Chapter 1

Introduction

1.1 Background

It is a fundamental problem in quantum statistical physics to clarify how the dynamics of quantum systems are affected by dissipation mechanisms from a microscopic point of view [1]. This problem is important in the fields of quantum optics [2, 3, 4, 5], spin relaxation [6, 7, 8] , condensed matter [9, 10, 11] and chemical physics [12, 13]. Recent developments in technology enable us to treat real quantum mechanical systems with dissipation effects.

Dissipation effects are inevitable because these quantum systems are not closed, but are in fact open systems. They are open to the surrounding environment which has an infinitely large number of degrees of freedom. Thus, energy transferred from the system to the environment can never be transferred back again, that is, energy is dissipated from the quantum system. Dissipation also causes fluctuations in the system.

There have been a large number of theoretical approaches to this problem. One phenomenological method introduces a fluctuating force which is described by classical stochastic processes [14]. This formalism has had great success in the field of spin relaxation and condensed matter, where fluctuations are considered to be rather strong. However the classical stochastic method is not suitable for treating low temperatures where the dissipative system must be treated fully quantum mechanically.

An approach dividing the whole system into the system and the reservoir is the most successful method for describing the quantum dissipative system from the microscopic point of view [1]. In this approach, the quantum dissipative system is considered to be coupled weakly with a reservoir consisting of many degrees of freedom. Let us first describe the whole system, the system plus reservoir, quantum mechanically. Then we eliminate the reservoir variables from the equation of motion of the whole system by tracing over the reservoir variables. We thus obtain equations of motion for the variables of the quantum system under the effect of coupling with reservoir.

There are several ways to treat the dynamics of quantum dissipative systems. We describe three of the main methods: the Langevin equation approach [3], the master equation approach [15], and the path integral (functional integral) approach [11]. In this thesis, we employ the master equation approach to formulate the theory. Before concentrating on our theory, let us briefly consider these three approaches.

The formulation of dissipative quantum systems in quantum optics was initiated by Senitzky [16] using the Langevin equation approach, and was developed further in connection with lasers in the 1960s. In the Langevin approach, the effect of the reservoir is introduced by noise operators which causes diffusion effects in the Heisenberg equations of motion for the operators of the quantum system. Recently, the Langevin approach has been developed by Gardiner [3, 17] using the mathematics of quantum stochastic calculus formulated by Hudson and Parthasarathy [18]. The advantage of the Langevin equation approach for quantum systems is its resemblance to the corresponding classical equations. But the Langevin equation is nonlinear in most cases and it is quite difficult to solve the operator equation.

The Schrödinger picture is an alternative to the Heisenberg description of quantum mechanics. The Schrödinger equation is an equation of motion for state vectors or wave functions. The corresponding equation for a quantum dissipative system is the master equation, the equation of motion for the density operator of the quantum system. Historically, the master equation approach originated in the field of the spin relaxation theory in the 1950s (WBR theory [19, 20]), earlier than the Langevin equation approach. In the 1960s, the master equation approach developed rapidly in the field of quantum optics, strongly connected with laser theory [15].

In many quantum optical phenomena, the time scale of the reservoir variables is much shorter than that of the quantum system variables. In this situation, the Markovian approximation is reasonable. The master equation in the Markovian approximation is much more tractable for real physical systems in quantum optics than the corresponding Langevin equation. Although it is not easy to solve the master equation directly, since the master equation is a differential equation of operators, it can be cast into a c-number differential equation. The principal method which has been used for decades is to map the master equation into a Fokker-Planck equation using the coherent states proposed by Glauber [21] and Sudarshan [22]. However, in this thesis, we do not use this mapping method with coherent states. We develop another method to solve the master equation.

In 1963, a formalism of the dissipative quantum problem using path integrals [23] was proposed by Feynman and Vernon [24]. They formally solved the equation of motion for the density operator of the quantum system by the path integral method, where the effects of the reservoir variables were introduced as influence functionals. Since the 1980s, the path integral approach has further been developed in the field of condensed matter physics to study macroscopic tunnelling problems, related to a quantum mechanical device, the SQUID (superconducting quantum interference device) [25, 26]. One of the main interests is to derive a quantum friction term which corresponds to classical friction. The advantage of this approach is that it is easy to handle non-Markovian cases. But generally, it is difficult to solve the path integral analytically beyond very simple systems with the Gaussian property. Monte-Carlo numerical simulations are usually necessary to evaluate the path integrals.

Simple quantum dissipative systems such as a damped harmonic oscillator or a dissipative two level atom have been well investigated by the approaches described above. However, real physical systems often cannot be modelled as a single dissipative system. Rather, they are considered to be coupled dissipative systems, composed of mutually interacting subsystems. For instance, in quantum optics, atoms interacting with a light field has been of central interest since the birth of the laser [15, 27]. If the interaction between the two subsystems is weak, it may be a good approximation to treat the composite quantum dissipative system as the combination of independently dissipative systems. This approach for the coupled dissipative system has been successfully applied to conventional problems in quantum optics. When the constituent subsystems interact with each other weakly, the above mentioned approach is qualitatively justified [28] and indeed, most of real existing systems were of this type.

In quantum optics, however, owing to recent developments in technological artifice, it is becoming possible to prepare strongly coupled quantum systems like cavity QED systems [29], micromasers [30], and laser cooling systems [31]. One of the aims of these researches is to exploit the quantum nature of these systems as quantum devices. In most cases, real physical systems are open systems, and thus, dissipation effects are inevitable. The central problem is to find how the dissipation destroys the quantum nature and to control the dissipative effects. Therefore, the study of strongly coupled quantum dissipative systems is now quite important.

The conventional treatment, which is appropriate for weakly coupled quantum dissipative systems, fails for strongly coupled systems. Insufficiency is revealed clearly when we examine the equilibrium state of a strongly coupled system [1, 32]. With the conventional treatment, the strongly coupled system relaxes independently to the product states of each constituent subsystem at thermal equilibrium, instead of relaxing to the correct canonical distribution of the coupled system. This kind of treatment cannot give the correct equilibrium distribution of the coupled quantum system because the introduced relaxation mechanisms have no information on the coupling between the subsystems.

In this thesis, we formulate a new relaxation theory for strongly coupled quantum systems which relax to the correct thermal equilibrium using the master equation approach. We study strongly coupled quantum dissipative systems by examining their relaxation dynamics toward the correct thermal equilibrium from off-equilibrium initial conditions.

1.2 Organization of the thesis

From chapter 2 to chapter 4, we formulate the relaxation theory for the dissipative Jaynes-Cummings model. In chapter 2, we investigate the case with a special dissipation mechanism. With this dissipation mechanism, the system relaxes to the thermal equilibrium of each subspace. In chapter 3, we generalize the relaxation model obtained in chapter 2 and formulate a relaxation theory which describes the strongly coupled systems evolving in time to the correct thermal equilibrium state. We investigate the relaxation dynamics of the diagonal elements in the boson quantum numbers. In chapter 4, we study another relaxation model, the strongly coupled spin $\frac{1}{2}$ system. In chapter 5, we give a short conclusion.

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