外　国　語　要　旨

Finsler/Kawaguchi Lagrangian formulation and its application to field theories and string theory

Ryoko Yahagi

The action of a relativistic free particle and the Nambu-Goto action represent a length of the world-line and an area of the world-sheet, respectively. These are geometrical objects, which are defined regardless of the choice of parameters. Inherently, physical laws are parametrization invariant as well. However, the standard Lagrangian formulation is not constructed so that general actions become geometrical objects. In this thesis, a novel geometrical formalism for Lagrangian formulation by using Finsler/Kawaguchi geometry, which is a generalization of Riemannian geometry, is established. Its applications to field theories, general relativity, and string theory are demonstrated.

The definition of Finsler geometry and its relation to dynamical system is given in chapter 2, and it is extended to Kawaguchi geometry and field theory in chapter 3. The formalism is constructed by connecting Lagrangians and Finsler/Kawaguchi metrics. Furthermore, new definitions of Killing vector field and Lie derivative on Finsler/Kawaguchi manifold are proposed. Noether's theorem is expressed by these definitions. A feature specific to this formulation is the indifferent nature of dynamical variables and time parameter for dynamical systems, or fields and spacetime parameters for field theories. It means that the conservation law of energy/energy-momentum, which is associated with time/spacetime variation, appears as one of the Euler-Lagrange equations along with equations of motion. The following examples are displayed: potential system, scalar field, Dirac field, electromagnetic field, and Maxwell-Dirac field. The last example has U(1) symmetry and it is reconstructed on the Kawaguchi manifold. It is also discussed how to treat the gauge symmetry in the Kawaguchi Lagrangian context. The important point in this formalism is that there is no distinction between internal and external symmetries, and both of them are derived from Lie derivatives on the Kawaguchi manifold.

In chapter 4, the field theory which includes second-order derivatives such as gravitational theory and its relation to Kawaguchi geometry are examined. Unlike usual field theories, there is no conclusive definition of the energy-momentum current of gravity. However, as mentioned above, the conservation law of energy-momentum current comes up as an Euler-Lagrange equation in the Kawaguchi Lagrangian formalism. It is clear that this current represents the conserved quantity for the translation invariance since it is derived from the variation of spacetime parameters. In the definition of the current, there is an ambiguity in the choice of exact terms and one of them is chosen

in this thesis, which turns into gauge invariant form of the current. Therefore, it can be said that the current selected is the most probable candidate of the energy-momentum of gravity. As an example, the current is calculated in the Schwarzschild spacetime.

In chapter 5, application to string theory is discussed. The fact that spacetime parameters and fields are on the same level and that the formulation has the reparametrization invariance generates more symmetries than the standard Lagrangian formulation. Consequently, there is a chance to find a new duality between certain two theories. Firstly, two known dualities, string-scalar duality and membrane-scalar duality on 4-dimentional spacetime, are checked at action level. A new theory that might be dual to the superstring theory is also suggested by exchanging a fermionic field for a Grassmann-valued coordinate function.

In this thesis, Lagrangian formulation is reconstructed in a geometrical manner and it is explained that there are some application to non-trivial problems such as the energy-momentum of gravity or duality. Since it contains various symmetry transformations which do not exist in the standard formalism, more analytical progress is expected.