外 国 語 要 旨

Title: Hydrodynamics on non-commutative space

Name: Mayumi Saitou

In particle physics, string and brane theories are strong candidates of unified theories, including gravity, since they have no divergences. In some cases, Nambu brackets emerge in an action function of a D-brane. A Nambu bracket is a generalized Poisson bracket for a triplet of canonical variables as a formal generalization of a canonical pair. D-branes are extended objects in string theory, and their trajectory consists of "world volume" in phase space. So, it's natural to express the action in terms of Nambu brackets. Although a number of studies are carried out so far on Nambu bracket, it is still difficult to quantize the dynamics of a three-dimensional phase space theory.

Recently, Y. Nambu formulated a new hydrodynamics in which incompressible fluid dynamics is connected to Hamiltonian dynamics in terms of area preserving diffeomorphism. If the equations of motion of two-dimensional fluid are expressed using Poisson brackets, then it is allowed to understand a stream function plays a role of Hamiltonian. According to this standpoint, three-dimensional incompressible fluid theory can be related to the dynamics of Nambu brackets.

In this paper, we investigate a hydrodynamics on non-commutative space based on the formulation of dynamics by Nambu. Replacing Poisson or Nambu brackets by Moyal brackets with a parameter θ, a new hydrodynamics on non-commutative space is derived. It has an additional term of $O(\theta^2)$, which does not exist in the usual Navier-Stokes equation.

In hydrodynamics, to introduce Moyal brackets corresponds to a kind of quantization procedure regarding position coordinates x and y, so that it makes the position coordinates to non-commutative ones. This procedure may be a step toward finding the hydrodynamics of granular materials whose minimum size is given by θ. In order to examine the non-commutative effect, I compare the behaver of flows which have different size of θ by computer simulation. In all the discussions in this paper, incompressibility and non-relativistic flow are supposed.

Chapter 1 reviews the definition of the Nambu bracket and the new description of hydrodynamics by Nambu.

In chapter 2 we introduce the non-commutativity using the Star (Moyal) product to Nambu's hydrodynamics. Ultimately, we derive the additional terms to the Navier-Stokes equation caused by the non-commutativity. In two-dimensional hydrodynamics the Moyal bracket is a

usual one, but in three-dimensional case we have to consider a generalized Moyal bracket and its product.

Chapter 3 discusses the Star product, and compare it with the path integral quantization through a calculation of expectation value to clarify the uncertainty relation. We find that to take the Star product is related to the path integral quantization in a toy model.

Chapter 4 leads the uncertainty relation when the Nambu dynamics is quantized. Using the action of the Nambu dynamics given by Takhtajan and Nambu, we explicitly demonstrate the 3D case in terms of the closed string theory.

Chapter 5 shows two analytic results about the equation of motion derived in Chapter 2. First, to search the tendencies of the additional terms, we plot the force-field caused by the additional terms under characteristic flows such as radiated one, rotated one and one around a board, which are described by the simple stream functions. The next analysis is to compute time evolution of flow. In this calculation, the boundary condition has two slits which is the big one and the small one and jam the flow.　This aim is to elucidate the difference when the parameter θ is changed against the width between the slits. According to our simulation, the bigger θ is, the less amount of flow from the narrow slit.

The last chapter summarize what we did.