Summary of the thesis

Analysis of quantum walks by the generating functions

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Recently, limit theorems have been derived analytically, and the asymptotic behavior has been clarified for several kinds of quantum walks(QWs). There are two distinct types of QWs, one is the continuous-time walk, and the other is the discrete-time one. In this thesis, we treat the discrete-time walk in one dimension. The QWs can be defined as a quantum analogue of the classical random walks. As the classical random walks have been utilized to analyze many kinds of transport phenomena such as diffusion phenomena, the QWs have been expected to play important roles in the world of quantum scale. Some of the 1-dimensional discrete-time QWs have a characteristic property, that is, there is a coexistence of localized behavior and the ballistic spreading where the probability distribution shapes like an inverted bell. The property is characterized by two measures: the time-averaged limit measure describing localization, and the weak limit measure describing the ballistic spreading.

These days, it has been reported that we can construct several measures from the QWs in one dimension, such as the stationary measure [5]. As Konno et al. [5] suggested, the time-averaged limit and stationary measures are closely related to each other for some one-dimensional QWs with one defect in one dimension. We focus on the mathematical expressions of localization here.

Actually, it is often rather difficult to analyze QWs analytically even for the simplest one-dimensional models. In fact, as for one-dimension, rigorous results for localization have been obtained only for the following special cases: Space-homogeneous QWs, 3-state Grover Walk, and the QWs with one defect. Thus, making rigorous asymptotic analyses on the QWs other than the three types is of a great importance in the theoretical study of QWs.

We studied the two topics as follows. We should remark that "the time-space generating function method" gives the time-averaged limit measure, and "the splitted generating function method(the SGF method)" gives the stationary measure, respectively. Up to now, it has not been clarified the types of the QWs that are appropriate for the two methods.

(1) The limit theorems of the Wojcik model

We have investigated "the Wojcik model" introduced and studied by Wojcik *et al.* [4], which is a two-state QW in one dimensional, having a single phase at the origin. They solved the eigenvalue problem for each two-step by the recurrence formula, and reported that giving a phase at one point causes an astonishing effect for localization.

As for the Wojcik model, we obtained the time-averaged limit measure using the time-space generating function method [2]. Then, we solved the eigenvalue problem for each time step, taking advantage of the SGF method. The analysis is more efficient than that of Ref. [4]. From the solution of the problem, we derived a stationary measure [1]. Owing to these two approaches, we obtained the mathematical expressions for localization of the Wojcik model, such as the time-averaged and stationary measures.

Furthermore, we also got the time-averaged limit measure at the origin by the pass counting and CGMV methods. The results have agreed with that by the time-space generating function method, which suggests that the Wojcik model is appropriate for the time-space generating function method.

(2) Limit theorems of the two-phase QW

We have studied a position-dependent QW on the line which we assign two different quantum coins to positive and negative parts respectively. We call the model "the two-phase QW".

First, we obtained the time-averaged limit measure [3]. Then, we solved the eigenvalue problem, and derived a stationary measure of the two-phase QW [3]. The analytical methods are mainly based on the time-space generating function and the SGF methods, respectively. We also investigated the relation between the two measures. Furthermore, we simulated the probability distribution and the time-averaged probability.

Our studies will contribute to construct the analytical platform and have deep understanding from the viewpoint of the gauging theory for the space- inhomogeneous QWs. In addition, it can be hoped that the exact results of the QWs will be applied to quantum computer and the analyses of quantum systems in the future.

【References】

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