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Enumeration of the spatial 2-bouquet graphs up to flat vertex isotopy
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A spatial graph is a graph embedded in $\mathbb{R}^3$, and spatial graph theory has been considered not so much a part of graph theory as an extension of knot theory. In knot theory, classifying all knots and links is a basic theme and over six billion knots and links have been tabulated. There also in spatial graph theory exist earlier studies on classification of some spatial graphs. J. Simon enumerated θ-curves with up to five crossings and K_4-graphs with up to four crossings. R. Litherland provided a table of prime θ-curves with up to seven crossings without proof, and H. Moriuchi gave it a proof. Moriuchi also enumerated all the prime handcuff graphs with up to seven crossings. In these studies, spatial graphs are classified up to ambient isotopy as well as in knot theory. On the other hand, spatial graphs can be also classified up to flat vertex isotopy.

We shall recall a flat vertex isotopy. A spatial graph $\tilde{G}$ embedded in $\mathbb{R}^3$ is called a *flat vertex spatial graph* if for each vertex v of $\tilde{G}$ there exists a small neighborhood B_v of v and a plane P_v in $\mathbb{R}^3$ such that $\tilde{G} \cap B_v \subset P_v$. Two flat vertex spatial graphs $\tilde{G_1}$ and $\tilde{G_2}$ are *flat vertex isotopic* if there exists an ambient isotopy between them leaving the image of $\tilde{G_1}$ to be a flat vertex spatial graph at any level of the isotopy. The ambient isotopy in this case is called a *flat vertex isotopy.* It is easy to see that there is no difference between the classification up to flat vertex isotopy and the one up to ambient isotopy for spatial trivalent graphs. (Note that θ-curves, K_4-graphs and handcuff graphs are all trivalent.) The simplest graph which may cause the difference between these two classifications is a *2-bouquet graph*, namely a connected graph which has exactly one 4-valent vertex and no other vertices.

The aim of this paper is to enumerate all the prime flat vertex spatial 2-bouquet graphs with up to six crossings: We construct a diagram of a flat vertex spatial 2-bouquet graph from a diagram of a 2-string tangle by connecting each end of a 4-valent flat vertex to that of the 2-string tangle diagram without increasing the number of crossings. It turns out that an arbitrary flat vertex spatial 2-bouquet graph can be obtained by this construction. We call a spatial 2-bouquet graph *prime* if an arbitrary 2-sphere which intersects the graph at two points divides it into a trivial arc and something. Because this definition is similar to that of a prime 2-string tangle, we can see the following facts: A 2-bouquet constructed from a prime 2-string tangle is prime, and a 2-bouquet constructed from a non-prime 2-string tangle is not prime. Therefore it is enough to prepare all prime 2-string tangles in order to obtain all prime flat vertex spatial 2-bouquet graphs. H. Yamano gave a method of constructing prime 2-string tangles from algebraic tangles, and Moriuchi made tables of algebraic tangles with seven crossings or less. From these results, we obtain all the prime 2-string tangles with up to six crossings, and construct all the prime flat vertex spatial 2-bouquet graphs with up to six crossings. We next distinguish them by using Yamada polynomial, which is a one-variable Laurent polynomial associated to each spatial graph diagram.

Our main result is

Main Theorem. There exist exactly 51 flat vertex isotopy classes of the prime spatial 2-bouquet graphs with up to six crossings.

It follows easily from this theorem that the difference between the classification up to flat vertex isotopy and the one up to ambient isotopy actually occurs with spatial 2-bouquet graphs.