On the Turbulent Diffusion in the Atmosphere Near the Ground[1)]

Jiro Sakagami (坂上治郎)

Department of Physics, Faculty of Science,
Ochanomizu University, Tokyo

Introduction

Owing to the recent development of theoretical researches on turbulence, many theoretical reports on the turbulent diffusion have been published.[(1)] However almost all of them deal with standard deviations of spatial distributions of diffusive quantities, and some papers, in which the spatial distribution-functions are given explicitly, deal with the phenomena in an isotropic field, so there has been scarcely any result which is available to analysing the diffusion phenomena in the domain where effects of boundaries cannot be neglected, just as those in the domain within several meters high from the ground. On the other hand, some explicit formulae useful for such a domain are often necessary to the practical problems. So contrary to the deductive way which starts from the mechanism of the turbulent diffusion and which the researches of the turbulent diffusion hitherto have followed, we intended to obtain in phenomenalistic way those explicit formulae for the diffusion near the ground which may unify the experimental data.

Differential equation

a) Referring to the results obtained by the researches on turbulence, we assume that the horizontal diffusion occurs as in the vorticity transport theory,[(2)] and that vertical diffusion occurs in the momentum transport theory.[(3)]

b) The vertical diffusion coefficient is assumed to be a linear function of height from the ground ($K_z = bz$), and the two horizontal diffusion coefficients are assumed to be equal and constant ($K_x = K_y = a$).

c) We consider a coordinate system which moves along with the mean wind whose velocity is u (m/sec.) and we take x-axis along the mean wind direction leeward, z-axis vertically from the ground and y-axis perpendicularly to them. The coordinates of a point source at the time $t=0$ be $(0, 0, h)$ and the concentration of the diffusive quantities be C.

[1)] Contribution from Department of Physics, Faculty of Science, Ochanomizu University, No. 22.

d) Then, we obtain the next differential equation:

$$\frac{\partial C}{\partial t}=a\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)+\frac{\partial}{\partial z}\left(bz\frac{\partial C}{\partial z}\right). \tag{1}$$

If we put

$$C=\frac{e^{-\frac{x^2+y^2}{4at}}}{4at}\phi(t,z), \tag{2}$$

we get

$$\frac{\partial\phi}{\partial t}=\frac{\partial}{\partial z}\left(bz\frac{\partial\phi}{\partial z}\right). \tag{3}$$

Substituting $\phi(z,t)=f(t)g(z)$ in eq. (3), we obtain

$$f'/f=-\lambda'^2, \qquad \therefore\quad f=e^{-\lambda'^2 t}, \tag{4}$$

and

$$g''+\frac{g'}{2}+\frac{\lambda'^2}{bz}g=0, \tag{5}$$

where λ' is an arbitrary constant.

Putting $\sqrt{bz}=\varphi$, a particular solution of eq. (5) is

$$g=J_0\left(\frac{2\lambda'}{b}\varphi\right), \tag{6}$$

J_0 being the 1st kind Bessel function of the order zero. Y_0 is not adoptable because of its anomaly at $\varphi=0$. Then the general solution becomes

$$\phi(\varphi,t)=\int_0^\infty A(\lambda')e^{-\lambda'^2 t}J_0\left(\frac{2\varphi}{b}\lambda'\right)d\lambda'. \tag{7}$$

Assuming $\phi=\Psi(\varphi)$ at $t=0$ and $\frac{2}{b}\lambda'=\lambda''$,

$$\phi(\varphi,0)=\Psi(\varphi)=\int_0^\infty A(\lambda')J_0\left(\frac{2\varphi}{b}\lambda'\right)d\lambda'=\int_0^\infty A\left(\frac{b}{2}\lambda''\right)J_0(\varphi\lambda'')\frac{b}{2}d\lambda''. \tag{8}$$

Putting $\frac{b}{2}A\left(\frac{b}{2}\lambda''\right)\equiv B(\lambda)$, we get

$$\psi(\varphi,0)=\int_0^\infty B(\lambda)J_0(\varphi\lambda)d\lambda. \tag{9}$$

Hankel's inverse relation gives

$$\Phi(\varphi)=\int_0^\infty J_0(\lambda\varphi)\lambda d\lambda\int_0^\infty \Psi(\alpha)J_0(\lambda\alpha)\alpha d\alpha, \tag{10}$$

and comparing (9) with (10), we obtain

$$B(\lambda)=\lambda\int_0^\infty \Psi(\alpha)J_0(\lambda\alpha)\alpha d\alpha \ , \tag{11}$$

so we get

$$\phi(\varphi, t)=\int_0^\infty e^{-\frac{b^2}{4}\lambda^2 t}\lambda J_0(\lambda\varphi)\,d\lambda\int_0^\infty \Psi(\alpha)J_0(\lambda\alpha)\alpha d\alpha$$

$$=\int_0^\infty \Psi(\alpha)\alpha d\alpha\int_0^\infty J_0(\lambda\alpha)\,J_0(\lambda\varphi)\,e^{-\frac{b^2\lambda^2}{4}t}\lambda d\lambda \,. \tag{12}$$

Instantaneous point source

We may consider the case of an instantaneous point source. When $t=0$, $\Phi(\varphi)d\varphi$ is finite only where z is close to h, namely where φ is close to $\sqrt{bh}(\equiv\alpha_0)$, and it is equal to zero elsewhere. Putting $\int_0^\infty \Psi(\alpha)\alpha d\alpha = D$, eq. (12) becomes

$$\phi(\varphi, t)=D\int_0^\infty J_0(\lambda\alpha_0)J_0(\varphi\lambda)e^{-\frac{b^2\lambda^2}{4}t}\lambda d\lambda$$

$$=D\int_0^\infty e^{-\frac{b^2\lambda^2}{4}t}\lambda d\lambda\,\frac{1}{\pi}\int_0^\pi J_0(\lambda R)d\theta$$

(where $R=\sqrt{\varphi^2+\alpha_0^2-2\varphi\alpha_0\cos\theta}$)

$$=\frac{D}{\pi}\int_0^\pi d\theta\int_0^\infty e^{-\frac{b^2\lambda^2}{4}t}J_0(\lambda R)d\lambda$$

$$=\frac{D}{\pi}\frac{2}{b^2t}\int_0^\pi e^{-\frac{R^2}{b^2t}}d\theta=\frac{2D}{\pi b^2t}e^{-\frac{\varphi^2+\alpha_0^2}{b^2t}}\int_0^\pi e^{-\frac{2\alpha_0\varphi}{b^2t}\cos\theta}\,d\theta$$

$$=\frac{2D}{\pi b^2t}e^{-\frac{\varphi^2+\alpha_0^2}{b^2t}}\pi J_0\left(i\,\frac{2\alpha_0\varphi}{b^2t}\right)$$

$$=\frac{2D}{b^2t}e^{-\frac{h+z}{b^2t}}J_0\left(i\,\frac{2\sqrt{hz}}{bt}\right)\,. \tag{13}$$

Therefore the final form of $\phi(z, t)$ is

$$\phi(z, t)=\frac{2D}{b^2t}e^{-\frac{h+z}{bt}}J_0\left(i\,\frac{2\sqrt{hz}}{bt}\right)\,. \tag{14}$$

The factor containing x and y in C, eq. (2), satisfies the initial condition of the instantaneous point source, and (14) satisfies the boundary condition that there is no flux through the ground. So the solution of eq. (1) is

$$C=E\frac{e^{-\frac{x^2+y^2}{4at}}}{4at}\,\frac{2}{bt}\,e^{-\frac{h+z}{bt}}J_0\left(i\,\frac{2\sqrt{hz}}{bt}\right)\,, \tag{15}$$

where E is a constant.

Total amount of the diffusive quantities be M, and we get,

$$M=\int_0^\infty dz \int_{-\infty}^\infty dy \int_{-\infty}^\infty Cdx$$
$$=\frac{E}{2ab^2t^2}\int_{-\infty}^\infty e^{-\frac{x^2}{4at}}dx \int_{-\infty}^\infty e^{-\frac{y^2}{4at}}dy \int_0^\infty e^{-\frac{h+z}{bt}} J_0\left(i\frac{2\sqrt{hz}}{bt}\right)dz$$
$$=E\,\frac{2\pi}{bt}\int_0^\infty e^{-\frac{h+z}{bt}} J_0\left(i\frac{2\sqrt{hz}}{bt}\right)dz=\frac{2E\pi}{b} \tag{16}$$

So E is determined by

$$E=\frac{bM}{2\pi}\,. \tag{17}$$

Putting (17) in (15), we obtain

$$C=\frac{M}{4ab\pi t^2}\,e^{-\frac{x^2+y^2}{4at}-\frac{h+z}{bt}} J_0\left(i\frac{2\sqrt{hz}}{bt}\right). \tag{18}$$

Arranging the instantaneous point sources adequately in time and space, we can constitute every kind of source, so eq. (18) is the most fundamental equation for the diffusion phenomena.

Formula referred to the coordinate system fixed to the ground

Now we refer to a coordinate system fixed to the ground. The point of the ground directly under the source at $t=0$ is taken as the origin. Eq. (18) becomes

$$C=\frac{M}{4ab\pi t^2}\,e^{-\frac{(x-ut)^2+y^2}{4at}-\frac{h+z}{bt}} J_0\left(i\frac{2\sqrt{hz}}{bt}\right). \tag{19}$$

Continuous line source

A line source is supposed to be set along y-axis from $-s$ to $+s$, and the rate of emission per unit length and per unit time be q, which is assumed to be constant in time and space. We obtain

$$C=\frac{q}{4ab\pi}\int_0^t \frac{e^{-\frac{(x-u(t-\zeta))^2}{4a(t-\zeta)}-\frac{h+z}{b(t-\zeta)}}}{(t-\zeta)^2} J_0\left(i\frac{2\sqrt{hz}}{b(t-\zeta)}\right)d\zeta \int_{-s}^s e^{-\frac{(y-\eta)^2}{4a(t-\zeta)}}d\eta$$
$$=\frac{q}{\sqrt{4a\pi}b}\int_0^t \frac{e^{-\frac{(x-u(t-\zeta))^2}{4a(t-\zeta)}-\frac{h+z}{b(t-\zeta)}}}{(t-\zeta)^{3/2}} J_0\left(i\frac{2\sqrt{hz}}{b(t-\zeta)}\right)d\zeta$$
$$\times\frac{\Phi((y+s)/\sqrt{4a(t-\zeta)})-\Phi((y-s)/\sqrt{4a(t-\zeta)})}{2}\,, \tag{20}$$

where
$$\Phi(r)=\frac{2}{\sqrt{\pi}}\int_0^r e^{-u^2}du\,. \tag{21}$$

We put $t-\zeta=\xi$, then we obtain

$$C=\frac{q}{\sqrt{4a\pi b}}\int_0^t \frac{e^{-\frac{(x-u\xi)^2}{4a\xi}-\frac{h+z}{b\xi}}}{\xi^{3/2}} J_0\left(i\frac{2\sqrt{hz}}{b\xi}\right)d\xi$$

$$\times\frac{\Phi((y+s)/\sqrt{4a\xi})-\Phi((y-s)/\sqrt{4a\xi})}{2} \qquad t\leqq\lambda \tag{22-1}$$

$$=\frac{q}{\sqrt{4a\pi b}}\int_{t-\lambda}^t \frac{e^{-\frac{(x-u\xi)^2}{4a\xi}-\frac{h+z}{b\xi}}}{\xi^{3/2}} J_0\left(i\frac{2\sqrt{hz}}{b\xi}\right)d\xi$$

$$\times\frac{\Phi((y+s)/\sqrt{4a\xi})-\Phi((y-s)/\sqrt{4a\xi})}{2} \qquad t\geqq\lambda\,, \tag{22-2}$$

where λ is the time of duration of the source.

As the equations (22-1) and (22-2) have not convenient forms, we want to obtain approximate formulae.

Among the terms in the integrand, the term $\dfrac{e^{-\frac{(x-u\xi)^2}{4a\xi}}}{\xi^{3/2}}$ varies with ξ far more remarkably than any other terms, so we take ξ_0 which makes the value of that term maximum :

$$\xi_0=\sqrt{\left(\frac{x}{u}\right)^2+\left(\frac{3}{4}\frac{4a}{u^2}\right)^2}-\frac{3}{4}\frac{4a}{u^2}\,, \tag{23}$$

and we put this value in all ξ's other than the term $x-u\xi$, then we obtain by integration

$$C\doteqdot\frac{q}{B_0}e^{-\frac{h+z}{B_0}}J_0\left(i\frac{2\sqrt{hz}}{B_0}\right)\frac{\Phi((y+s)/\sqrt{A_0})-\Phi((y-s)/\sqrt{A_0})}{2}$$

$$\times\frac{1}{u}\frac{\Phi(x/\sqrt{A_0})-\Phi((x-ut)/\sqrt{A_0})}{2} \qquad t\leqq\lambda \tag{24-1}$$

$$\doteqdot\frac{q}{B_0}e^{-\frac{h+z}{B_0}}J_0\left(i\frac{2\sqrt{hz}}{B_0}\right)\frac{\Phi((y+s)/\sqrt{A_0})-\Phi((y-s)/\sqrt{A_0})}{2}$$

$$\times\frac{1}{u}\frac{\Phi((x-ut+u\lambda)/\sqrt{A_0})-\Phi((x-ut)/\sqrt{A_0})}{2} \qquad t\geqq\lambda\,, \tag{24-2}$$

where $A_0=4a\xi_0$ and $B_0=b\xi_0$.

When t and λ are not so small, both equations are reduced to the next equation :

$$C\doteqdot\frac{q}{B_0}e^{-\frac{h+z}{B_0}}J_0\left(i\frac{2\sqrt{hz}}{B_0}\right)\frac{\Phi((y+s)/\sqrt{A_0})-\Phi((y-s)/\sqrt{A_0})}{2}\frac{1}{u}\,. \tag{25}$$

If $f\geqq 3\sqrt{A_0}$, the concentration on the x-axis is given by

$$C \doteqdot \frac{q}{uB_0} e^{-\frac{h+z}{B_0}} J_0\left(i\frac{2\sqrt{hz}}{B_0}\right). \tag{26}$$

If we put $a=\alpha u$ and $b=\beta u$ (cf. Appendix), eq. (23) becomes

$$\xi_0 = \frac{x}{u}\left\{\sqrt{1+\left(\frac{1}{x}\frac{3}{4}4\alpha\right)^2}-\frac{1}{x}\frac{3}{4}4\alpha\right\}. \tag{27}$$

So we get

$$A_0 = 4\alpha\left\{\sqrt{1+\left(\frac{1}{x}\frac{3}{4}4\alpha\right)^2}-\frac{1}{x}\frac{3}{4}4\alpha\right\},$$

$$B_0 = \beta\left\{\sqrt{1+\left(\frac{1}{x}\frac{3}{4}4\alpha\right)^2}-\frac{1}{x}\frac{3}{4}4\alpha\right\},$$

and we can see that they are independent of u.

Comparison with the experiment

Though there is scarcely any report or datum of the diffusion experiment near the ground available for the criticism of the theory,

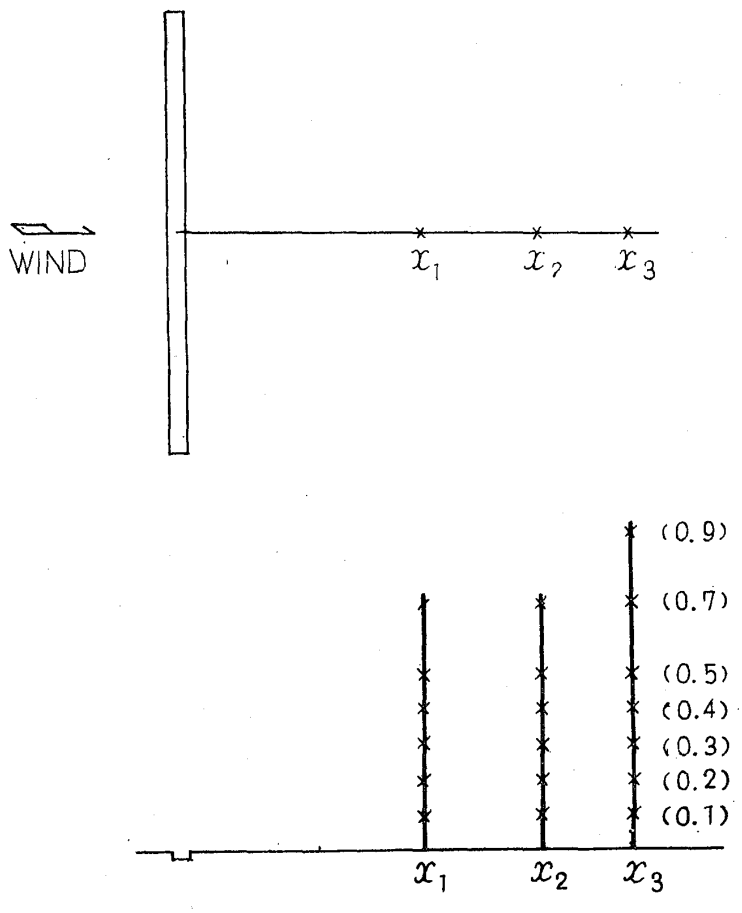

Fig. 1.

fortunately we had a chance to use the date of the thermal-diffusion experiments from a line source, which had been carried as a basic research on fire-prevention.[1]

a) *Experiment.* A narrow linear channel, 5 m in length, was dug on the beach, and firewood was burnt in it. The temperatures at every measuring point were measured by thermo-junctions. The scheme of the experiments was shown in Fig. 1 and the results are shown in Table 1. As the rate of emission of heat was not constant in each experiment and even at every instant during each experiment, the absolute

Table 1.[2]

		x \ z	0.1	0.2	0.3	0.4	0.5	0.7	0.9
A	1536 Aug. 13	3	16.1	22.5	31.6	20.8	16.0	10.0	
		4	15.8	19.7	25.2	18.2	15.4	8.6	
		5	110.5		136.6	132.3	127.6	81.4	64.4
		T_{air}	31	30	30	30	29	28	
		u	3.3		3.6		4.9	4.6	
B	1526 Aug. 14	2.5	14.0	21.7	35.0	31.9	24.2	15.7	
		4	6.6	10	14.1	16	10.1	8.5	
		5	40·8		51.1	61.6	61.2	53.4	47.0
		T_{air}	30	29	29	29	29	27	
		u	27		2.9		3.3	3.8	
C	1305 Aug. 15	2.5	10	20	29	21	—	<20	
		4	10.1	29.9	38.9	43.5	—	25.4	
		5	40.3		36.0	41.2	—	48.8	46.2
		T_{air}	30	30	29	29	29	28	
		u	1.7		1.9		1.9	2.3	2.4

values of the temperature were out of consideration, but only their spatial distributions were significant. So we plotted in Fig. 2 values of $\log\theta$ against z (m) for every x (m), where θ (°C) is the temperature difference between the measured temperature and the air one. As in the experiment the source was the woodfire, the air heated by it had an ascending tendency, so that the height of the source became effectively different from the real height. Therefore, when we analyse the data, we must choose such an effective height for each x as is consistent with the vertical distribution of the temperature.

[1] These experiments were carried on at Omaezaki, Shizuoka Prefecture, in August 1952, by S. Yokoi, T. Sekine and others, Architectural Research Institute. The report of these experiments has not been published yet.

[2] The data for $x=2.5$, 3 and 4 m were the mean values during a 10 minute interval and those for $x=5$ m were taken from the records on bromide papers. To secure the simultaneousity of the reading from the papers, we chose the peaks of the records, so the values for $x=5$ m are relatively larger than those for other x.

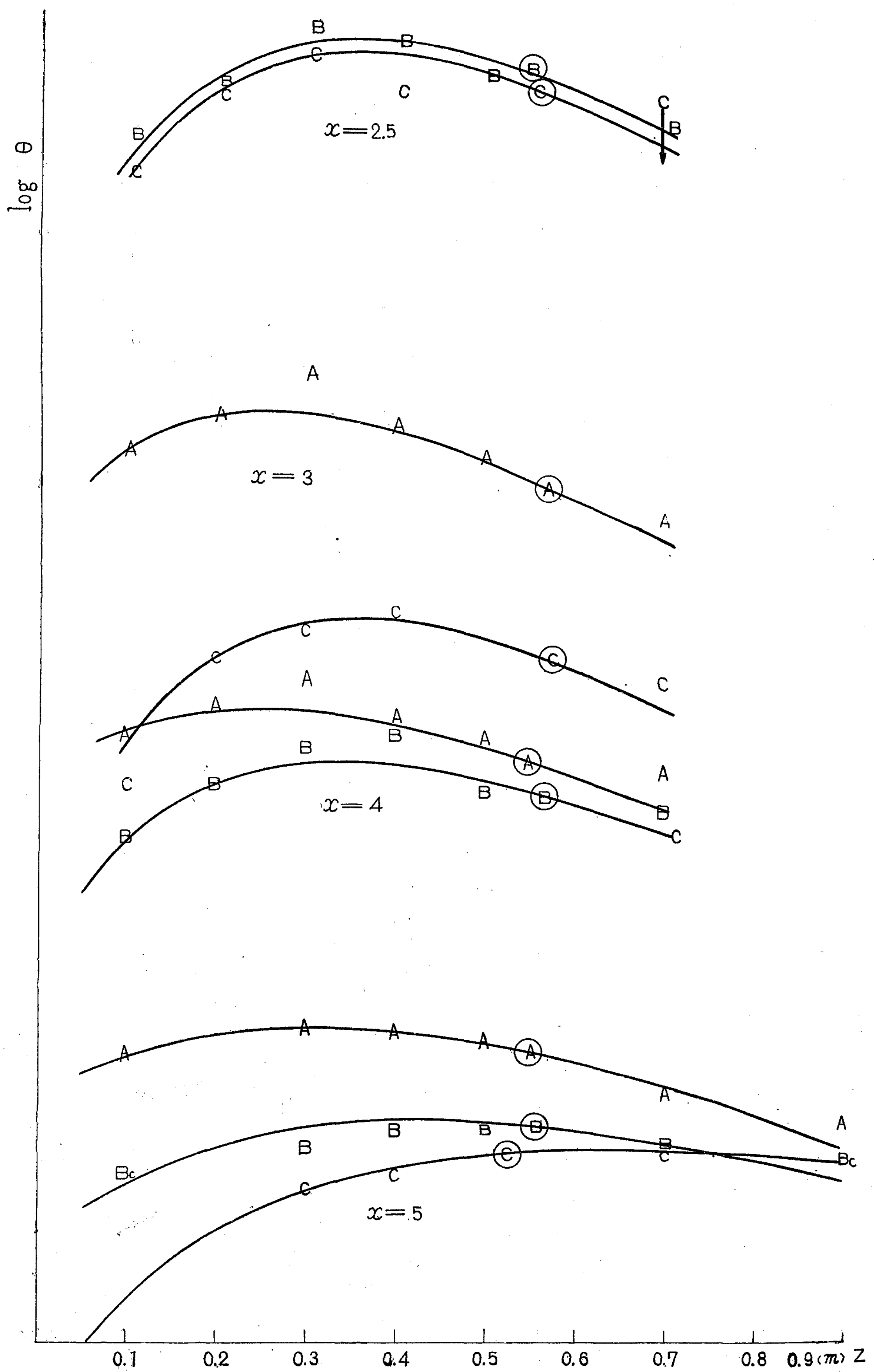

Fig. 2.

Calculating numerically eq. (26) for several sets of h, $B_0=b\xi_0$ and z, we can obtain the curves of $\log\theta$–z. Superposing the experimental curves on these calculated ones, we can examine whether or not the theory holds good, and determine probable values for h and B_0. The probable curves which were selected by this procedure are also shown in Fig. 2, and the values of h and B_0 are shown in Table 2. We can conclude that eq. (26) represents the vertical distributions of θ faithfully.

Table 2.

	x	2.5	3	4	5
	ξ	1.6	2.1	3.0	4.0
A	h		0.3	0.3	0.4
	B_0		0.1	0.1	0.15
B	h	0.4		0.4	0.5
	B_0	0.07		0.1	0.1
C	h	0.4		0.4	0.7
	B_0	0.07		0.07	0.15

To calculate the vertical diffusion coefficient $b=\beta u$, we use eq. (27) which shows the relation between ξ_0 and x. This equation contains a parameter α which is proportional to the horizontal diffusion coefficient whose value could not be determined by the above experiments. Fortunately the values of $u\xi_0$ for each x vary within only a small range even when the value of α varies to some extent about the value $4\alpha=1.5$ which has been observed by the previous experiments[1] (Table 3), so we adopt the value $4\alpha=1.5$.

Table 3.
$u\xi_0$

α \ x	2.5	3	4	5	10	20	30
1.0	1.86	2.34	3.32	4.31			
1.5	1.62	2.08	3.03	4.00	8.94	18.91	28.90
2.0	1.41	1.86	2.77	3.72			

Putting for each x the value of B_0 against ξ_0, we obtain Fig. 3, and we can estimate the value of β by the method of least squares and obtain $\beta=0.033$[2]. Owing to the small range of x, we cannot determine

1) Cf. Appendix.

2) This value is more larger than that which has been observed by the previous experiments: $\beta=0.02$. The reason may be that the source was woodfire.

β with satisfactory accuracy by the data used here.

Relation between diffusion coefficient and observation period or scale

That the diffusion coefficients have a tendency that the longer the period of the observation is, the larger their measured values are, has been pointed out theoretically and experimentally by many authors.[4] We want to consider about this fact.

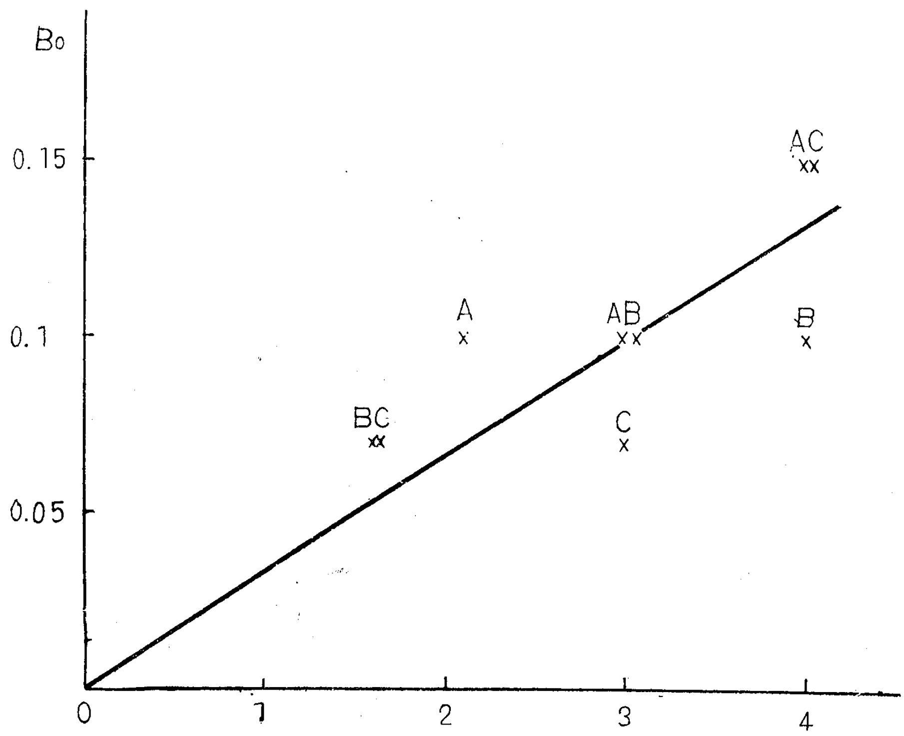

Fig. 3.

a) *Treatment of data in the Eulerian way.* The y-ward distribution of the time mean or the accumulative amount of the observed concentration emitted from a continuous point source has a tendency to become broader, namely the resulted value of K_y becomes larger as the period of the observation becomes longer. This is a result from the fact that, among the spectra of turbulence which take part in the diffusion, the components of longer periods become efficient as the period of the observation becomes longer. However, even in those cases we have experienced that the simultaneous y-ward distributions of the concentration are constant during a whole period of the observation, except that the position of mode of the distribution (namely mean wind direction) may vary at each instant. In other words, the values of K_y are constant during the observation, no matter how long its period may be.

If this displacement of the position of the mode occurs frequently in a sufficient degree and at random during the whole observation period, it is naturally an elementary constituent of the diffusion mechanism, and we must consider the diffusion coefficient including this effect. But in this circumstance the diffusion coefficient finally becomes independent of the observation period. Conversely, when we consider that the diffusion coefficient depends on the period of the observation, we do not regard the amount of the displacement of the mean wind direction as a physico-statistical quantity. On the other hand, if the diffusion coefficient ought to be physical quantity, its value must be reproducible in various cases. So the phenomena whose occurrences are not so reproducible, such as the slow displacement of the mean wind direction during the observation period, should not be regarded as primary elements of the diffusion phenomena, and they should be taken into consideration as the corrective effects.

b) *Treatment of data in the Lagrangean way.* When we observe the simultaneous y-ward distributions of the concentrations in order to avoid the additional effect described above, and calculate the value of K_y for every x, we obtain the relation between K_y and x.

In his famous paper, Richardson adopted the standard deviation of the positions of particles from their mean as the "scale", l, in the case of the diffusion of the particles.[5] In the case of a continuous point source, y-ward distribution of the concentrations is expressed by e^{-y^2/A_0} so the diffusion coefficient is measured by $a=\dfrac{A_0u}{4x}$, and the standard deviation is given by $A_0^{1/2}/2^{1/2}$. When x is not so close to the source A_0 is approximately proportional to x (cf. Table 3), so l becomes to proportional to $x^{3/4}$. According Richardson's results, a is proportional to $l^{4/3}$, so we get

$$a\propto(x^{1/2})^{4/3}=x^{2/3}\,.$$

So when we write K_{50} and K_{200} for K at the distances $x=50$ m and $x=200$ m respectively, we get

$$\frac{K_{200}}{K_{50}}=\left(\frac{200}{50}\right)^{2/3}=2.5\,.$$

This ratio is too large to be regarded as appropriate (cf. Appendix).

c) In the diffusion phenomena which appear most frequently in the practical problems and occur within the period of several minutes and in the region within several meters high from the ground and several hundred meters long in x and y directions, in such phenomena it seems to be adequate that the diffusion coefficients are assumed to be independent of time and space, and that the displacement of the mean wind direction is not included in the diffusion mechanism, but is treated correctively as the change of position with time of the field in which the diffusion takes place.

Conclusion

1) Eq. (18) and the formulae which can be derived from it, seem to represent the concentration distribution sufficiently, at least in the first approximation.

2) The data of the diffusion phenomena should be treated with care, and it is important to distinguish the essential factors and the corrective ones.

Future plannings

It can not be expected to perform a detailed diffusion experiment even on a several deca-meter scale. So we are now carrying on an experiment of the thermal diffusion on a several cm scale by the schlieren method, and intend to examine a) the relation between the diffusion coefficient and the meteorological data, and b) the relation between K_y and K_z near the ground.

The author wishes to express his sincere thanks to Mr. S. Yokoi for his good-will to allow to use his valuable data of the thermal diffusion, and to Miss M. Matsuda for her various assistances to this research.

Literature

(1) Taylor, G. I: Proc. London Math. Soc. **20** (1921) 196.
Sutton, O. G: Proc. Roy. Soc. (A) **135** (1932) 143.
Batchelor, G. K: Austr. J. Sci. Res. (A) **2** (1949) 437. Q. J. Roy. Met. Soc. **76** (1950) 133.
Inoue, E: J. Met. Soc. Japan **28** (1950) 441, **29** (1951) 32.
Ogura, Y: Geophysical Notes, Geophys. Inst., Tokyo Univ., Japan **3** No. 26 (1950) 1. J. Met. Soc. Japan **30** (1952) (1) 23, (2) 21, **32** (1) (1954) 22.
Inoue, E., Frenkiel, F. N., Lettau, H., Lowry, P. H., Record, F. A: Internal Symposium on Atmospheric Turbulence in the Boundary Layer, Geoghy. Res. Papers. No. 19 M. I. T. (1952) 397.
(2) Fage, A., Falkner, V. M: Proc. Roy. Soc. (A) **135** (1926) 702.
(3) Prandtl, L: Z. angew. Math. Mech. **5** (1925) 136. Proc. II Int. Congr. App. Mech. Zürich (1926) 62.
Nikuradse, J: V. D. I. Forschungsheft Nr. 356 (1932)
(4) Richardson, L. F: Proc. Roy. Soc. (A) **110** (1926) 709.
Sutton, O. G: Proc. Roy. Soc. (A) **135** (1932) 143.
Yamamoto, G., Shiotani, M: J. Met. Soc. Japan **25** (1947) 120.
Inoue, E: J. Met. Soc. Japan **30** (1952) 255.
(5) Richardson, L. F: ditto.

Appendix

From 1935 to the end of the World War II, the author engaged in measuring and analysing the diffusion of the gaseous matters of various kinds of sources, within the region ranging from several meters to several kilometers in the horizontal direction and 5 meters high from the ground. On the day of the end of the War, all reports and data were completely burnt during the author's absence, so there is no numerical datum which is available at present. Nevertheless it may have some meanings to describe the results which remain in his memorandum. But they have lost their scientific values, so we want to summarize them in Appendix.

1) The diffusion coefficients a and b were proportional to u, and the values of 4α and β were 1.5 and 0.02 respectively.

2) Within the range of $x=1000$ ca m (surely 400 m), α and β were constant. At least, such a large difference as $K_{200}/K_{50}=2.5$ could not be experienced.

3) In the case of the instantaneous 3-dimensional sources[1] on the ground, the vertical distributions of the initial concentrations in the sources were in the form of $e^{-z/\nu}$[2] and those of the concentrations at leeward points were always in the form of $e^{-z/\mu}$. If we consider z-direction only, we get from eq. (9)

$$C\propto\int_0^\infty \frac{1}{B}e^{-\frac{\gamma}{\nu}}e^{-\frac{\gamma+z}{B}}J_0\left(i\frac{2\sqrt{\gamma z}}{B}\right)d\gamma=\frac{\nu}{\nu+B}e^{-\frac{z}{\nu+B}},$$

and this was another verification of the theory.

4) We sprinkled a certain amount of volatile liquid in a short-cropped area of 2 meter-squares, and measured at its center the vertical distribution of the concentration of $z=0.1$, 0.2, 0.3, 0.4 and 0.5 m in every hour throughout a whole day. The results were always in the form of $e^{-\omega z}$, and the values of ω were almost always the same, in spite of various conditions of wind (0.5∼3 m/sec.) and temperature (inversion or lapse).

(Received June 17, 1954)

1) A cylindrical vessel (180 l ca) made of tin plate containing volatile liquid was exploded by an explosive.

2) The distributions of the initial concentration in the source were measured by properly protected wash-bottles and the test papers arranged 3-dimensionally near the source.