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Numerical simulation of flow between two rotating coaxial circular cylinders having different temperature

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Abstract

In order to investigate the meandering mechanism of the westerly wind, a flow between two coaxial cylinders with temperature difference is performed. When the radius and the height of the fluid in coaxial cylinder are fixed, the parameters that govern the flow are the Reynolds number and the buoyancy parameter. Therefore, the difference between the flow patterns can be investigated by systematically changing these two parameters. In the simulation, the nonlinear terms in the basic equations are approximated by the third order upwind difference in order to perform stable calculations even for high Reynolds number flows. As a result, steady Couette flow, wavy steady flow with wave number of 2 to 5 and unsteady flow with the wave number which change in time appear as flow patterns. These patterns are summarized by Reynolds number and buoyancy parameter in a table.

1. Introduction

In recent years, abnormal weather such as torrential downpour or hot summer frequently occurs. One of these causes is deviation from the average of the atmospheric pressure arrangement due to meandering of the westerly wind. As a meandering model of the westerly wind, experiments have been conducted to give a temperature difference to a rotating coaxial cylinder.

A famous study of flow instability in this geometry is an experiment by Taylor[1]. In the study, Taylor investigated the change in the flow pattern by filling the gap between the rotating two coaxial cylinders with a fluid, giving a rotational speed difference between the inner and outer cylinders. As a result, for example, when the outer cylinder is stationary and the inner cylinder rotation speed is increased, the flow pattern changes through a Couette flow, a rolled vortex (Taylor vortex) flow, a wavy Taylor vortex flow, and finally turbulence. Coles[2] performed in comprehensive detailed experiment. Among many researches by numerical simulation on this flow, it is noteworthy that Kawamura and Iwatsu[3] numerically reproduced up to the wavy Taylor vortex flow despite the work was very early.

The flow becomes even more interesting flow if temperature (buoyancy) is taken into consideration with the same geometry. In particular, by keeping the rotational angular velocity of the inner and outer cylinders the same and by adding a temperature difference to the inner and outer cylinders, the meandering mechanism of the westerly wind can be explained to some extent in laboratory experiments. Although it is ideal to examine the polar westerly wind in the spherical shell, it is impossible to realize the gravity facing the center of the sphere by ground experiments, so that it is substituted by cylinders. Regarding this, there is a detailed experiment by Uryu[4], and as the angular velocity is increased, a flow in the form of a wave meandering in the circumferential direction of the cylinder is observed, and it is reported that the number of waves increases with angular velocity.

For numerical simulation corresponding to this experiment, there is research by Ukaji et al. [5] [6] They performed both indoor experiments and numerical simulations. Although the qualitative behaviors of heat transport coincided with each other, quantitatively there was a shift in the results of experiments and numerical simulations. It is said that it originated from the fluid resistance due to equipments such as probes and the leakage of heat from the upper and lower surfaces.

Pattern change of flow by plural parameters has been investigated mainly by experiments so far. While there are differences between experiments and numerical simulations, there are few cases where flow patterns are examined by simulation for a wide range of parameters.

Therefore, in this study, numerical simulation is performed by matching geometric shapes based on experiments of Uryu. The parameter dependence of the flow pattern is investigated by systematically changing the two parameters (Reynolds number) and buoyancy parameter α (= Grashoff Number / (Reynolds number) ^{2}) which appear in the governing equation.

2. Numerical Method

As basic equations, dimensionless incompressible Navier-Stokes equations under Boussinesq approximation and heat equation are used.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{2.1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{\partial p}{\partial x}+\frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) \tag{2.2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{\partial p}{\partial y}+\frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right) \quad (2.3)$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\frac{1}{Re}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)+\alpha T \quad (2.4)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\frac{1}{RePr}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right) \quad (2.5)$$

Here, (u, v, w) is flow velocity, p is pressure, T is temperature, Re is Reynolds number, α is buoyancy parameter that is $\alpha = Gr / Re^2$ where Gr is Grashoff number. Pr is Prandtl number. Also, z is taken in the vertical direction. Thus, the equation includes two parameters Re and α.

The parameter α works only on the z component and represents the effect of buoyancy. Normally, the above equations are expressed in a cylindrical coordinate system, but in this study, a generalized coordinate system is used for future expansion.

After expressing these basic equations by generalized coordinates, they are solved numerically by using the fractional step method. For the approximation of the nonlinear term of the formulas (2.2) to (2.5), the third order accurate upstream difference is used. The other spatial differential terms in the equations including the Poisson equation for the pressure used in the fractional step method are approximated by the second order center difference. Euler explicit method is used for time integration.

The computational domain is a region surrounded by coaxial circular cylinders with radii of 1 and 2 as shown in Figure 2.1, and the height of the column is 1. This is similar to the equipment used in the experiment. This computational region is divided into grid system based on cylindrical coordinates. The number of grids is 81 in the circumferential direction and 30 in both the radial and the vertical directions. Uniform grid system is employed in each direction.

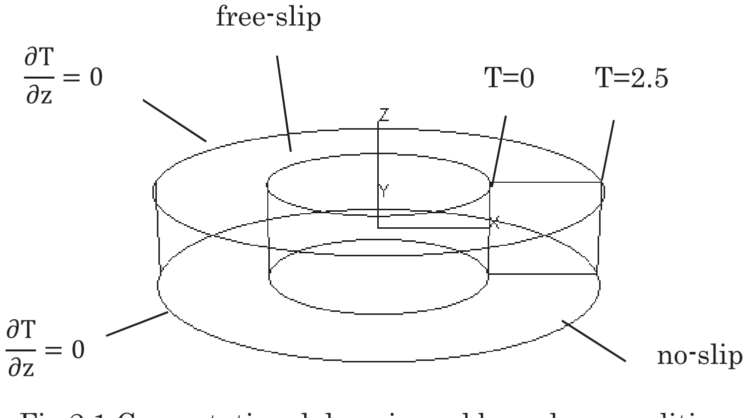


Fig.2.1 Computational domain and boundary conditions

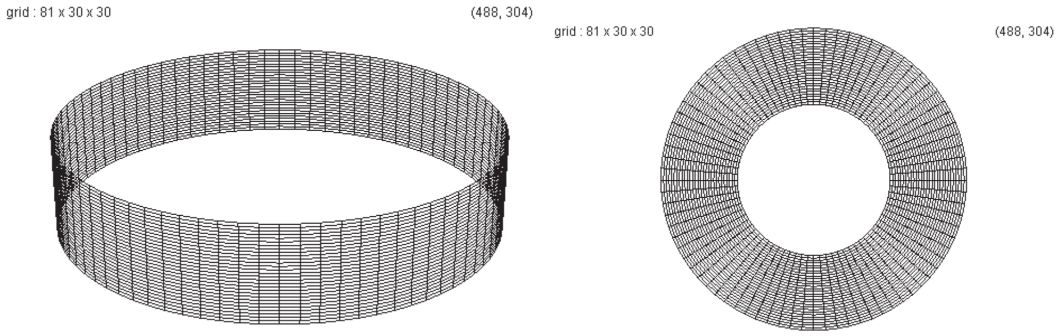


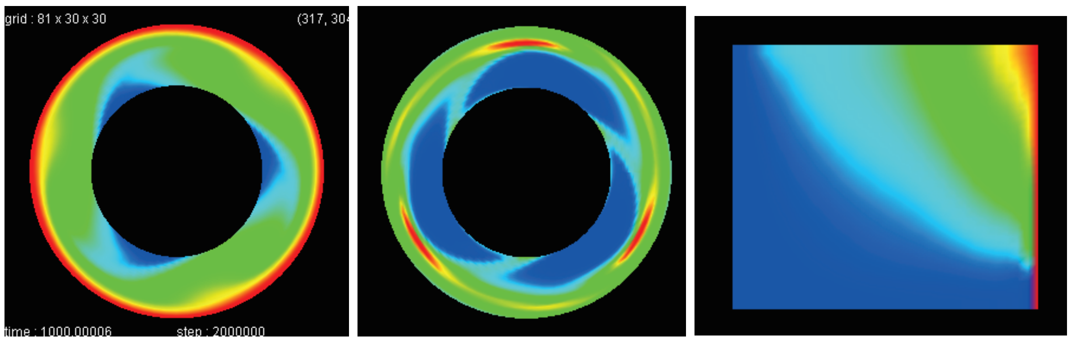
Fig. 2.2 Computational grids in a constant-radius plane and a horizontal plane

Boundary conditions are as follows: no-slip conditions on the side and bottom surfaces (the inner velocity $V_\theta = 1$, the outer velocity $V_\theta = 2$ and linearly changed in the radial direction from 1 to 2 at bottom.), and slip conditions on the top surface. The outside temperature is set to 2.5 and the inside temperature is set to 0, and the bottom and top surface are insulated. The pressure gradient on each boundary is equal to 0. The velocity field of the rigid body is given in whole region as initial condition. The initial temperature is set to the average value of the inner and outer surfaces.

Calculation is performed with Re varied in the range of 500 to 5000 and α in the range of 0.1 to 4. Δt is 1/2000, and it is done until 200,0000 steps per case. This is equivalent to rotating the aquarium 125 turns.

3. Results

Figure 3.1 shows an example of typical calculation for Re = 3000, α = 0.44.



(a) (b) (c)

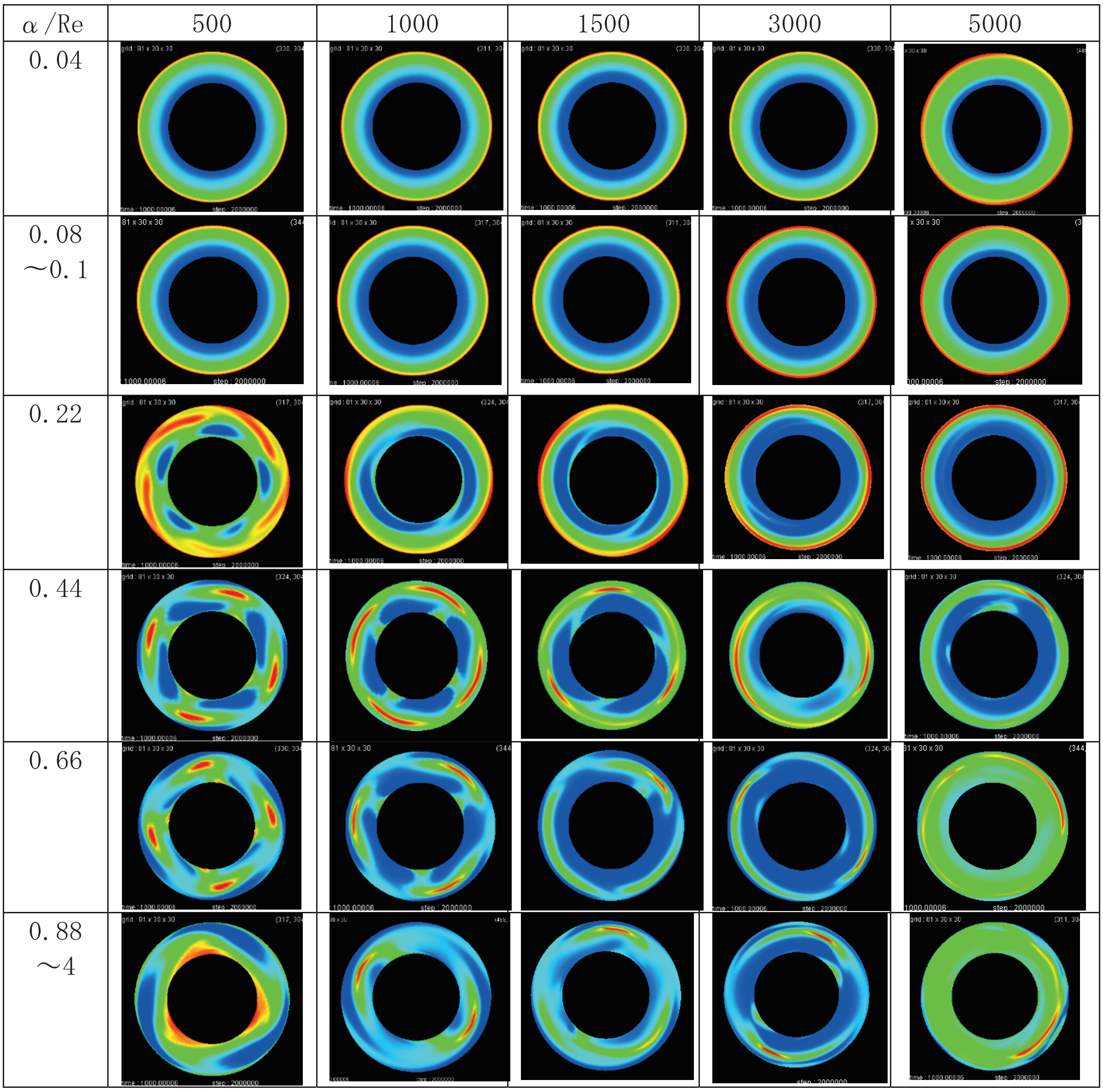
Figure 3.1

(a) Temperature distribution in a horizontal plane

(b) Distribution of vorticity of vertical direction in a horizontal plane

(c) Temperature distribution in a vertical plane

Figure. 3.1 (a) shows the temperature distribution in the horizontal cross section below two lattices from the top surface, and Figure.3.1 (b) shows the vorticity distribution of the vertical direction in the same surface. Figure.3.1 (c) shows the temperature distribution in a vertical section. In Figure.3.1 (a), a waveform close to the experiment is observed, but since the vorticity is better understood in the structure of the flow, the result of vorticity will be shown thereafter.



α: buoyancy parameter Re: Reynolds number

Figure.3.2 Distribution of vorticity of vertical direction in a horizontal plane for various α and Re

Figure 3.2 shows the vorticity distribution corresponding to Figure.3.1 (b) in the final state for various Re and α. The upper column shows the Reynolds number Re and the left column shows the buoyancy parameter α.

Focusing on buoyancy parameters, following tendency is seen.

α ≤0.1: No wave is generated ·

α ≥0.22: Waves are generated in the low Reynolds number range

Figure 3.2 is one snapshot of the flow, so after observing time development of the flows it is found out they can be classified into 5 patterns shown in Table 3.1.

	500	1000	1500	3000	5000
0.22	A	B	B	A	A’
0.44	A	A	A	A	D
0.66	A	A	C	A	D
0.88-4	A	A	A	A	D

Table 3.1 Classification of flow pattern

Here, symbols in the table indicate that they are in the following states.

S: Stable(like coquette flow)

A: Waveform stabilized within 30, 0000 to 100, 0000 steps

A ': Stability and instability are repeated.

B: Waveform of wave number 2 is seen, but after the number of steps is 60, 0000, the wave number becomes 4. After the number of steps is 70, 0000, the wave number becomes 3 and it gradually decreases to 2 until the step number 170, 0000 .

C: The waveform is not clear.

D: It is an unstable and a disturbed state.

4. Discussion

Increasing parameter α corresponds to increasing the buoyancy that causes the effect of eliminating the temperature difference becoming large. Therefore, the flow becomes unstable. On the other hand, enlarging Re corresponds to increasing the flow velocity, but as a result, the Coriolis force also increases. If it is not rotating, the flow becomes unstable when the flow velocity is large, but Coriolis force stabilizes the flow, so it is considered that increasing Re will not lead to instability directly. From Table 3.1, we can see that wave is occurring in the region where Re is small and α is larger than a certain number.

If the temperature difference is suppressed too much due to the effect of rotation, heat is accumulated in the fluid in a certain place, so that the distribution of the vorticity tends to become uniform and the effect of heat and the effect of rotation alternately appear sometimes. This is consistent to the fact that uniform distribution appears when Re $\geqq$ 5000, $\alpha \geqq$ 0.66 as shown in Table 3.1.

5. Summary and future works

In this paper, the flow in the rotating coaxial circular cylinder with the temperature difference is investigated in detail by systematically changing the two parameters governing the flow. As a result, it is possible to reproduce a steady Couette flow, a steady flow having a wave number of 2 to 5, an irregular flow, etc. numerically. These patterns are summarized by Reynolds number and buoyancy parameter in a table.

Future works are to examine the influence of geometric parameters such as ratio of radius to height and deformation of the top surface due to centrifugal force in order to examine what kind of waveform is observed. As a result, more realistic simulation can be performed, and reproduction of westerly wind will be carried out.

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