6 Central part Lock Acquisition

Central part lock Double modulation and demodulation is applied for central part locking. This double modulation and demodulation merits is that there is no disturbance of central part locking even if the carrier is resonant at the arms. It is because the displacement information is earned by the beat signals [figure30].

Problem There are carrier and two upper and lower sidebands. When carrier is resonance in the arm cavities, if the end mirror were roaming, a sideband could have the possibility to disturb the central part lock by resonating at the cavity.

Idea for solution Therefore as the alternative idea for holding the lock of central part, here is one solution. The feeding back signal from RSE port to the end test mass make the Fabry Perot cavity length antiresonant with sidebands.

For realization of central part lock acquisition, the strategy is as follow.

1. End test mass constrain with locked central part
2. End test mass constrain with unlocked central part
3. Development of lock acquisition from constrained end test mass

As the first step, in this section, testing 1.the end test mass constraint with locked central part.

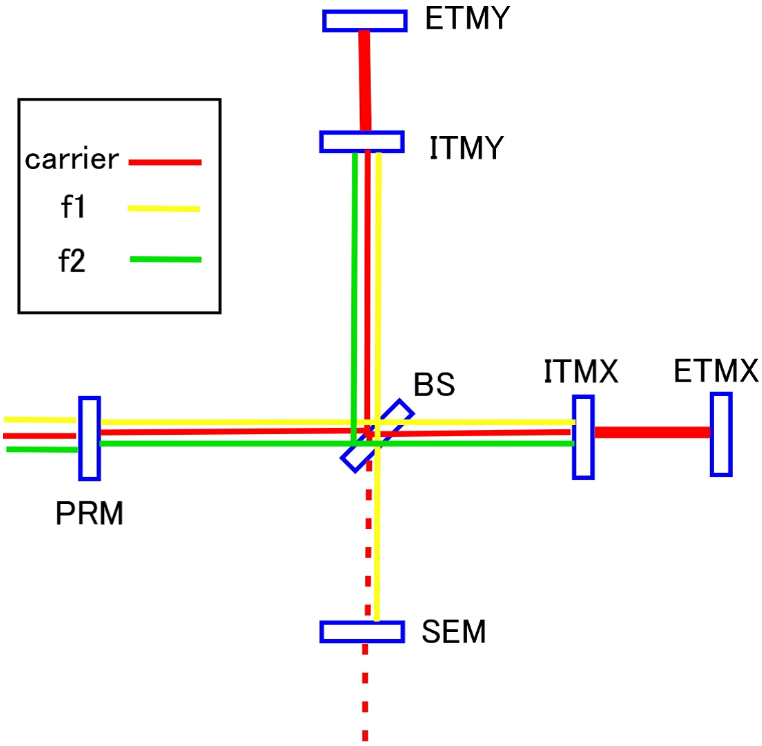

Figure 29: Resonant conditions

6.1 Double modulation

KAGRA use double modulation, PM-PM. Therefore RSE signal are gained from PRM Pick port, Reflection port of PRM and SRM port. At those ports, there are DC signal and signals that demodulated by I phase and Q phase. Those signals contain the cavity length in some proportional rate.

Modulation Injected lights E_{inc} is represented as

$$\begin{aligned} E_{inc} &= E_1 e^{1\Omega t} e^{im_1 \cos\omega_1 t}(1 + m_2 \cos\omega_2 t) \\ &= E_1 e^{1\Omega t}[\sum_{n_1=-\infty}^{\infty} i^{n_1} J_{n_1}(m_1) e^{in_1\omega_1 t}] \left(1 + \frac{m_2}{2} e^{i\omega_2 t} + \frac{m_2}{2} e^{-i\omega_2 t}\right) \\ E_{inc} &= E_1 i^{|n_1|} J_{|n_1|}(m_1) \left(\frac{m_2}{2}\right)^{|n_2|} \end{aligned} \quad (48)$$

$E_{inc,--}$	$E_{inc,-0}$	$E_{inc,-+}$
$iJ_1(m_1)\frac{m_2}{2}$	$iJ_1(m_1)$	$iJ_1(m_1)\frac{m_2}{2}$
$E_{inc,0-}$	$E_{inc,00}$	$E_{inc,0+}$
$J_0(m_1)\frac{m_2}{2}$	$J_0(m_1)$	$J_0(m_1)\frac{m_2}{2}$
$E_{inc,+-}$	$E_{inc,+0}$	$E_{inc,++}$
$iJ_0(m_1)\frac{m_2}{2}$	$iJ_1(m_1)$	$iJ_0(m_1)\frac{m_2}{2}$

Table 5: Coefficients of each light

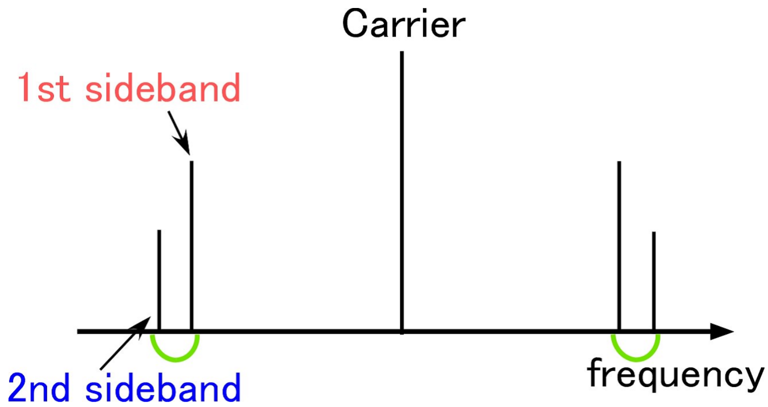

Figure 30: Double modulation

$J(m)$ is the Bessel function. After the double modulation, PMω_1, AMω_2, the absolute amplitude of each light are

$$\begin{pmatrix} ia_{++} & ia_{+0} & ia_{++} \\ a_{0+} & a_{00} & a_{0+} \\ ia_{++} & ia_{+0} & ia_{++} \end{pmatrix}$$

a_{00}, a_{+0}, a_{0+}, a_{++} are absolute amplitude of carrier, ω_1 sideband, ω_2 sideband and sidebands sideband. In this case, relation $a_{00}a_{++} = a_{0+}a_{+0}$ is realized.

Demodulation By the approximation calculation using up to 1st Bessel function, the electric field is

$$E_{in}^{+} + E_{in}^{-} = E_1 e^{-\Omega t}\left(1 + im\sin\omega_m t\right) \tag{49}$$

In RSE control with double modulation, there are single demodulation and double demodulation. In addition, there are two type of modulation phase, in phase and quadrature phase.

single demodulation

$$\begin{aligned} V_\omega^{(I)} &= \left(\tilde{I}_\omega e^{i\omega t} + \tilde{I}_{-\omega} e^{-i\omega t}\right) V_0 \cos i\omega t \\ &= \left(\tilde{I}_\omega e^{i\omega t} + \tilde{I}*_\omega e^{-i\omega t}\right) (e^{i\omega t} + e^{-i\omega t}) \frac{V_0}{2} \\ &= \left(\tilde{I}_\omega + \tilde{I}*_\omega\right) \frac{V_0}{2} + O(e^{i2\omega t}) \\ \propto Re(\tilde{I}_\omega) \end{aligned} \tag{50}$$

$$\begin{aligned} V_\omega^{(Q)} &= \left(\tilde{I}_\omega e^{i\omega t} + \tilde{I}_{-\omega} e^{-i\omega t}\right) V_0 \sin i\omega t \\ &= \left(\tilde{I}_\omega e^{i\omega t} + \tilde{I}*_\omega e^{-i\omega t}\right) (e^{i\omega t} - e^{-i\omega t}) \frac{V_0}{2i} \\ &= \left(-\tilde{I}_\omega + \tilde{I}*_\omega\right) \frac{V_0}{2i} + O(e^{i2\omega t}) \\ \propto Im(\tilde{I}_\omega) \end{aligned} \tag{51}$$

Parameter	Value
L_+	4000
L_-	0
l_+	57.676
l_-	0.025
l_s	56.028
Modulation 1	9.1MHz
Modulation 2	45.5MHz

Table 6: Parameters for constraint end test mass

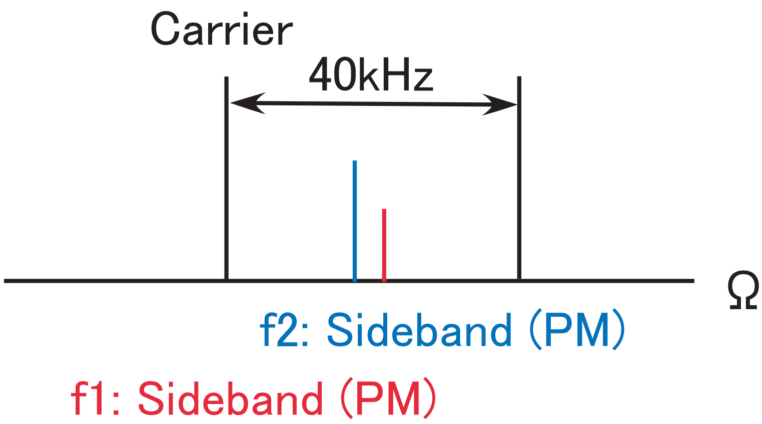

Figure 31: Sideband condition

6.2 Constraint of end test mass

The final goal is to constrain both X (inline), Y (perpendicular) end mirrors of FP cavity with central part lock holding. Free spectral range is estimated by

$$\frac{c}{2L} \propto 40\text{kHz} \tag{52}$$

On the other hand, both modulation 1, 2 are on the mid of this spectral range. When using 1064nm light, the sideband would resonate at Fabry Perot cavity by $\Delta = 0.25\mu m$. Therefore here is the process of end test mass constraint within this Δ.

1. Constraint of end test mass X
2. Constraint of end test mass Y
3. Constraint of end test masses X, Y

This test being complex, therefore firstly constrain end test mass X, secondly end test mass Y and finally constrain both end mirrors.

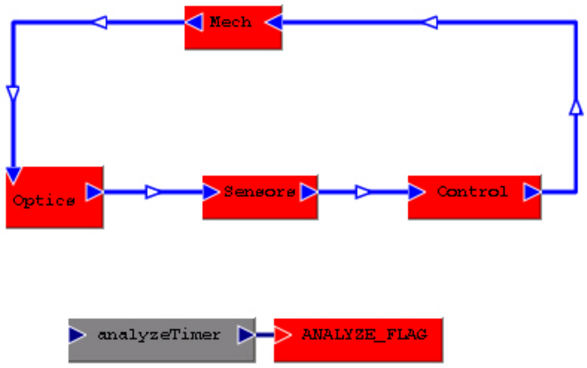

Figure 32: Model on e2e for constraint

6.3 Simulation model for constraint

The simulation model for constrain is composed of Mech.box, Optics.box, Sensor.box and Control.box.

Mech.box Mechanic calculation. Input is force, output is position.

Optics.box RSE configuration. Calculate the signals at each port. Input is position and output is field at every port.

Sensor.box Demodulation the all signals by I phase and Q phase or acquiring DC signal. Input is field and output is signal.

Control.box Calculation actuation force from signal. Servo default setting is that UGF is 300Hz. Input is signal and output is force

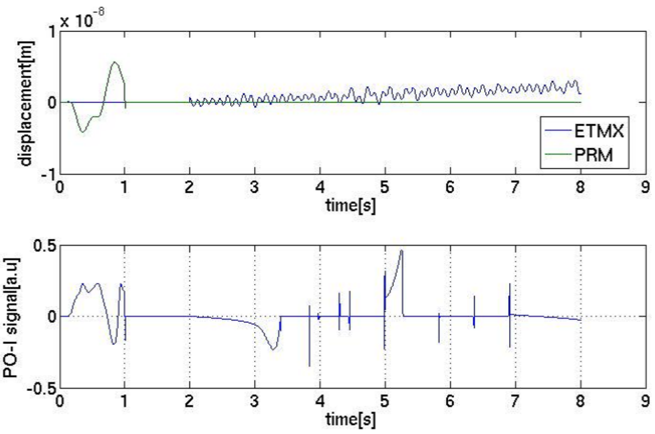

Figure 33: (Above) Locked (t>1) PRM and moving (1e-7m/s, t>2) ETMX, (Below) PO-I signal for constraint

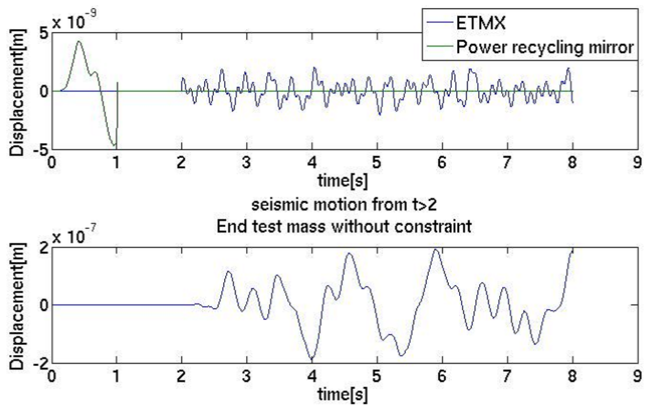

Figure 34: (Above) Locked (t>1) PRM and moving (Hanford×10, t>2) ETMX, (Below) Displacement of end test mass without constraint

ETMX \ PRM	REFL-I	REFL-Q	AS-I	AS-Q	PO-I	PO-Q
REFL-I					○	○
REFL-Q			○	○		
AS-I				○		
AS-Q			○			
PO-I			○	○		
PO-Q			○	○	○	

Table 7: Combinations of signals for end test mass constraint and central part lock

6.4 End test mass X constraint

A step for constraint end test mass X is follow.

1. Make PRM (or SEM, BS) moving randomly, then at 1 second feed back moderate signal to the PRM (or SEM, BS) to central part lock.
2. Make end test mass X moving (1e-7m/s or randomly) to see the central part unlock by resonating sideband at FP cavity.
3. Constrain the end test mass X by a signal not to resonate sideband at FP cavity.

Constraint with shaking Power Recycling Mirror (PRM) The results are figure33. At this time, the PO-I signal become gradually larger according to the end test mass move by 1×10^{-7} m/s. Using this signal to the end test mass, achieve the constraint of this.

Constraint and holding lock with shaking PRM could be achieved by some other signals. The signals combination is above table.

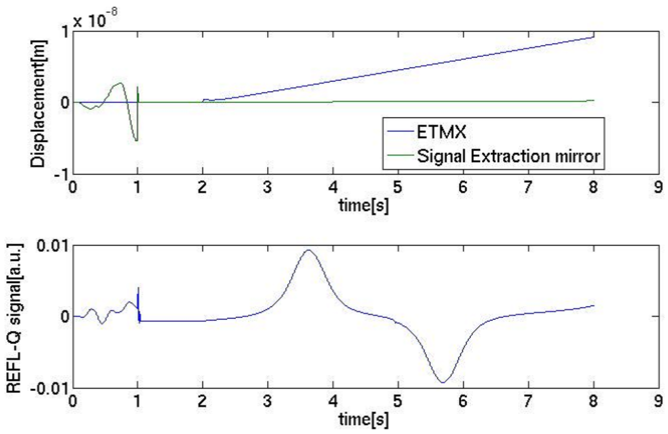

Figure 35: Locked Signal Extraction Mirror with constrained ETMX: (Above) Locked (t>1) SEM and moving (1e-7m/s, t>2) ETMX, (Below) Locked (t>1) SEM and moving (Hanford×10, t>2) ETMX

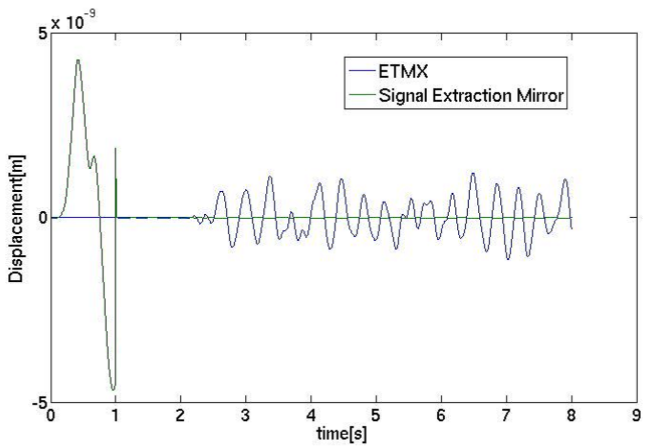

Figure 36: Position of locked Signal Extraction Mirror and constrained ETMX

ETMX \ SEM	REFL-I	REFL-Q	AS-I	AS-Q	PO-I	PO-Q
REFL-I					○	○
REFL-Q						
AS-I						
AS-Q						
PO-I						○
PO-Q						

Table 8: Combinations of signals for end test mass constraint and central part lock

Constrain with shaking Signal Extraction Mirror (SEM) Holding lock with shaking SEM could be achieved by REFL-I signal. Using this signal, when there are disturbance of 10 times of Hanford site, it's achieved to constrain and hold lock.

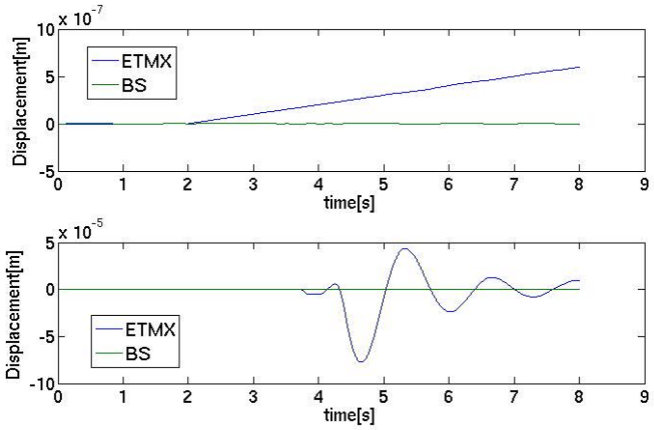

Figure 37: Locked Beam Splitter with constrained ETMX: (Above) Locked (t>1) BS and moving (1e-7m/s, t>2) ETMX, (Below) Locked (t>1) SEM and moving (Hanford×10, t>2) ETMX

BS \ ETMX	REFL-I	REFL-Q	AS-I	AS-Q	PO-I	PO-Q
REFL-I						
REFL-Q						
AS-I						○
AS-Q						
PO-I						
PO-Q	○					○

Table 9: Combinations of signals for end test mass constraint and central part lock

Constrain with shaking Beam splitter (BS) For beam splitter it seems not to need any constraint force.

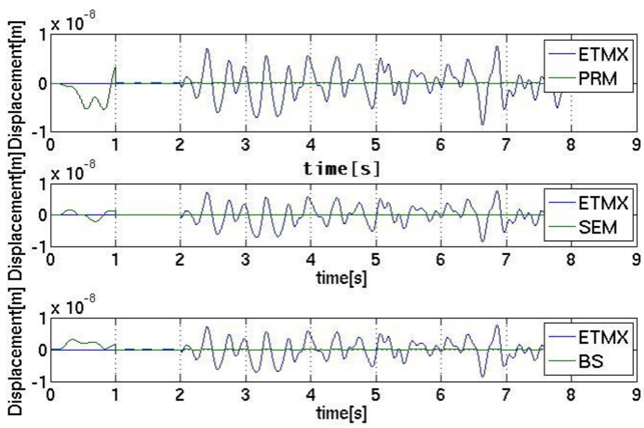

Figure 38: Locked PRM, SEM, and BS with constrained ETMX

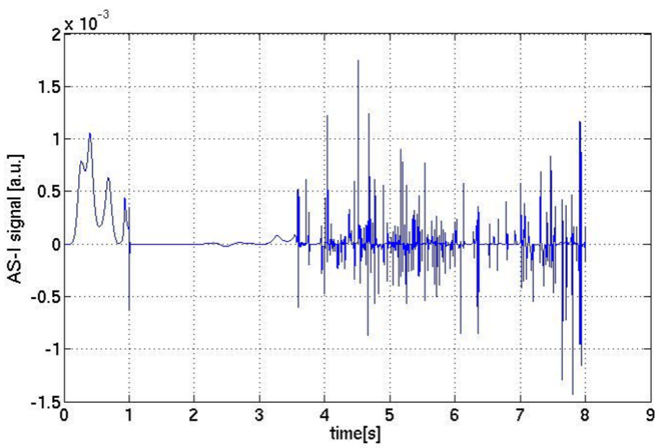

Figure 39: Signal for constraint ETMX

Constraint with shaking PRM, SEM and BS Using the signals for central part lock and for constraint end test mass, tried to lock the free central part (Hanford×1, t>1) and to constrain end test mass (t>2). The signals for PRM, SEM, BS and ETMY are REFL-I, PPick-I, REFL-Q and AS-I.

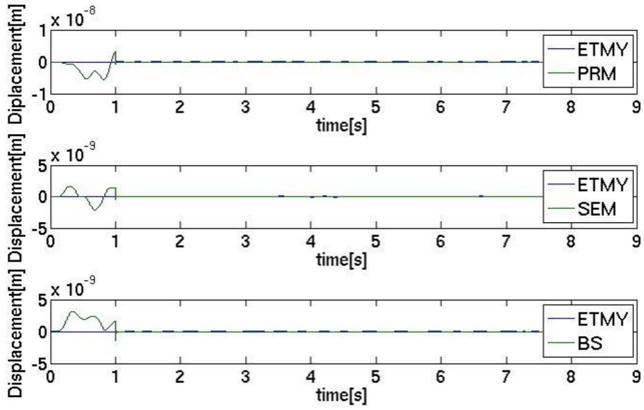

Figure 40: Locked PRM, SEM, and BS with constrained ETMY

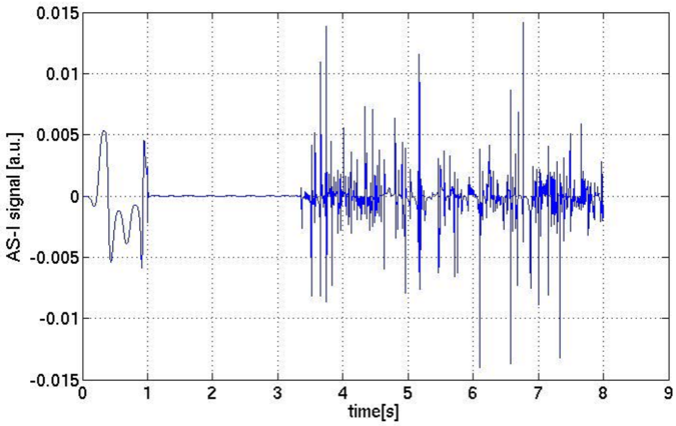

Figure 41: Signal for constraint ETMY

6.5 End test mass Y constraint

See if it is able to realize in the perpendicular (Y) cavity. It is also achieved to constrain end test mass Y (ETMY) and hold central part locked. The signals for PRM, SEM, BS and ETMY are REFL-I, PPick-I, REFL-Q and AS-I.

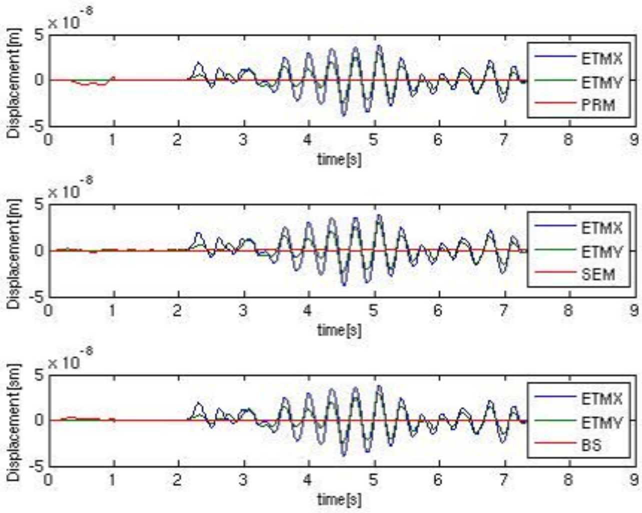

Figure 42: Constraint of ETMX and ETMY

6.6 End test mass X, Y constraint

The signals for end test mass is right inverted sing. Therefore combination of the signals for constraint both ETMX and ETMY, we could use those signals as common and differential length control. Signals from RSE are allotted to End Test Mass X and Y, PRM, SEM and BS. When PRM, SEM, and BS moving with Hanford $\times$ $1(t > 0)$, controlled by some signals and ETMX and Y with Hanford $\times$ $30(t > 2)$, feeding back moderate signals to ETMX and ETMY, it is attested to constrain them and central part hold locked.

6.7 Result of constraint end test mass

The step for constraint end test mass and holding lock of central part is follow.

1. Constraint of end test mass X Initial state is that
   (a) Power Recycling Mirror is moving.
   (b) Signal Extraction Mirror is moving.
   (c) Beam Splitter is moving.
   (d) PRM, SEM and BS are moving separately.
2. Constraint of end test mass Y
3. Constraint of end test mass X and Y

From given initial state, by combination of 9 signals, we could see that

- Lock the Central part.
- The End Test Mass disturbance make the central part unlocked.
- The End Test Mass could be constrained and central part held Locked.

All this process was succeeded in, and first step for RSE lock acquisition has been achieved.