1, -\frac{1}{2}}\left(\frac{\alpha}{\beta}, \beta\right) - \frac{1}{9} \left\{ 2P_{1, \frac{3}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right) + 6P_{1, \frac{5}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right) + 12P_{1, \frac{7}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right) \right. \right. \\
& + 18P_{1, \frac{9}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right) + 18P_{1, \frac{1}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right) + 9P_{1, -\frac{1}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right) \Big\} - P_{3, -\frac{1}{2}}\left(\frac{\alpha}{\beta}, \beta\right) \\
& + \frac{1}{9} \left\{ 2P_{3, \frac{3}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right) + 6P_{3, \frac{5}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right) + 12P_{3, \frac{7}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right) + 18P_{3, \frac{9}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right) \right. \\
& \left. \left. + 18P_{3, \frac{1}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right) + 9P_{3, -\frac{1}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right) \right\} \right]
\end{aligned}$$

$$\begin{aligned}
& +18P_{3, \frac{1}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right)+9P_{3, -\frac{1}{2}}\left(\frac{\alpha}{\beta}+2, \beta\right)\} \\
L_{\sigma\pi\pi\bar{h}} &= \frac{1}{R} \frac{18}{5} \beta^{\frac{1}{2}} \left[P_{1, -\frac{1}{2}}\left(\frac{\beta}{\alpha}, \alpha\right) - \frac{1}{9} \left\{ 2\left(\frac{\beta}{\alpha}\right)^5 P_{1, \frac{3}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right) + 6\left(\frac{\beta}{\alpha}\right)^4 P_{1, \frac{7}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right) \right. \right. \\
& + 12\left(\frac{\beta}{\alpha}\right)^3 P_{1, \frac{5}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right) + 18\left(\frac{\beta}{\alpha}\right)^2 P_{1, \frac{3}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right) + 18\frac{\beta}{\alpha} P_{1, \frac{1}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right) + 9P_{1, -\frac{1}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right) \} \\
& - P_{3, -\frac{1}{2}}\left(\frac{\beta}{\alpha}, \alpha\right) + \frac{1}{9} \left\{ 2\left(\frac{\beta}{\alpha}\right)^5 P_{3, \frac{3}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right) + 6\left(\frac{\beta}{\alpha}\right)^4 P_{3, \frac{7}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right) + 12\left(\frac{\beta}{\alpha}\right)^3 P_{3, \frac{5}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right) \right. \\
& \left. \left. + 18\left(\frac{\beta}{\alpha}\right)^2 P_{3, \frac{3}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right) + 18\frac{\beta}{\alpha} P_{3, \frac{1}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right) + 9P_{3, -\frac{1}{2}}\left(3\frac{\beta}{\alpha}, \alpha\right) \right\} \right] \\
L_{\sigma s\pi\pi\bar{\pi}} &= \frac{1}{R} \frac{18}{5\sqrt{3}} \beta^{\frac{1}{2}} \left[P_{1, \frac{1}{2}}(1, \beta) - \frac{1}{9} \left\{ 2P_{1, \frac{11}{2}}(3, \beta) + 6P_{1, \frac{9}{2}}(3, \beta) + 12P_{1, \frac{7}{2}}(3, \beta) + 18P_{1, \frac{5}{2}}(3, \beta) \right. \right. \\
& \left. \left. + 18P_{1, \frac{3}{2}}(3, \beta) + 9P_{1, \frac{1}{2}}(3, \beta) \right\} - P_{3, \frac{1}{2}}(1, \beta) + \frac{1}{9} \left\{ 2P_{3, \frac{11}{2}}(3, \beta) + 6P_{3, \frac{9}{2}}(3, \beta) \right. \right. \\
& \left. \left. + 12P_{3, \frac{7}{2}}(3, \beta) + 18P_{3, \frac{5}{2}}(3, \beta) + 18P_{3, \frac{3}{2}}(3, \beta) + 9P_{3, \frac{1}{2}}(3, \beta) \right\} \right] \\
L_{\sigma\pi\pi\bar{s}} &= \frac{1}{R} \frac{18}{5\sqrt{3}} \beta^{\frac{1}{2}} \left[Q_{1, -\frac{1}{2}}(1, \beta) - \frac{1}{9} \left\{ 2Q_{1, \frac{9}{2}}(3, \beta) + 6Q_{1, \frac{7}{2}}(3, \beta) + 12Q_{1, \frac{5}{2}}(3, \beta) + 18Q_{1, \frac{3}{2}}(3, \beta) \right. \right. \\
& \left. \left. + 18Q_{1, \frac{1}{2}}(3, \beta) + 9Q_{1, -\frac{1}{2}}(3, \beta) \right\} - Q_{3, -\frac{1}{2}}(1, \beta) + \frac{1}{9} \left\{ 2Q_{3, \frac{9}{2}}(3, \beta) + 6Q_{3, \frac{7}{2}}(3, \beta) \right. \right. \\
& \left. \left. + 12Q_{3, \frac{5}{2}}(3, \beta) + 18Q_{3, \frac{3}{2}}(3, \beta) + 18Q_{3, \frac{1}{2}}(3, \beta) + 9Q_{3, -\frac{1}{2}}(3, \beta) \right\} \right] \\
L_{\sigma\sigma\pi\pi\bar{\pi}} &= \frac{1}{R} \left[\frac{6}{5} \beta^{\frac{1}{2}} \left[P_{0, \frac{1}{2}}(1, \beta) - \frac{1}{9} \left\{ 2P_{0, \frac{11}{2}}(3, \beta) + 6P_{0, \frac{9}{2}}(3, \beta) + 12P_{0, \frac{7}{2}}(3, \beta) + 18P_{0, \frac{5}{2}}(3, \beta) \right. \right. \right. \\
& \left. \left. + 18P_{0, \frac{3}{2}}(3, \beta) + 9P_{0, \frac{1}{2}}(3, \beta) \right\} \right] \\
& + \frac{6}{7} \beta^{\frac{1}{2}} \left[P_{2, \frac{1}{2}}(1, \beta) - \frac{1}{9} \left\{ 2P_{2, \frac{11}{2}}(3, \beta) + 6P_{2, \frac{9}{2}}(3, \beta) + 12P_{2, \frac{7}{2}}(3, \beta) + 18P_{2, \frac{5}{2}}(3, \beta) \right. \right. \\
& \left. \left. + 18P_{2, \frac{3}{2}}(3, \beta) + 9P_{2, \frac{1}{2}}(3, \beta) \right\} \right] \\
& - \frac{72}{35} \beta^{\frac{1}{2}} \left[P_{4, \frac{1}{2}}(1, \beta) - \frac{1}{9} \left\{ 2P_{4, \frac{11}{2}}(3, \beta) + 6P_{4, \frac{9}{2}}(3, \beta) + 12P_{4, \frac{7}{2}}(3, \beta) + 18P_{4, \frac{5}{2}}(3, \beta) \right. \right. \\
& \left. \left. + 18P_{4, \frac{3}{2}}(3, \beta) + 9P_{4, \frac{1}{2}}(3, \beta) \right\} \right] \right] \\
L_{\sigma\pi\pi\bar{\sigma}} &= \frac{1}{R} \left[\frac{18}{5\sqrt{3}} \beta^{\frac{3}{2}} \left[P_{1, -\frac{1}{2}}(1, \beta) - \frac{1}{9} \left\{ 2P_{1, \frac{9}{2}}(3, \beta) + 6P_{1, \frac{7}{2}}(3, \beta) + 12P_{1, \frac{5}{2}}(3, \beta) + 18P_{1, \frac{3}{2}}(3, \beta) \right. \right. \right. \\
& \left. \left. + 18P_{1, \frac{1}{2}}(3, \beta) + 9P_{1, -\frac{1}{2}}(3, \beta) \right\} \right] - P_{3, -\frac{1}{2}}(1, \beta) + \frac{1}{9} \left\{ 2P_{3, \frac{9}{2}}(3, \beta) + 6P_{3, \frac{7}{2}}(3, \beta) \right. \\
& \left. \left. + 12P_{3, \frac{5}{2}}(3, \beta) + 18P_{3, \frac{3}{2}}(3, \beta) + 18P_{3, \frac{1}{2}}(3, \beta) + 9P_{3, -\frac{1}{2}}(3, \beta) \right\} \right] \\
& - \frac{6}{5} \beta^{\frac{1}{2}} \left[P_{0, \frac{1}{2}}(1, \beta) - \frac{1}{9} \left\{ 2P_{0, \frac{11}{2}}(3, \beta) + 6P_{0, \frac{9}{2}}(3, \beta) + 12P_{0, \frac{7}{2}}(3, \beta) + 18P_{0, \frac{5}{2}}(3, \beta) \right. \right. \\
& \left. \left. + 18P_{0, \frac{3}{2}}(3, \beta) + 9P_{0, \frac{1}{2}}(3, \beta) \right\} \right] \\
& - \frac{6}{7} \beta^{\frac{1}{2}} \left[P_{2, \frac{1}{2}}(1, \beta) - \frac{1}{9} \left\{ 2P_{2, \frac{11}{2}}(3, \beta) + 6P_{2, \frac{9}{2}}(3, \beta) + 12P_{2, \frac{7}{2}}(3, \beta) + 18P_{2, \frac{5}{2}}(3, \beta) \right. \right. \\
& \left. \left. + 18P_{2, \frac{3}{2}}(3, \beta) + 9P_{2, \frac{1}{2}}(3, \beta) \right\} \right] \right]
\end{aligned}$$

$$\begin{aligned}
& + \frac{72}{35} \beta^{\frac{3}{2}} \left[P_{4, \frac{1}{2}}(1, \beta) - \frac{1}{9} \left\{ 2P_{4, \frac{11}{3}}(3, \beta) + 6P_{4, \frac{5}{2}}(3, \beta) + 12P_{4, \frac{7}{2}}(3, \beta) + 18P_{4, \frac{5}{2}}(3, \beta) \right. \right. \\
& \left. \left. + 18P_{4, \frac{5}{2}}(3, \beta) + 9P_{4, \frac{1}{2}}(3, \beta) \right\} \right]
\end{aligned}$$

Table XXVII Formulae for Ionic Integrals $L_{\alpha\beta\gamma\delta}$
in terms of $A_n(\alpha)$ and $B_n(\alpha)$

$$L_{hhhh\bar{h}} = \frac{1}{R} \frac{1}{2} \alpha^3 \left[2A_1(\alpha) - \frac{1}{2} \left\{ \alpha \left(A_2(2\alpha)B_0(\alpha) - A_0(2\alpha)B_2(\alpha) \right) + 2 \left(A_1(2\alpha)B_0(\alpha) - A_0(2\alpha)B_1(\alpha) \right) \right\} \right]$$

$$\begin{aligned}
L_{hhhh\bar{s}} = \frac{1}{R} \frac{1}{4\sqrt{3}} \alpha^{\frac{3}{2}} \beta^{\frac{5}{2}} & \left[A_2(r)B_0(\delta) - 2A_1(r)B_1(\delta) + A_0(r)B_2(\delta) - \frac{1}{2} \left\{ \alpha \left(A_3(\mu)B_0(\alpha+\delta) \right. \right. \right. \\
& - A_2(\mu)B_1(\alpha+\delta) - A_1(\mu)B_2(\alpha+\delta) + A_0(\mu)B_3(\alpha+\delta) \left. \left. \left. \right) + 2 \left(A_2(\mu)B_0(\alpha+\delta) \right. \right. \right. \\
& \left. \left. \left. - 2A_1(\mu)B_1(\alpha+\delta) + A_0(\mu)B_2(\alpha+\delta) \right) \right\} \right]
\end{aligned}$$

$$\begin{aligned}
L_{hsh\bar{h}} = \frac{1}{R} \frac{1}{4\sqrt{3}} \alpha^{\frac{3}{2}} \beta^{\frac{5}{2}} & \left[A_2(r)B_0(-\delta) - A_0(r)B_2(-\delta) - \frac{1}{2} \left\{ \alpha \left(A_3(\mu)B_0(r) + A_2(\mu)B_1(r) - A_1(\mu)B_2(r) \right. \right. \right. \\
& \left. \left. \left. - A_0(\mu)B_3(r) \right) + 2 \left(A_2(\mu)B_0(r) - A_0(\mu)B_2(r) \right) \right\} \right]
\end{aligned}$$

$$\begin{aligned}
L_{hsh\bar{s}} = \frac{1}{R} \frac{1}{4} \alpha^{\frac{3}{2}} \beta^{\frac{5}{2}} & \left[-A_0(r)B_1(\delta) + A_1(r) \left\{ B_0(\delta) + B_2(\delta) \right\} - A_2(r)B_1(\delta) - \frac{\alpha}{2} \left\{ -A_0(\mu)B_2(\alpha+\delta) \right. \right. \\
& + A_1(\mu)B_3(\alpha+\delta) + A_2(\mu)B_0(\alpha+\delta) - A_3(\mu)B_1(\alpha+\delta) \left. \left. \right\} - \left\{ -A_0(\mu)B_1(\alpha+\delta) \right. \right. \\
& \left. \left. + A_1(\mu) \left(B_0(\alpha+\delta) + B_2(\alpha+\delta) \right) - A_2(\mu)B_1(\alpha+\delta) \right\} \right]
\end{aligned}$$

$$\begin{aligned}
L_{h\bar{s}h\bar{h}} = \frac{1}{R} \frac{1}{4} \alpha^{\frac{3}{2}} \beta^{\frac{5}{2}} & \left[-A_0(r)B_1(-\delta) + A_1(r) \left\{ B_1(-\delta) - B_2(-\delta) \right\} + A_2(r)B_1(-\delta) \right. \\
& \left. - \frac{\alpha}{2} \left\{ -A_0(\mu)B_2(r) - A_1(\mu)B_3(r) + A_2(\mu)B_0(r) + A_3(\mu)B_1(r) \right\} \right. \\
& \left. - \left\{ -A_0(\mu)B_1(r) + A_1(\mu) \left(B_0(r) - B_2(r) \right) + A_2(\mu)B_1(r) \right\} \right]
\end{aligned}$$

$$\begin{aligned}
L_{hsh\bar{s}} = \frac{1}{R} \frac{1}{24} \beta^5 & \left[2A_3(\beta) - \frac{2}{3}A_1(\beta) - \frac{1}{2} \left\{ \alpha \left(A_4(2r)B_0(\alpha) - 2A_2(2r)B_2(\alpha) + A_0(2r)B_4(\alpha) \right) \right. \right. \\
& \left. \left. + 2 \left(A_3(2r)B_0(\alpha) - A_2(2r)B_1(\alpha) - A_1(2r)B_2(\alpha) + A_0(2r)B_3(\alpha) \right) \right\} \right]
\end{aligned}$$

$$\begin{aligned}
L_{h\bar{s}h\bar{s}} = \frac{1}{R} \frac{1}{8\sqrt{3}} \beta^5 & \left[-\frac{2}{3}A_0(\beta) + 2A_2(\beta) - \frac{\alpha}{2} \left\{ -A_0(2r)B_3(\alpha) - A_1(2r) \left(B_2(\alpha) - B_4(\alpha) \right) \right. \right. \\
& + A_2(2r) \left(B_1(\alpha) + B_3(\alpha) \right) + A_3(2r) \left(B_0(\alpha) - B_2(\alpha) \right) - A_4(2r)B_1(\alpha) \left. \left. \right\} - \left\{ -A_0(2r)B_2(\alpha) \right. \right. \\
& \left. \left. + A_1(2r)B_3(\alpha) + A_2(2r)B_0(\alpha) - A_3(2r)B_1(\alpha) \right\} \right]
\end{aligned}$$

$$\begin{aligned}
L_{h\bar{s}h\bar{s}} = \frac{1}{R} \frac{1}{8\sqrt{3}} \beta^5 & \left[\frac{2}{3}A_0(\beta) + \frac{2}{3}A_2(\beta) - \frac{\alpha}{2} \left\{ A_0(2r)B_3(\alpha) - A_1(2r) \left(B_2(\alpha) - B_4(\alpha) \right) \right. \right. \\
& - A_2(2r) \left(B_1(\alpha) + B_3(\alpha) \right) + A_3(2r) \left(B_0(\alpha) - B_2(\alpha) \right) + A_4(2r)B_1(\alpha) \left. \left. \right\} - \left\{ A_0(2r)B_2(\alpha) \right. \right. \\
& \left. \left. - A_1(2r) \left(2B_1(\alpha) - B_3(\alpha) \right) + A_2(2r) \left(B_0(\alpha) - 2B_2(\alpha) \right) + A_3(2r)B_1(\alpha) \right\} \right]
\end{aligned}$$

$$L_{\hbar\sigma\hbar\bar{\sigma}} = \frac{1}{R} \frac{1}{8} \beta^5 \left[2A_1(\beta) - \frac{2}{3}A_3(\beta) - \frac{\alpha}{2} \left\{ -A_0(2r)B_2(\alpha) + A_2(2r)(B_0(\alpha) + B_4(\alpha)) - A_4(2r)B_2(\alpha) \right\} \right. \\ \left. - \left\{ -A_0(2r)B_1(\alpha) + A_1(2r)B_0(\alpha) + A_2(2r)B_3(\alpha) - A_3(2r)B_2(\alpha) \right\} \right]$$

$$L_{\hbar\pi\hbar\bar{\pi}} = \frac{1}{R} \frac{1}{16} \beta^5 \left[\frac{4}{3}A_3(\beta) - \frac{4}{3}A_1(\beta) - \frac{\alpha}{2} \left\{ (B_0(\alpha) - B_2(\alpha))(A_4(2r) - A_2(2r)) \right. \right. \\ \left. \left. - (B_2(\alpha) - B_4(\alpha))(A_2(2r) - A_0(2r)) \right\} - \left\{ (B_0(\alpha) - B_2(\alpha))(A_3(2r) - A_1(2r)) \right. \right. \\ \left. \left. - (B_1(\alpha) - B_3(\alpha))(A_2(2r) - A_0(2r)) \right\} \right]$$

$$L_{\hbar\hbar s\bar{s}} = \frac{1}{R} 4\sqrt{3} \frac{\alpha^{\frac{3}{2}}\beta^{\frac{5}{2}}}{(2r)^4} \left[2A_1(\alpha) - \frac{1}{6} \left\{ r^2(A_3(\mu)B_0(r) + A_2(\mu)B_1(r) - A_1(\mu)B_2(r) - A_0(\mu)B_3(r)) \right. \right. \\ \left. \left. + 4r(A_2(\mu)B_0(r) - A_0(\mu)B_2(r)) + 6(A_1(\mu)B_0(r) - A_0(\mu)B_1(r)) \right\} \right]$$

$$L_{\hbar\hbar s\bar{s}} = \frac{1}{R} \frac{1}{3} \frac{\alpha^2\beta^5}{(2r)^4} \left[6A_2(r)B_0(\delta) - 12A_1(r)B_1(\delta) + 6A_0(r)B_2(\delta) - \left\{ r^2(A_4(2r)B_0(\alpha) - 2A_2(2r)B_2(\alpha)) \right. \right. \\ \left. \left. + A_0(2r)B_4(\alpha) + 4r(A_3(2r)B_0(\alpha) - A_2(2r)B_1(\alpha) - A_1(2r)B_2(\alpha) + A_0(2r)B_3(\alpha)) \right. \right. \\ \left. \left. + 6(A_2(2r)B_0(\alpha) - 2A_1(2r)B_1(\alpha) + A_0(2r)B_2(\alpha)) \right\} \right]$$

$$L_{\hbar s s \bar{h}} = \frac{1}{R} \frac{1}{3} \frac{\alpha^3\beta^5}{(2r)^4} \left[6A_2(r)B_0(-\delta) - 6A_0(r)B_2(-\delta) - \left\{ r^2(A_4(2r)B_0(\beta) + 2A_3(2r)B_1(\beta) - 2A_1(2r)B_3(\beta)) \right. \right. \\ \left. \left. - A_0(2r)B_4(\beta) + 4r(A_3(2r)B_0(\beta) + A_2(2r)B_1(\beta) - A_1(2r)B_2(\beta) - A_0(2r)B_3(\beta)) \right. \right. \\ \left. \left. + 6(A_2(2r)B_0(\beta) - A_0(2r)B_2(\beta)) \right\} \right]$$

$$L_{\hbar\hbar s\bar{\sigma}} = \frac{1}{R} 2\sqrt{3} \frac{\alpha^3\beta^5}{(2r)^4} \left[-A_0(r)B_1(\delta) + A_1(r) \left\{ B_0(\delta) + B_2(\delta) \right\} - A_2(r)B_1(\delta) - \frac{r^2}{6} \left\{ -A_0(2r)B_3(\alpha) \right. \right. \\ \left. \left. - A_1(2r)(B_2(\alpha) - B_4(\alpha)) + A_2(2r)(B_1(\alpha) + B_3(\alpha)) + A_3(2r)(B_0(\alpha) - B_2(\alpha)) - A_4(2r)B_1(\alpha) \right\} \right. \\ \left. - \frac{2}{3}r \left\{ -A_0(2r)B_2(\alpha) - A_1(2r)B_3(\alpha) + A_2(2r)B_0(\alpha) - A_3(2r)B_1(\alpha) \right\} - \left\{ -A_0(2r)B_1(\alpha) \right. \right. \\ \left. \left. + A_1(2r)(B_0(\alpha) + B_2(\alpha)) - A_2(2r)B_1(\alpha) \right\} \right]$$

$$L_{\hbar\sigma s\bar{h}} = \frac{1}{R} 2\sqrt{3} \frac{\alpha^3\beta^5}{(2r)^4} \left[-A_0(r)B_1(-\delta) + A_1(r)(B_0(-\delta) - B_2(-\delta)) + A_2(r)B_1(-\delta) \right. \\ \left. - \frac{r^2}{6} \left\{ -A_0(2r)B_3(\beta) - A_1(2r)(B_2(\beta) + B_4(\beta)) + A_2(2r)(B_1(\beta) - B_3(\beta)) \right. \right. \\ \left. \left. + A_3(2r)(B_0(\beta) + B_2(\beta)) + A_4(2r)B_1(\beta) \right\} - \frac{2}{3}r \left\{ -A_0(2r)B_2(\beta) - A_1(2r)B_3(\beta) + A_2(2r)B_0(\beta) \right. \right. \\ \left. \left. + A_3(2r)B_1(\beta) \right\} - \left\{ -A_0(2r)B_1(\beta) + A_1(2r)(B_0(\beta) - B_2(\beta)) + A_2(2r)B_1(\beta) \right\} \right]$$

$$L_{\hbar s s \bar{s}} = \frac{1}{R} \frac{1}{6\sqrt{3}} \frac{\alpha^{\frac{3}{2}}\beta^{\frac{15}{2}}}{(2r)^4} \left[12A_3(\beta) - 4A_1(\beta) - \left\{ r^2(A_3(\lambda)B_0(r) + A_4(\lambda)B_1(r) - 2A_3(\lambda)B_2(r)) \right. \right. \\ \left. \left. - 2A_2(\lambda)B_3(r) + A_1(\lambda)B_4(r) + A_0(\lambda)B_5(r) \right\} + 4r(A_4(\lambda)B_0(r) - 2A_2(\lambda)B_2(r) + A_0(\lambda)B_4(r)) \right. \\ \left. + 6(A_3(\lambda)B_0(r) - A_2(\lambda)B_1(r) - A_1(\lambda)B_2(r) + A_0(\lambda)B_3(r)) \right\} \right]$$

$$L_{\hbar s s \bar{\sigma}} = \frac{1}{R} \frac{\alpha^{\frac{3}{2}}\beta^{\frac{15}{2}}}{(2r)^4} \left[-\frac{2}{3}A_0(\beta) + 2A_2(\beta) - \frac{r^2}{6} \left\{ -A_0(\lambda)B_4(r) - A_1(\lambda)(2B_3(r) - B_5(r)) + 2A_2(\lambda)B_4(r) \right. \right. \\ \left. \left. + 2A_3(\lambda)B_5(r) \right\} \right]$$

$$\begin{aligned}
& +2A_3(\lambda)B_1(r) + A_4(\lambda)(B_0(r) - 2B_2(r)) - A_5(\beta+r)B_1(r) \left\} - \frac{2}{3}r \left\{ -A_0(\lambda)B_3(r) \right. \right. \\
& - A_1(\lambda)(B_2(r) - B_4(r)) + A_2(\lambda)(B_1(r) + B_3(r)) + A_3(\lambda)(B_0(r) - B_2(r)) - A_4(\lambda)B_1(r) \left. \right\} \\
& - \left. \left\{ -A_0(\lambda)B_2(r) + A_1(\lambda)B_3(r) + A_2(\lambda)B_0(r) - A_3(\lambda)B_1(r) \right\} \right]
\end{aligned}$$

$$\begin{aligned}
L_{h\sigma s\bar{s}} = \frac{1}{R} \frac{\alpha^{\frac{3}{2}} \beta^{\frac{15}{2}}}{(2r)^4} & \left[\frac{2}{3}A_0(\beta) + \frac{2}{3}A_2(\beta) - \frac{r^2}{6} \left\{ A_3(\lambda)B_1(r) + A_4(\lambda)B_0(r) - 2A_3(\lambda)B_3(r) - 2A_2(\lambda)B_2(r) \right. \right. \\
& + A_1(\lambda)B_3(r) + A_0(\lambda)B_4(r) \left. \right\} - \frac{2}{3}r \left\{ A_0(\lambda)B_3(r) - A_1(\lambda)(B_2(r) - B_4(r)) - A_2(\lambda)(B_1(r) + B_3(r)) \right. \\
& + A_3(\lambda)(B_0(r) - B_2(r)) + A_4(\lambda)B_1(r) \left. \right\} - \left\{ A_0(\lambda)B_2(r) - A_1(\lambda)(2B_1(r) - B_3(r)) \right. \\
& \left. \left. + A_2(\lambda)(B_0(r) - 2B_2(r)) + A_3(\lambda)B_1(r) \right\} \right]
\end{aligned}$$

$$\begin{aligned}
L_{h\sigma s\bar{\sigma}} = \frac{1}{R} \sqrt{3} \frac{\alpha^{\frac{3}{2}} \beta^{\frac{15}{2}}}{(2r)^4} & \left[2A_1(\beta) - \frac{2}{3}A_3(\beta) - \frac{r^2}{6} \left\{ -A_0(\lambda)B_3(r) - A_1(\lambda)B_2(r) + A_2(\lambda)(B_1(r) + B_3(r)) \right. \right. \\
& + A_3(\lambda)(B_0(r) + B_4(r)) - A_4(\lambda)B_3(r) - A_5(\lambda)B_2(r) \left. \right\} - \frac{2}{3}r \left\{ -A_0(\lambda)B_2(r) \right. \\
& + A_2(\lambda)(B_0(r) + B_4(r)) - A_4(\lambda)B_2(r) \left. \right\} - \left\{ -A_0(\lambda)B_1(r) + A_1(\lambda)B_0(r) + A_2(\lambda)B_3(r) \right. \\
& \left. \left. - A_3(\lambda)B_2(r) \right\} \right]
\end{aligned}$$

$$\begin{aligned}
L_{h\pi s\bar{\pi}} = \frac{1}{R} \frac{\sqrt{3}}{2} \frac{\alpha^{\frac{3}{2}} \beta^{\frac{15}{2}}}{(2r)^4} & \left[\frac{4}{3}A_3(\beta) - \frac{4}{3}A_1(\beta) - \frac{r^2}{6} \left\{ B_0(r)(A_5(\lambda) - A_3(\lambda)) + B_1(r)(A_4(\lambda) - A_2(\lambda)) \right. \right. \\
& - B_2(r)(A_5(\lambda) - A_1(\lambda)) - B_3(r)(A_4(\lambda) - A_0(\lambda)) + B_4(r)(A_3(\lambda) - A_1(\lambda)) \\
& + B_5(r)(A_2(\lambda) - A_0(\lambda)) \left. \right\} - \frac{2}{3}r \left\{ (B_0(r) - B_2(r))(A_4(\lambda) - A_2(\lambda)) \right. \\
& - (B_2(r) - B_4(r))(A_2(\lambda) - A_0(\lambda)) \left. \right\} - \left\{ (B_0(r) - B_2(r))(A_3(\lambda) - A_1(\lambda)) \right. \\
& \left. \left. - (B_1(r) - B_3(r))(A_2(\lambda) - A_0(\lambda)) \right\} \right]
\end{aligned}$$

$$\begin{aligned}
L_{s\bar{h}s\bar{h}} = \frac{1}{R} \frac{1}{48} \alpha^3 & \left[48A_1(\alpha) - \left\{ \beta^3(A_4(2r)B_0(\beta) + 2A_3(2r)B_1(\beta) - 2A_1(2r)B_3(\beta) - A_0(2r)B_4(\beta)) \right. \right. \\
& + 6\beta^2(A_3(2r)B_0(\beta) + A_2(2r)B_1(\beta) - A_1(2r)B_2(\beta) - A_0(2r)B_3(\beta)) + 18\beta(A_2(2r)B_0(\beta) \\
& \left. \left. - A_0(2r)B_2(\beta)) + 24(A_1(2r)B_0(\beta) - A_0(2r)B_1(\beta)) \right\} \right]
\end{aligned}$$

$$\begin{aligned}
L_{s\bar{h}s\bar{s}} = \frac{1}{R} \frac{1}{96\sqrt{3}} \alpha^{\frac{3}{2}} \beta^{\frac{15}{2}} & \left[24A_2(r)B_0(\delta) - 48A_1(r)B_1(\delta) + 24A_0(r)B_2(\delta) - \left\{ \beta^3(A_5(\lambda)B_0(r) \right. \right. \\
& + A_4(\lambda)B_1(r) - 2A_3(\lambda)B_2(r) - 2A_2(\lambda)B_3(r) + A_1(\lambda)B_4(r) + A_0(\lambda)B_5(r)) \\
& + 6\beta^2(A_4(\lambda)B_0(r) - 2A_2(\lambda)B_2(r) + A_0(\lambda)B_4(r)) + 18\beta(A_3(\lambda)B_0(r) - A_2(\lambda)B_1(r) \\
& \left. \left. - A_1(\lambda)B_2(r) + A_0(\lambda)B_3(r)) + 24(A_2(\lambda)B_0(r) - 2A_1(\lambda)B_1(r) + A_0(\lambda)B_2(r)) \right\} \right]
\end{aligned}$$

$$\begin{aligned}
L_{s\bar{s}s\bar{s}} = \frac{1}{R} \frac{1}{96\sqrt{3}} \alpha^{\frac{3}{2}} \beta^{\frac{15}{2}} & \left[24A_2(r)B_0(-\delta) - 24A_0(r)B_2(-\delta) - \left\{ \beta^3(A_5(\lambda)B_0(\beta - \delta) \right. \right. \\
& + 3A_4(\lambda)B_1(\beta - \delta) + 2A_3(\lambda)B_2(\beta - \delta) - 2A_2(\lambda)B_3(\beta - \delta) - 3A_1(\lambda)B_4(\beta - \delta) - A_0(\lambda)B_5(\beta - \delta) \left. \right\}
\end{aligned}$$

$$\begin{aligned}
& +6\beta^2 \left(A_4(\lambda)B_0(\beta-\delta) + 2A_3(\lambda)B_1(\beta-\delta) - 2A_1(\lambda)B_3(\beta-\delta) - A_0(\lambda)B_4(\beta-\delta) \right) \\
& +18\beta \left(A_3(\lambda)B_0(\beta-\delta) + A_2(\lambda)B_1(\beta-\delta) - A_1(\lambda)B_2(\beta-\delta) - A_0(\lambda)B_3(\beta-\delta) \right) \\
& +24 \left(A_2(\lambda)B_0(\beta-\delta) - A_0(\lambda)B_2(\beta-\delta) \right) \Big]
\end{aligned}$$

$$\begin{aligned}
L_{s h s \bar{\sigma}} = & \frac{1}{R} \frac{1}{4} \alpha^{\frac{3}{2}} \beta^{\frac{5}{2}} \left[-A_0(r)B_1(\delta) + A_1(r) \left\{ B_0(\delta) + B_2(\delta) \right\} - A_2(r)B_1(\delta) - \frac{\beta^3}{24} \left\{ -A_0(\lambda)B_4(r) \right. \right. \\
& - A_1(\lambda) \left(2B_3(r) - B_5(r) \right) + 2A_2(\lambda)B_4(r) + 2A_3(\lambda)B_1(\lambda) + A_4(\lambda) \left(B_0(r) - 2B_2(r) \right) - A_5(\lambda)B_1(r) \Big\} \\
& - \frac{\beta^2}{4} \left\{ -A_0(\lambda)B_3(r) - A_1(\lambda) \left(B_2(r) - B_4(r) \right) + A_2(\lambda) \left(B_1(r) + B_3(r) \right) + A_3(\lambda) \left(B_0(r) - B_2(r) \right) \right. \\
& - A_4(\lambda)B_1(r) \Big\} - \frac{3}{4} \beta \left\{ -A_0(\lambda)B_2(r) + A_1(\lambda)B_3(r) + A_2(\lambda)B_0(r) - A_3(\lambda)B_1(r) \right\} \\
& \left. \left. - \left\{ -A_0(\lambda)B_1(r) + A_1(\lambda) \left(B_0(r) + B_2(r) \right) - A_2(\lambda)B_1(r) \right\} \right] \right]
\end{aligned}$$

$$\begin{aligned}
L_{s \sigma s \bar{h}} = & \frac{1}{R} \frac{1}{4} \alpha^{\frac{3}{2}} \beta^{\frac{5}{2}} \left[-A_0(r)B_1(-\delta) + A_1(r) \left\{ B_0(-\delta) - B_2(-\delta) \right\} + A_2(r)B_1(-\delta) \right. \\
& - \frac{\beta^3}{24} \left\{ -A_0(\lambda)B_4(\beta-\delta) - A_1(\lambda) \left(2B_3(\beta-\delta) + B_5(\beta-\delta) \right) - 2A_2(\lambda)B_4(\beta-\delta) + 2A_3(\lambda)B_1(\beta-\delta) \right. \\
& + A_4(\lambda) \left(B_0(\beta-\delta) + 2B_2(\beta-\delta) \right) + A_5(\lambda)B_1(\beta-\delta) \Big\} - \frac{\beta^2}{4} \left\{ -A_0(\lambda)B_3(\beta-\delta) \right. \\
& - A_1(\lambda) \left(B_2(\beta-\delta) + B_4(\beta-\delta) \right) + A_2(\lambda) \left(B_1(\beta-\delta) - B_3(\beta-\delta) \right) + A_3(\lambda) \left(B_0(\beta-\delta) + B_2(\beta-\delta) \right) \\
& + A_4(\lambda)B_1(\beta-\delta) \Big\} - \frac{3}{4} \beta \left\{ -A_0(\lambda)B_2(\beta-\delta) - A_1(\lambda)B_3(\beta-\delta) + A_2(\lambda)B_0(\beta-\delta) + A_3(\lambda)B_1(\beta-\delta) \right\} \\
& \left. \left. - \left\{ -A_0(\lambda)B_1(\beta-\delta) + A_1(\lambda) \left(B_0(\beta-\delta) - B_2(\beta-\delta) \right) + A_2(\lambda)B_1(\beta-\delta) \right\} \right] \right]
\end{aligned}$$

$$\begin{aligned}
L_{s s s \bar{s}} = & \frac{1}{R} \left(\frac{1}{24} \right)^2 \beta^5 \left[48A_3(\beta) - 16A_1(\beta) - \left\{ \beta^3 \left(A_6(2\beta)B_0(\beta) + 2A_5(2\beta)B_1(\beta) - A_4(2\beta)B_2(\beta) \right. \right. \right. \\
& - 4A_3(2\beta)B_3(\beta) - A_2(2\beta)B_4(\beta) + 2A_1(2\beta)B_5(\beta) + A_0(2\beta)B_6(\beta) \Big) + 6\beta^2 \left(A_5(2\beta)B_0(\beta) \right. \\
& + A_4(2\beta)B_1(\beta) - 2A_3(2\beta)B_2(\beta) - 2A_2(2\beta)B_3(\beta) + A_1(2\beta)B_4(\beta) + A_0(2\beta)B_5(\beta) \Big) \\
& + 18\beta \left(A_4(2\beta)B_0(\beta) - 2A_2(2\beta)B_2(\beta) + A_0(2\beta)B_4(\beta) \right) + 24 \left(A_3(2\beta)B_0(\beta) - A_2(2\beta)B_1(\beta) \right. \\
& \left. \left. \left. - A_1(2\beta)B_2(\beta) + A_0(2\beta)B_3(\beta) \right) \right\} \right]
\end{aligned}$$

$$\begin{aligned}
L_{s s s \bar{\sigma}} = & \frac{1}{R} \frac{1}{8\sqrt{3}} \beta^5 \left[-\frac{2}{3} A_0(\beta) + 2A_2(\beta) - \frac{\beta^3}{24} \left\{ -A_0(2\beta)B_5(\beta) - A_1(2\beta) \left(3B_4(\beta) - B_6(\beta) \right) \right. \right. \\
& - A_2(2\beta) \left(2B_3(\beta) - 3B_5(\beta) \right) + 2A_3(2\beta) \left(B_2(\beta) + B_4(\beta) \right) + A_4(2\beta) \left(3B_1(\beta) - 2B_5(\beta) \right) \\
& + A_5(2\beta) \left(B_0(\beta) - 3B_2(\beta) \right) - A_6(2\beta)B_1(\beta) \Big\} \\
& - \frac{\beta^2}{4} \left\{ -A_0(2\beta)B_4(\beta) - A_1(2\beta) \left(2B_3(\beta) - B_5(\beta) \right) + 2A_2(2\beta)B_4(\beta) + 2A_3(2\beta)B_1(\beta) \right. \\
& + A_4(2\beta) \left(B_0(\beta) - 2B_2(\beta) \right) - A_5(2\beta)B_1(\beta) \Big\} \\
& \left. - \frac{3}{4} \beta \left\{ -A_0(2\beta)B_3(\beta) - A_1(2\beta) \left(B_2(\beta) - B_4(\beta) \right) + A_2(2\beta) \left(B_1(\beta) + B_3(\beta) \right) \right\} \right]
\end{aligned}$$

$$\begin{aligned}
& + A_3(2\beta)(B_0(\beta) - B_2(\beta)) - A_4(2\beta)B_1(\beta) \Big\} - \left\{ -A_0(2\beta)B_2(\beta) + A_1(2\beta)B_3(\beta) + A_2(2\beta)B_0(\beta) \right. \\
& \left. - A_3(2\beta)B_1(\beta) \right\} \Big] \\
L_{ss\sigma s} = & \frac{1}{R} \frac{1}{8\sqrt{3}} \beta^5 \left[\frac{2}{3} A_0(\beta) + \frac{2}{3} A_2(\beta) - \frac{\beta^3}{24} \left\{ A_0(2\beta)B_5(\beta) + A_1(2\beta)(B_4(\beta) + B_6(\beta)) \right. \right. \\
& - A_2(2\beta)(2B_3(\beta) - B_5(\beta)) - 2A_3(2\beta)(B_4(\beta) + B_2(\beta)) + A_4(2\beta)(B_1(\beta) - 2B_3(\beta)) \\
& + A_5(2\beta)(B_0(\beta) + B_2(\beta)) + A_6(2\beta)B_1(\beta) \Big\} \\
& - \frac{\beta^2}{4} \left\{ A_0(2\beta)B_4(\beta) + A_1(2\beta)B_5(\beta) - 2A_2(2\beta)B_2(\beta) - 2A_3(2\beta)B_3(\beta) + A_4(2\beta)B_0(\beta) \right. \\
& + A_5(2\beta)B_1(\beta) \Big\} - \frac{3}{4} \beta \left\{ A_0(2\beta)B_3(\beta) - A_1(2\beta)(B_2(\beta) - B_4(\beta)) - A_2(2\beta)(B_1(\beta) + B_3(\beta)) \right. \\
& + A_3(2\beta)(B_0(\beta) - B_2(\beta)) + A_4(2\beta)B_1(\beta) \Big\} - \left\{ A_0(2\beta)B_2(\beta) - A_1(2\beta)(2B_1(\beta) - B_3(\beta)) \right. \\
& \left. + A_2(2\beta)(B_0(\beta) - 2B_2(\beta)) + A_3(2\beta)B_1(\beta) \right\} \Big] \\
L_{s\sigma s\sigma} = & \frac{1}{R} \frac{1}{8} \beta^5 \left[2A_1(\beta) - \frac{2}{3} A_3(\beta) - \frac{\beta^3}{24} \left\{ -A_0(2\beta)B_4(\beta) - 2A_1(2\beta)B_3(\beta) + A_2(2\beta)B_6(\beta) \right. \right. \\
& + 2A_3(2\beta)(B_1(\beta) + B_5(\beta)) + A_4(2\beta)B_0(\beta) - 2A_5(2\beta)B_3(\beta) - A_6(2\beta)B_2(\beta) \Big\} - \frac{\beta^2}{4} \left\{ -A_0(2\beta)B_3(\beta) \right. \\
& - A_1(2\beta)B_2(\beta) + A_2(2\beta)(B_1(\beta) + B_5(\beta)) + A_3(2\beta)(B_0(\beta) + B_4(\beta)) - A_4(2\beta)B_3(\beta) \\
& - A_5(2\beta)B_2(\beta) \Big\} - \frac{3}{4} \beta \left\{ -A_0(2\beta)B_2(\beta) + A_2(2\beta)(B_0(\beta) + B_4(\beta)) - A_4(2\beta)B_2(\beta) \right\} \\
& \left. - \left\{ -A_0(2\beta)B_1(\beta) + A_1(2\beta)B_0(\beta) + A_2(2\beta)B_3(\beta) - A_3(2\beta)B_2(\beta) \right\} \right] \\
L_{s\pi s\pi} = & \frac{1}{R} \frac{\beta^5}{16} \left[\frac{4}{3} \left\{ A_3(\beta) - A_1(\beta) \right\} \right. \\
& - \frac{\beta^3}{24} \left\{ (A_6(2\beta) - A_4(2\beta))B_0(\beta) + 2(A_5(2\beta) - A_3(2\beta))B_1(\beta) - (A_6(2\beta) - A_4(2\beta))B_2(\beta) \right. \\
& - 2(A_5(2\beta) - A_1(2\beta))B_3(\beta) - (A_2(2\beta) - A_0(2\beta))B_4(\beta) \\
& + 2(A_3(2\beta) - A_1(2\beta))B_5(\beta) + (A_2(2\beta) - A_0(2\beta))B_6(\beta) \Big\} \\
& - \frac{\beta^2}{4} \left\{ (A_5(2\beta) - A_3(2\beta))B_0(\beta) + (A_4(2\beta) - A_2(2\beta))B_1(\beta) - (A_5(2\beta) - A_1(2\beta))B_2(\beta) \right. \\
& - (A_4(2\beta) - A_0(2\beta))B_3(\beta) + (A_3(2\beta) - A_1(2\beta))B_4(\beta) + (A_2(2\beta) - A_0(2\beta))B_5(\beta) \Big\} \\
& - \frac{3}{4} \beta \left\{ (A_4(2\beta) - A_2(2\beta))B_0(\beta) - (A_4(2\beta) - A_2(2\beta))B_2(\beta) + (A_2(2\beta) - A_0(2\beta))B_4(\beta) \right\} \\
& - \left\{ (A_3(2\beta) - A_1(2\beta))B_0(\beta) - (A_2(2\beta) - A_0(2\beta))B_1(\beta) - (A_3(2\beta) - A_1(2\beta))B_2(\beta) \right. \\
& \left. + (A_2(2\beta) - A_0(2\beta))B_3(\beta) \right\} \Big]
\end{aligned}$$